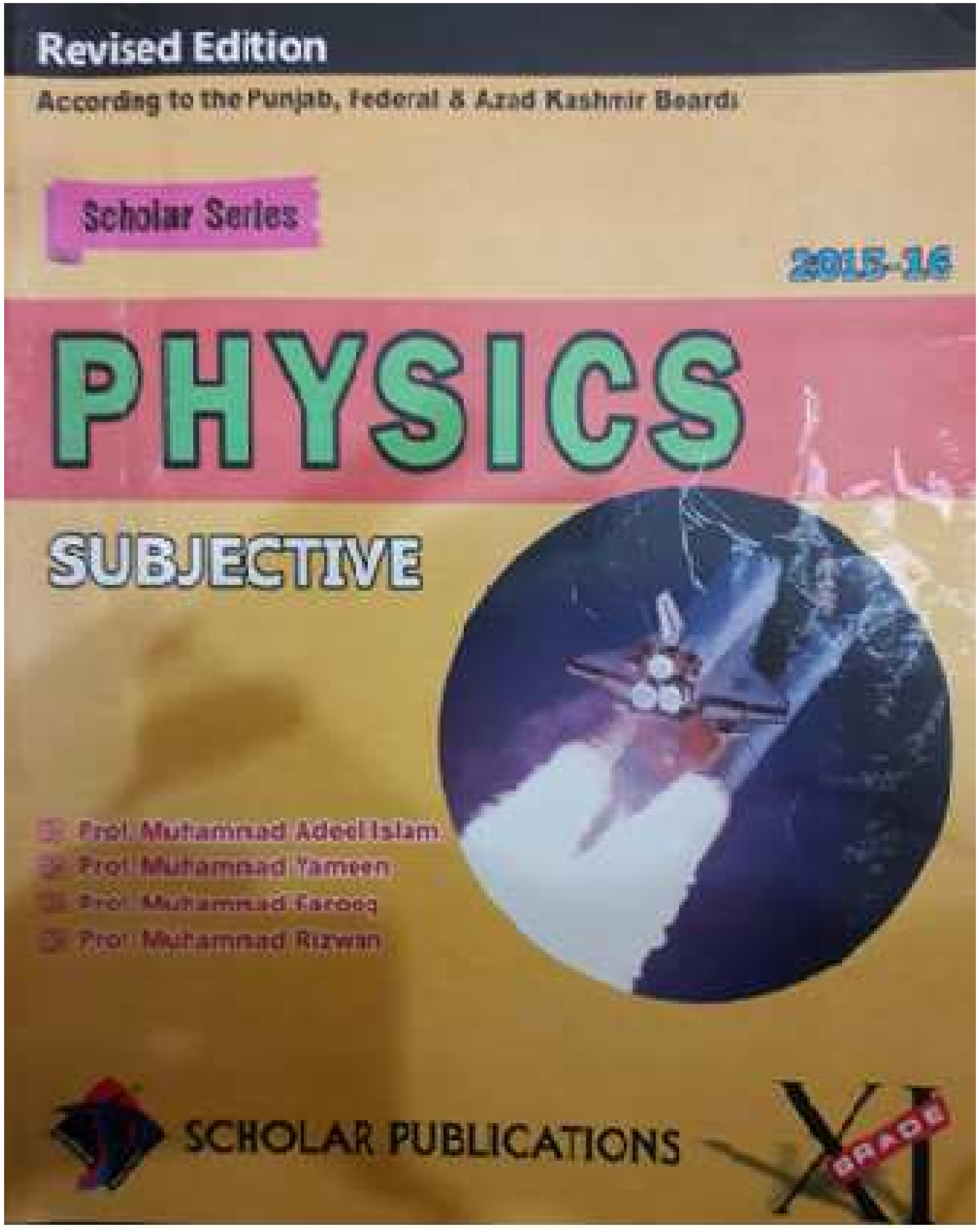


11TH HELPING BOOK PDF

Chapter 1

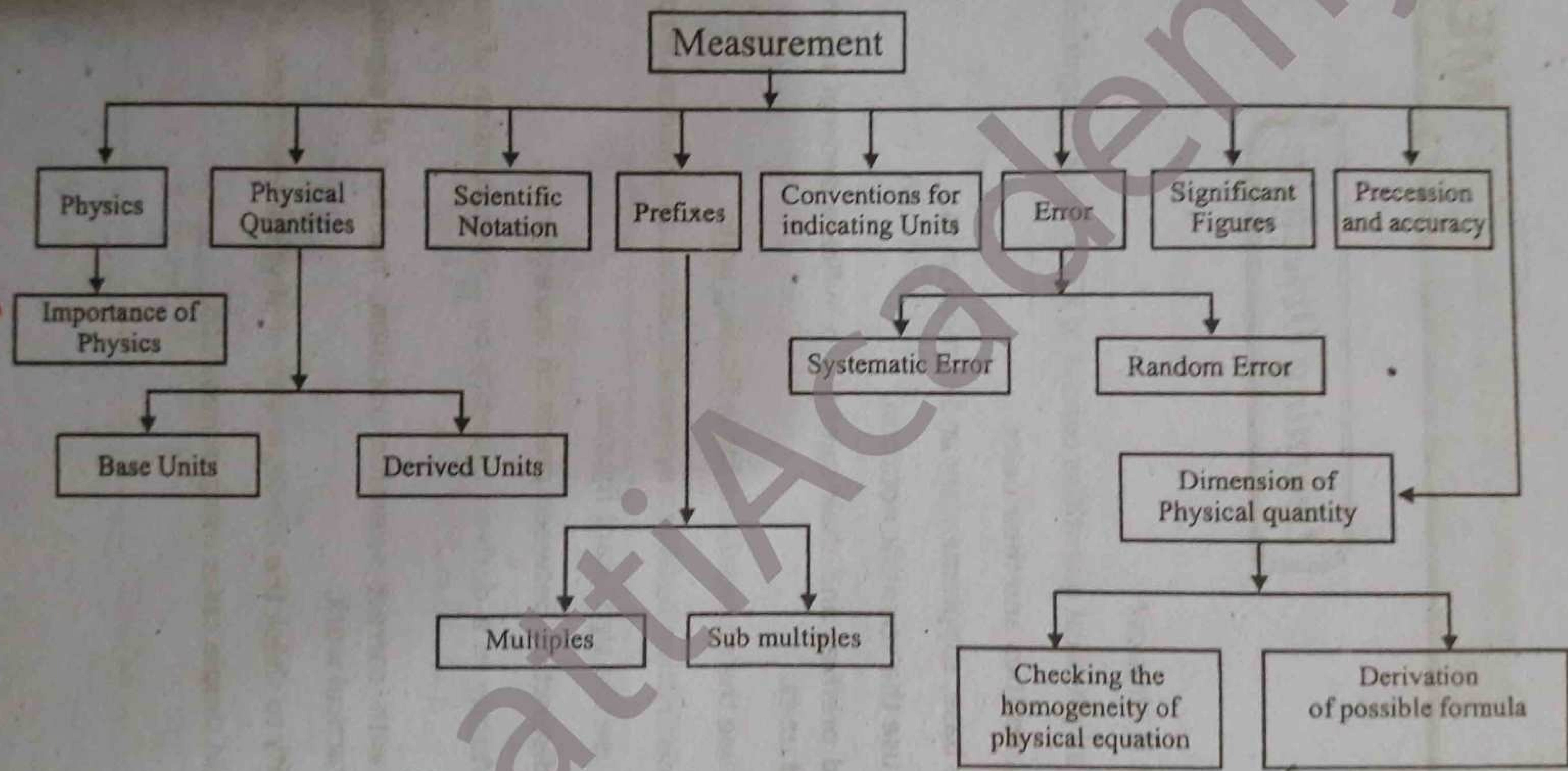
MEASUREMENTS

Learning Objectives

1. Understand what is Physics?
2. Understand that all physical quantities consist of a numerical magnitude and a unit.
3. Recall the base quantities and their units.
4. Describe and use base, supplementary and derived units.
5. Understand and use the scientific notation.
6. Use the standard prefixes and their symbols to indicate decimal sub-multiples or multiples to both base and derived units.
7. Understand and use the conventions for indicating units.
8. Understand the distinction between systematic errors and random errors.
9. Understand and use the significant figures.
10. Understand the distinction between precision and accuracy.
11. Assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties.
12. Quote answers with correct scientific notation, number of significant figures and units on all numerical and practical work.
13. Use dimensionality to check the homogeneity of physical equations.
14. Derive formulae in simple cases using dimensions.

Chapter No. 1

CONCEPT MAP



A man has always wanted to observe, think and reason about the world around him. Man tried to find ways to organize the disorder in the observed facts about the natural phenomena and material things in orderly manner which results for the single discipline of science, called ***natural philosophy***.

For Your Information
Science is the knowledge of logical reasoning.

Q.1 How can we classify the study of nature?

Ans.

Classification of the Study of Nature

The study of nature may be classified into two branches:

1. Biological Science

The science of **living** things is called Biological science.

2. Physical Science

The science of **non-living** things is called Physical science.

Q.2 What is physics?

Ans.

Physics

Physics the branch of science concerned with the **properties** of matter and energy and the **relationships** between them.

In other words, physics is basically the study of **how objects behave**.

Physics is an important and the basic part of physical science.

It is the **experimental** science.

Areas of Physics
- Mechanics
- Heat & Thermodynamics
- Electromagnetism
- Optics
- Sound
- Hydrodynamics
- General relativity
- Quantum mechanics
- Atomic physics
- Molecular physics
- Nuclear physics
- Solid–state physics
- Particle physics
- Super conductivity

Q.3 Describe the main frontiers of fundamental science.

Ans.

Main Frontiers of Fundamental Science

There are three main frontiers of fundamental science.

1) The world of **extremely large** i.e. universe
2) The world of the **extremely small** (i.e. particles such as electrons, protons, neutrons, mesons and others)
3) The world of **middle sized** things (from molecule at one extreme to the Earth at the other). It is the world of *complex matter*.

Interdisciplinary Areas of Physics
- Astrophysics
- Biophysics
- Chemical Physics
- Engineering Physics
- Geophysics
- Medical Physics
- Physical Oceanography

Q.4 Describe some new branches of physics. Also describe the role of Physics in developing technology.

Ans.

Branches of Physics

By the end of 19th century many physicists started believing that every thing about physics has been discovered. However, about the beginning of the 20th century many new experimental facts revealed that the laws formulated by the previous investigators need modification.

Do You Know?

Computer chips are made from wafers of the metalloid silicon, a semiconductor.

(1) Nuclear Physics

The branch of physics which deals with atomic ***nuclei*** is called nuclear physics.

(2) Particle Physics

The branch of physics which is concerned with the ***ultimate particles*** of which matter is composed is called particle physics.

(3) Relativistic Mechanics

The branch of physics which deals with velocities approaching that of light is called relativistic mechanics.

(4) Solid State Physics

The branch of physics which is concerned with the **structure** and **properties** of solids is called solid state physics.

Other branches of Physics

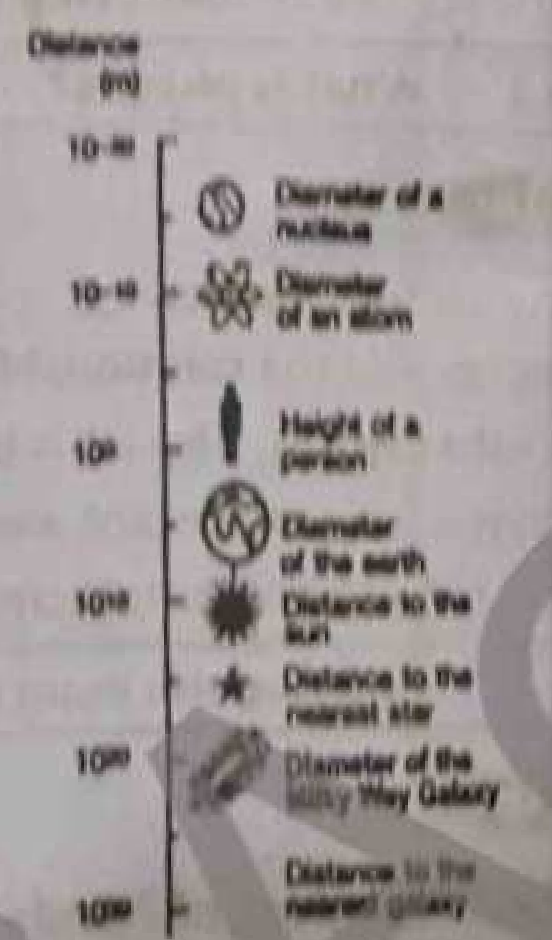
Physics is most fundamental of all sciences and provides other branches of science, basic principle and fundamental laws. This overlapping of physical and other fields gave birth to new branches

Role of Physics in Technology

- Physics also plays an important role in the development of technology and engineering.
- Science and technology are a vital force for change in the outlook of mankind.
- The information media and the fast means of communications have brought all parts of the world in close contact with one another.
- Events in one part of the world immediately reverberate round the globe.
- We are living the age of information technology.
- The computer networks are products of chips developed from the basic ideas of physics. The chips are made of **silicon**. Silicon can be obtained from **sand**.

It is up to us whether we make a sandcastle or a computer out of it.

For Your Information



Order for magnitude of same distance

<u>Astrophysics</u> The branch of physics concerned with the physical and chemical properties, **origin, and evolution of the celestial bodies.**
<u>Biophysics</u> The branch of physics which deals with the scientific study of **biological processes** in terms of the laws of physics. For example Echolocation in bats, stresses and strains in skeletal and muscular structures.
<u>Aerodynamics</u> The branch of physics which deals with the study of the movement of air and other **gases**. It includes the study of the **interactions of air with moving objects**, such as airplanes, and of the effects of moving air on stationary objects, such as buildings.
<u>Cosmology</u> The branch of physics which deals with the behavior of the material universe in its entirety. It is the one of widest subjects in the spectrum of physics.
<u>Physical chemistry</u> The branch of chemistry that is concerned with the **physical structure** of chemical compounds, the amount of energy they have, the way they **react** with other compounds, and the **bonds** that hold their atoms together
<u>Physical oceanography</u> It is the study of physical ***conditions*** and physical ***processes*** within the ocean, especially the motions and physical properties of ocean waters.
<u>Medical physics</u> *It is the application of physics to* **medicine**. It generally concerns physics as applied to **medical and radiotherapy.**
<u>Geophysics</u> *It is the physics of the Earth and its environment in space.* Its subjects include the shape of the Earth, its gravitational and magnetic fields, the dynamics of the Earth as a whole and of its component parts, the Earth's internal structure, composition and tectonics, the generation of magmas, volcanism and rock formation, the hydrological cycle including snow and ice, all aspects of the oceans, the atmosphere, ionosphere, magnetosphere and solar-terrestrial relations, and analogous problems associated with the moon and other planets.

<u>Engineering Physics (EP)</u> It is an academic degree, available mainly at the levels of B.Tech, B.Sc., M.Sc. and Ph.D. Unlike other engineering degrees (such as aerospace engineering or electrical engineering), EP does not necessarily include a particular branch of science or physics. Instead, EP provides a more thorough grounding in applied physics of any area chosen by the student (such as optics, nanotechnology, micro fabrication, mechanical engineering, electrical engineering, control theory, aerodynamics, energy or solid-state physics)
<u>Plasma</u> Plasma is a gas in which an important fraction of the atoms is *ionized*, so that the electrons and ions are separately free.
In physics and chemistry, **plasma** is a state of matter similar to gas in which a certain portion of the particles are ionized. The basic premise is that heating a gas dissociates its molecular bonds, rendering it into its constituent atoms. Further heating leads to ionization (a loss of electrons), turning it into plasma containing charged particles, positive ions and negative electrons.
The presence of a non-negligible number of charge carriers makes the plasma electrically conductive so that it responds strongly to electromagnetic fields. Plasma, therefore, has properties quite unlike those of solids, liquids, or gases and is considered a distinct state of matter. Like gas, plasma does not have a definite shape or a definite volume unless enclosed in a container; unlike gas, under the influence of a magnetic field, it may form structures such as filaments, beams and double layers. Some common plasma is stars and neon.
<u>Magneto hydrodynamics (MHD)</u> Magneto fluid dynamics or hydro magnetics is the branch of physics which studies the dynamics of electrically conducting fluids. Examples of such fluids include plasmas, liquid metals, and salt water. The word **magneto hydrodynamics (MHD) is derived from magneto- meaning magnetic field, and hydro- meaning liquid, and -dynamics meaning movement.** The idea of MHD is that magnetic fields can induce currents in a moving conductive fluid, which create forces on the fluid, and also change the magnetic field itself. It is also known as **space plasma physics**. It is the study of plasmas as they occur naturally in the universe.It is a fundamental part of the study of space weather and has important implications not only to understanding the universe, but also to practical every-day life, including the operation of communications and weather satellites. It is unique from other fields of astrophysics which study similar phenomenon, in that space physics utilizes measurements from high altitude rockets and spacecraft.
<u>Superfluidity</u> It is a state of matter in which the matter behaves like a fluid without viscosity and with infinite thermal conductivity. The substance, which looks like a liquid, will flow uncontrollably, and also will be at exactly the same temperature throughout itself. Superfluidity is the frictionless flow and other exotic behaviour observed in liquid helium at temperatures near absolute zero (−273.15 °C, or −459.67 °F), and (less widely used) similar frictionless behaviour of electrons in a superconducting solid.
<u>Superconductivity</u> It is an electrical resistance of exactly zero which occurs in certain materials below a characteristic temperature. Superconductivity is a phenomenon observed in several metals and ceramic materials. When these materials are cooled to temperatures ranging from near absolute zero (0 degrees Kelvin, -273 degrees Celsius) to liquid nitrogen temperatures (77 K, -196 C), their electrical resistance drops with a jump down to zero.
The temperature at which electrical resistance is zero is called the **critical temperature** (T_c) and varies with the individual material. For practical purposes, critical temperatures are achieved by cooling materials with either liquid helium or liquid nitrogen. Because these materials have no electrical resistance, meaning electrons can travel through them freely, they can carry large amounts of electrical current for long periods of time without losing energy as heat. Superconducting loops of wire have been shown to carry electrical currents for several years with no measurable loss. This property has implications for electrical power transmission, if transmission lines can be made of superconducting ceramics, and for electrical-storage devices.
<u>Optics</u> It is the branch of physics which involves the behavior and properties of light, including its interactions with matter and the construction of instruments that use or detect it. Optics usually describes the behavior of visible, ultraviolet, and infrared light.
<u>Hydrodynamics</u> The branch of science that deals with the dynamics of fluids, especially incompressible fluids, in motion. It is concerned with the mechanical properties of fluids. It tells that how quickly a object can travel in a fluid. **Example**: a person swimming in water.
<u>Electromagnetism</u> It is one of the four fundamental interactions in nature. The other three are the strong interaction, the weak interaction and gravitation. Electromagnetism is the force that causes the interaction between electrically charged particles; the areas in which this happens are called electromagnetic fields. Electromagnetism is responsible for practically all the phenomena encountered in daily life, with the exception of gravity. Ordinary matter takes its form as a result of intermolecular forces between individual molecules in matter. Electromagnetism is also the force which holds electrons and protons together inside atoms, which are the building blocks of molecules. This governs the processes involved in chemistry, which arise from interactions between the electrons inside and between atoms.

Q.5 What are physical quantities? Discuss its different types and way to measure the base quantities.

Ans.

Physical Quantities

All those quantities in terms of which **laws of physics** can be described are called physical quantities.

Types of Physical Quantities

Physical quantities are divided into:

(i) Base quantities (ii) Derived Quantities

Base Quantities

The minimum number of those physical quantities in terms of which other physical quantities can be defined are called base quantities.

Examples

length, mass, time etc.

Derived Quantities

Physical quantities whose definitions are based on other physical quantities are called derived quantities.

Examples

velocity, acceleration, momentum, force etc.

Measurement of Base Quantities

The measurement of base quantities involves two steps:

(i) The choice of the standard

(ii) The procedure for comparing the quantity to be measured with the Standard.

Properties of an Ideal Standard

An ideal standard has two principal characteristics;

(i) It is accessible (ii) It is invariable.

These two requirements are often incompatible and compromise has to be made between them.

Q.6 What is international system of units? Discuss the units upon which it is built up.

Ans.

International System of Units

In 1960, an international committee agreed on a set of definitions and standard to describe the physical quantities. The system that was established is called the system international (SI).

The system international is formed from three kinds of units:

(1) Base unit (2) Derived units (3) Supplementary units

(1) Base Units

There are seven base units for different physical quantities.

length, mass, time, temperature, electric current, luminous intensity, amount of substance.

No.	Physical Quantity	Symbol of quantity	SI Unit	Symbol of unit
(i)	Length	L	metre	m
(ii)	Mass	M	kilogram	kg
(iii)	Time	T	second	s
(iv)	Electric Current	I	ampere	A

For Your Information

(i) Meter The distance traveled by light in vacuum during a time of 1/299,792,458 second.

(ii) Kilogram It is defined as the mass of a Platinum (90%) and Iridium (10%) alloy cylinder, 3.9 cm in diameter and 3.9 cm in height. kept at the International Bureau of Weights and Measures in France. This mass standard was established in 1901.

(iii) Second The duration in which the outer most electron of the cesium. 133 atoms make 9,192,631,770 vibrations.

(iv) Kelvin It is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

(v) Ampere The unit of electric current is ampere. It is that constant current which if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section and placed a meter of length.

(vi) Candela The unit of luminous intensity is candela. It is defined as the luminous intensity in the perpendicular direction of a surface of 1/60000 square meter of a black body radiator at the solidification temperature of platinum under standard atmospheric pressure

(vii) Mole The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg if carbon 12. One mole of any substance contains 6.0225×10^{23} entities.

(v)	Temperature	T	kelvin	K
(vi)	Light intensity	L	candela	cd
(vii)	Amount of substance	n	mole	mol.

(2) Supplementary Units

The units which are neither base units nor derived units are called supplementary units and are oftenly used for geometrical quantities. They are:

(i) Plane angle (ii) The solid angle

Standard Definitions of Supplementary Units

(i) Radian

It is the plane angle between two radii of a circle which cut off on the circumference an arc equal in length to the radius of the circle. It is shown in figure.

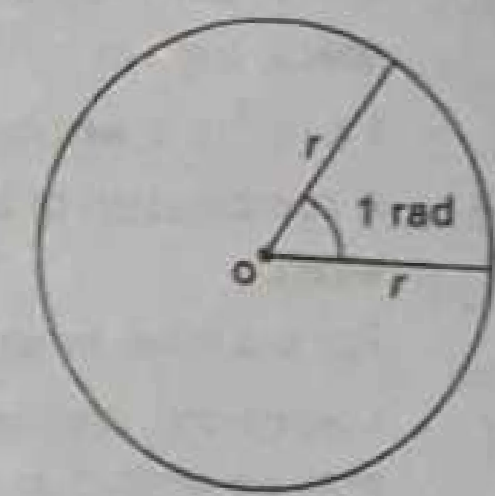


Fig 1

(ii) Steradian

It is the solid angle (three dimensional angle) subtended at the center of sphere by an area of surface equal to square the radius of the sphere. It is shown in figure.

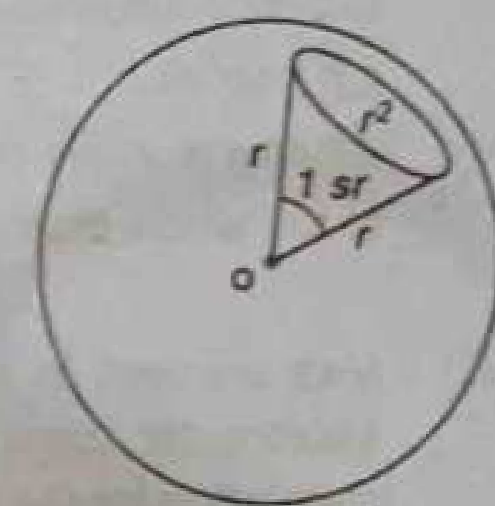


Fig 2

Physical Quantity	SI Unit	Symbol
Plane angle	Radian	rad
Solid angle	Steradian	sr

(3) Derived Units

The SI units for measuring all other physical quantities are derived from the base and supplementary units, such units are called derived units.

Physical Quantity	Unit	Symbol	In terms of base units
Force	Newton	N	$kg\,m\,s^{-2}$
Work	Joule	J	$Nm = kg\,m^{2}s^{-2}$
Power	Watt	W	$Js^{-1} = kg\,m^{2}s^{-3}$
Pressure	Pascal	Pa	$Nm^{-2} = kg\,m^{-1}s^{-2}$
Charge	Coulomb	C	As

Q.7 Write down the conventions for indicating the units. What are scientific notation?

Ans.

Conventions for Indicating the Units

Use of SI units requires special care, more particularly in writing prefixes:

(i) Full name of the unit does not being with a capital letter even if named after scientist e.g., newton.

(ii) The symbol of units after a scientist has initial capital such as N for newton.

(iii) The prefix should be written before the unit without any space, such as 1×10^{-3} m is written as 1mm.

(iv) A combination of base units is written each with one space apart.
For example, newton meter is written as Nm.

(v) Compound prefixes are not allowed. For example 1μμF may be written as 1pF.

(vi) A number such as 5.0×10^4 cm may be expressed in scientific notation as 5.0×10^2 m.

(vii) When a multiple of a base unit is raised to a power, the power applies to the whole multiple and not the base unit alone, Thus,

$$1\text{km}^2 = 1(\text{km})^2 = 1 \times 10^6 \text{ m}^2$$

(viii) Measurement in practical work should be recorded immediately in the most convenient unit. e.g., Micrometer screw gauges measurement in mm, and the mass of calorimeter in grams. But before calculation for the result, all measurements must be converted to the appropriate SI base unit.

SI Prefix	SI Symbol	FACTOR (Power)	FACTOR (Language)
exa	E	10^{18}	quintillion
peta	P	10^{15}	quadrillion
tera	T	10^{12}	trillion
giga	G	10^{9}	billion
mega	M	10^{6}	million
kilo	k	10^{3}	thousand
hecto	h	10^{2}	hundred
deca	da	10^{1}	ten
deci	d	10^{-1}	tenth
centi	c	10^{-2}	hundredth
milli	m	10^{-3}	thousandth
micro	μ	10^{-6}	millionth
nano	n	10^{-9}	billionth
pico	p	10^{-12}	trillionth
femto	f	10^{-15}	quadrillionth
atto	a	10^{-18}	quintillionth

Q.8 What is scientific notation?

Ans.

Scientific Notation

Numbers are expressed in standard form called scientific notation, which employs power of ten.

The internationally accepted practice is that there should be only one non-zero digit left of decimal. Thus the number 1347 should be written as 1.347×10^3 and .0023 should be expressed as 2.3×10^{-3}.

Q.9 What are the sources of errors in the measurement of a physical quantity? What are the types of errors? How can we reduce the error any measurement?

Ans.

Errors and Uncertainties

All physical measurements are uncertain and imprecise to some limit. There are three sources of errors

sources of errors

(i) Negligence or inexperience of a person.

(ii) faulty apparatus

(iii) Inappropriate method or technique.

For Your Information

	Interval (s)
Age of the universe	5×10^{17}
Age of the earth	1.4×10^{17}
One year	3.2×10^{7}
One day	8.6×10^{4}
Time between normal heartbeats	8×10^{-1}
Period of audible sound waves	1×10^{-3}
Period of typical radio waves	1×10^{-6}
Period of vibration of an atom in a solid	1×10^{-13}
Period of visible light waves	2×10^{-15}

Approximate Values of Some Time Intervals

The uncertainty is usually described as an error in measurement.

Types of Errors

There are two major types of errors

1. Random error
2. Systematic error

Random Error

Random error is said to take place when repeated measurements of the quantity, gives **different values** under the same conditions.

Causes

It is due to some **unknown** reasons.

Reduction of random Error

The random error can be reduced by taking several readings of same quantity and then taking their mean value.

Systematic Error

Systematic error occurs when all the measurements of particular quantity are affected equally, these give **consistent difference** in the readings.

Causes

The systematic error may occur due to

(i) Zero error in measuring instrument

(ii) Poor calibration of instrument

(iii) Incorrect calibration on the measuring instruments

Reduction of Systematic Error

Systematic error can be reduced by comparing the instrument with another instrument which is known to be more accurate. Thus, systematic error is reduced by applying a **correction factor** to all the reading taken on an instrument.

Q.10 What are scientific figures? How can we estimate the number of significant figure in the physical measurement and explain the way to rounding off data.

Ans.

Significant Figures

In any measurement, the accurately known digits and the first doubtful digit are called significant figures. OR

In other words, a significant figure is the one which is known to be reasonably reliable.

How to increase the number of significant figures

We can increase the number of significant figures in a measurement by improving the quality of our measuring instrument.

General Rules for deciding number of significant figures

(1) Digits

(i) All digits 1,2,3,4,5,6,7,8,9 are significant.

(ii) Zeros may or not be significant.

Rules for zeros

(i) A zero between two significant figures is itself significant.

(ii) Zeros to the **left** of significant figures are not significant.

For example,

None of the zeros in 0.0046 or 02.59 is significant.

(**Note**: these zeros are used only to locate decimal position)

(iii) Zeros to the **right** of significant figure may or may not be significant.

(iv) In decimal fraction, zeros to the right of significant figure are significant.

For example

All the zeros in 3.570 or 7.4000 are significant. However, in integers such as 8000 kg, the number of significant zeros is determined by the **accuracy** of the measuring instrument.

- If the measuring scale has a least count of 1 kg then there are four significant figures written in scientific notation as 8.000×10^3 kg.
- If the least count of the scale is 10 kg, then the number of significant figures will be 3 written in scientific notation as 8.00×10^3 kg.
- If the least count of the scale is 100 kg, then the number of significant figures will be 2 written in scientific notation as 8.0×10^3 kg.
- If the least count of the scale is 1000 kg, then the number of significant figures will be 1 written in scientific notation as 8×10^3 kg.

(v) When the measurement is recorded in scientific notation or standard form, the **figures other than the powers of ten** are significant figures.

For example,

A measurement recorded as 8.70×10^4 kg has three significant figures.

(2) Multiplication and division of Numbers

In multiplying or dividing number, keep a number of significant figures in the product or quotient not more than that contained in the ***least accurate factor***.

$$\frac{5.348\times10^{-2}\times3.64\times10^{4}}{1.336}=1.45768982\times10^{3}$$

As the factor 3.64×10^4, the least accurate in the above calculation had three significant figures, the answer should be written to three significant figures only.

(3) Addition or Subtraction of Numbers

In adding or subtracting number, the number of decimal places in the answer should be equal to the smallest number of decimal places in any of the quantities being added or subtracted.

In this case, the number of significant figures is not important.

It is the position of decimal that matters.

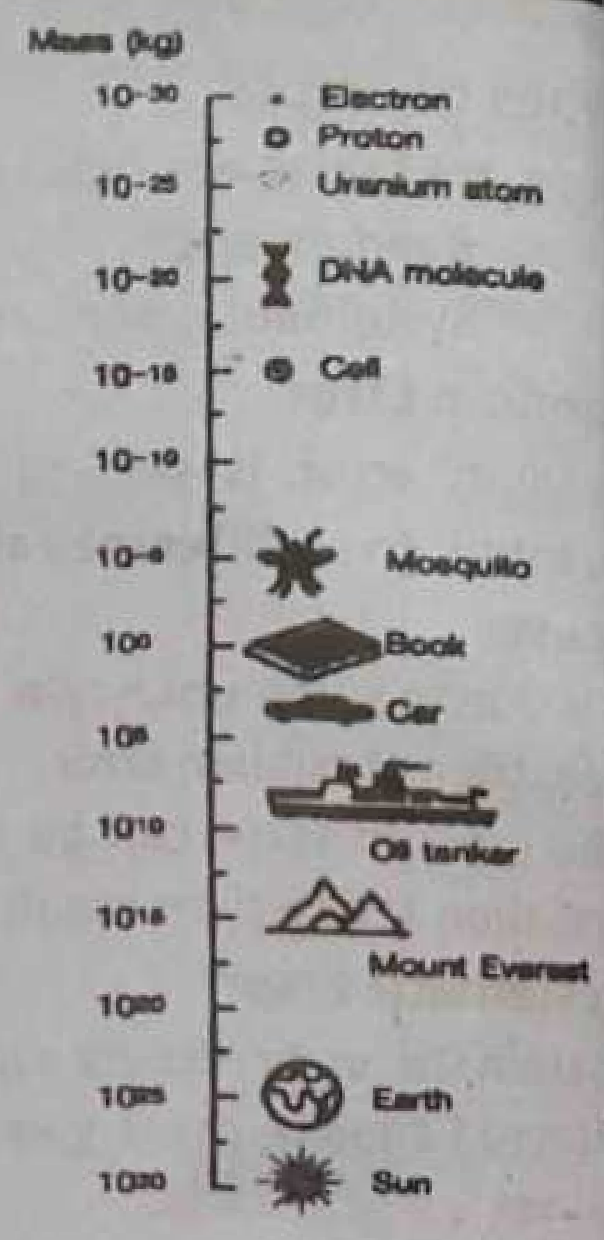
For example

Suppose we wish to add the following quantities expressed in meters.

(i)	72.1	(ii)	2.7543
	3.42		4.10
	0.003		1.273
	75.523		8.1273

Correct answer (i) 75.5m (ii) 8.13

Interesting Information



Order of magnitude of some masses.

Do You Know?

Mass can be thought of as a form of energy. In effect, the mass is highly concentrated form of energy. Einstein's famous equation, $E = mc^2$ means energy = mass × (speed of light)^2

According to this equation 1 kg mass is actually 9×10^{16} J.

EXPLANATION:

If m = 1kg, then

Energy = mc^2 = 1kg × $(3 \times 10^8\ ms^{-1})^2 = 9 \times 10^{16}$ J

In case (i) 72.1 have the smallest number of decimal places, thus the answer is rounded off to the same position which is 75.5m.

In case (ii) the number 4.10 has the smallest number of decimal places and hence, the answer is rounded off to the same decimal positions which is then 8.13 m.

Q.11 Describe the rules for rounding of data up to appropriate precision.

Ans.

Rounding Off Data

The non-significant figures should be deleted by using the following rules:

1. If the first digit dropped is less than 5, the last digit retained should remain unchanged.
2. If the first digit dropped is more than 5, the digit to be retained is increased by one.
3. If the digit to be dropped is 5, the previous digit which is to be retained is increased by one, if it is odd and retained as such if it is even. For example the following numbers are rounded off to three significant figures as follows;

43.75	is rounded off as	43.8
56.8546	is rounded off as	56.9
73.650	is rounded off as	73.6
64.350	is rounded off as	64.4

Q.12 What do understand by the terms precision and accuracy?

Ans.

Precision

Precision means how close the measured values are to each other.

A precise measurement is the one which has less **absolute uncertainty**. The precision of a measurement is determined by the **instrument** or device being used

The precision of a measurement depends upon the size of the unit you use to make a measurement. The smaller the unit, the more precise the measurement. The precision of a measurement describes the units you used to measure something.

For example, you might describe your height as 'about 6 feet'. That wouldn't be very precise. If however you said that you were '74 inches tall', that would be more precise.

Accuracy

Accuracy means how close a measured value (result) is to the actual (true) value.

The accuracy of a measurement is the difference between your measurement and the accepted correct answer. The bigger the difference, the less accurate your measurement. An accurate measurement is one which has less fractional or

For your information

We use many devices to measure physical quantities, such as length, time and temperature. They all have some limit of precision.

EXPLANATION:

Each device for measurement has some least count. The precision of the device depends on its least count, therefore, all devices have some limit of precision.

percentage error. The accuracy of a measurement depends on the fractional or percentage uncertainty in that measurement.

Example

When the object is recorded as 25.5 cm by using a meter rod having smallest division in millimeter, it is the difference of two reading of the initial and position. The uncertainty in the single reading as discussed before is taken as ±0.05 cm which is now double and called absolute uncertainty equal to ±0.1 cm. Absolute uncertainty, in effect, is equal to the least count of the measuring instrument. This is called precision:

Case (i)

Precision or absolute uncertainly (least count) = ±0.1 cm

Fractional uncertainty = $\frac{0.1\ \text{cm}}{25.5\ \text{cm}} = 0.004$

Percentage uncertainty = $\frac{0.1\ \text{cm}}{25.5\ \text{cm}} \times \frac{100}{100} = \frac{0.4}{100} = 0.4\%$

Case (ii)

Another measurement taken by vernier calipers with least count as 0.01 recoded as 0.45 cm. It has

Precision or absolute uncertainty (least count) = ±0.01 cm

Fractional uncertainty = $\frac{0.01\text{cm}}{0.45\text{cm}} = 0.02$

Percentage uncertainty = $\frac{0.1\text{cm}}{0.45\text{cm}} \times \frac{100}{100} = \frac{2.0}{100} = 2.0\%$

So the reading 25.5 cm taken by meter rule is although less precise but is more accurate having less percentage uncertainty or error.

Whereas the reading 0.45cm taken relative, measurement which important. The smaller a physical quantity, the more precise instrument should be used. Here the measurement 0.45 cm demands that a more precise instrument, such as micrometer screw gauge, with least count 0.001 cm, should have been used.

Q.13 How can you assess the total uncertainty in the final result?

Ans.

Assessment of Total Uncertainty in the Final Result

Every measurement has a degree of uncertainty associated with it.

The uncertainty derives from the measuring device and from the skill of the person doing the measuring. The total uncertainty in the final result can be found as follows

In Case of Addition and Subtraction

Absolute uncertainties are added.

For example

The distance 'x' found by the difference between two separate position measurements

For your information

These are not decoration pieces of glass but are the earliest known exquisite and sensitive thermometers, built by the Accadomia del Cinemonto (1657-1667), in Florence. They contained alcohol, some times, coloured red for easier reading.

$x_1 = 10.5 \pm 0.1$ cm

and

$x_2 = 26.8 \pm 0.1$ cm

The difference x between them is recorded as

$x = x_2 - x_1$

$= (26.8 \pm 0.1) - (10.5 \pm 0.1)$

$= 16.3 \pm 0.2$ cm

In Case of Multiplication and Division

Percentage uncertainties are added.

For example

The maximum possible uncertainty in the value of resistance R of conductor determined from the measurements of potential difference V and resulting current flow 'I' by using V= IR is found as follows;

V = 5.2 ± 0.1V

I = 0.84 ± 0.05A

The % age uncertainty for V $= \frac{0.1}{5.2} \times \frac{100}{100}$ = about 2%

The % age uncertainty for I $= \frac{0.05}{0.84} \times \frac{100}{100}$ = about 6%

Hence total uncertainty in the value of resistance R when V is divided by I is 8%.

The result is thus given as R $= \frac{5.2\text{V}}{0.84\text{A}}$ = 6.19 V/A

= 6.19 ohms with %age uncertainty of 8%

Because % uncertainty for V is 2% and for I is 6%. So,

Total uncertainty = 2% + 6% = 8%

Hence R= 6.2 ± 8% ohms

R= 6.2 ± 0.5 ohms [8% of 6.2 = 8/100 ×6.2 = 0.5]

In Case of Power Factor

Multiply the percentage uncertainty by that power.

For example

In the calculation of the volume of a sphere using $V=4/3(\pi r^3)$

% age uncertainty in V =3×% age uncertainty in radius r.

When the uncertainty is multiplied by power factor, then it increases the precision demand of measurement. If the radius of a small sphere is measured as 2.25cm by a vernier calipers with least count 0.01 cm. then the radius r is recorded as r = 2.25 ± 0.01 cm

Absolute uncertainty in r = Least count = ±0.01 cm

%age uncertainty in r $= \frac{0.01\text{cm}}{2.25\text{cm}} \times \frac{100}{100} = 0.4\%$

Total percentage uncertainty in V = 3 × 0.4 = 1.2%

Colour printing uses just four colours: Cyan, magenta, yellow and black to produce the entire range of colours. All the colours in this book have been made from just those four colours.

EXPLANATION:

Any colour can be produced by a suitable combination of red, green and blue colours. Two colours which produce white light, when mixed are termed as complementary colours. It is found that yellow and blue, or green and magenta, or red and peacock blue are complementary colours. The appearance of colours is a process of subtractive nature. Red glass absorbs all oilier colours excepting the red. Thus blue is white mixture of green and red. The colours obtained with paints and inks result from a subtractive proces.

For Your Information

ATOMIC CLOCK

The cesium atomic frequency standard at the Nations! Institute of Standard and Technology in Colorado (USA), .it Is the primary standard for the unit of time.

EXPLANATION:

To meet the need for a better standard of time, atomic clock has been developed. In this device, the frequencies associated with certain atomic transitions can be measured precisely. In 1967, the SI unit of time, the SECOND, was redefined by 13th General Conference on Weights and Measures.

using characteristic frequency of cesium-133 atom. According to this standard.

Thus volume $V = 4/3(\pi r^3)$

$= 4/3 \times (3.14) \times (2.25)^3$

$= 47.689\ cm^3$ with 1.2% uncertainty

Hence the result should be recorded as

$V = 47.7 \pm 0.6\ cm^3$

In Case of Average value of Many Measurement

(i) Find the average value of measured values.

(ii) Find deviation of each measured value from the average value.

(iii) The mean deviation is the uncertainty in the average value.

The six reading of the micrometer screw gauge to measure the diameters of a wire in mm are; 1.20, 1.22, 1.23, 1.19, 1.22, 1.21.

$$\text{Then Average} = \frac{1.20+1.22+1.23+1.19+1.22+1.21}{6}$$

$= 1.21$ mm

The deviation of the readings, which are the differences with out regards to the sign, between each reading and average values are 0.01, 0.01, 0.02, 0.02, 0.01, 0.00

$$\text{Mean of deviation} = \frac{0.01+0.01+0.02+0.02+0.01+0.00}{6}$$

$= 0.01$ mm

In Case of timing experiment

The uncertainty in the time period is found by dividing the least count of timing measurement instrument by the number of vibrations.

For example

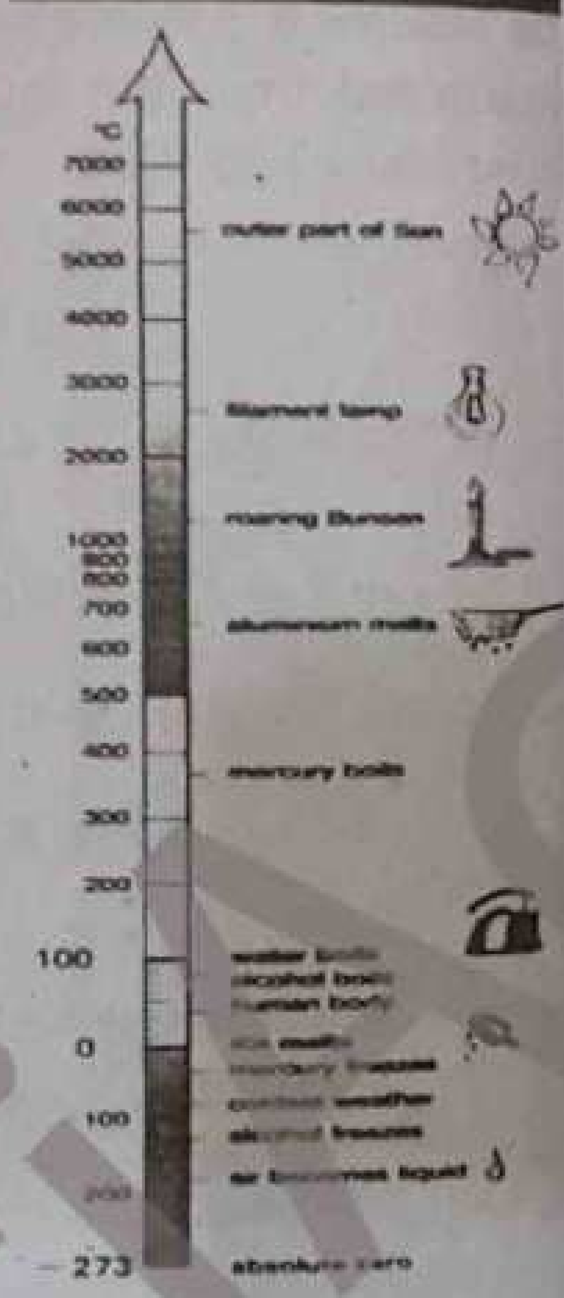
The time of 30 vibrations of a simple pendulum recorded by a stop watch accurately up to one tenth of second is 54.65. Thus, the time period is given by

$T = 54.6/30 = 1.82s$

$$\text{Uncertainty in time period} = \frac{\text{Least count}}{\text{No of vibrations}} = \frac{0.1s}{30}$$

Thus, time period T is written as $T = (1.82 \pm 0.003)s$

INTRUSTING INFORMATION



Some Specific Temperatures

Q.14 What do you understand by dimensions of physical quantities? Explain with examples. Also write its uses.

Ans.

Dimensions of Physical Quantities

The dimensions of a physical quantity represents nature of that physical quantity

Each basic physical quantity represented is by a specific symbol with in square brackets.

The dimensions of length, mass and time are [L], [M] and [T] respectively.

For Your Information

The pair of quantities which have same dimensions
* work, torque, energy
* impulse, momentum
* Angular momentum, Planck's constant
* Pressure, stress, elastic modulus
* radioactivity, frequency

(i)Speed

$$\text{As speed} = \frac{\text{Length}}{\text{Time}}$$

$$\text{Dimension of speed} = [v] = \frac{\text{Dimension of length}}{\text{Dimension of time}} = \frac{[L]}{[T]} = [LT^{-1}]$$

(ii)Acceleration

$$\text{As acceleration} = \frac{\text{Velocity}}{\text{Time}}$$

$$\text{Dimension of acceleration} = [a] = \frac{\text{Dimension of velocity}}{\text{Dimension of time}}$$

$$[a] = \frac{[LT^{-1}]}{[T]} = [LT^{-1}][T^{-1}] = [LT^{-2}]$$

(iii)Force

As force= mass × acceleration

Dimension of force= dimension of mass × dimension of acceleration

$[F] = [M][LT^{-2}]$

$[F] = [MLT^{-2}]$

Uses of Dimensions

Using the method of dimensions called the dimensional analysis; we can check the correctness of a given formula or an equation and can also derive it.

(i)Checking the homogeneity of the physical equation

In order to check the correctness of an equation, we are to show that dimension of the quantities on both sides if the equation is the same, irrespective of the form of the formulas. This is called the principle of homogeneity of dimensions.

(ii)Deriving the possible formula

The success of this method for deriving a relation for a physical quantity depends on the correct guessing of various factors on which the physical quantity depends.

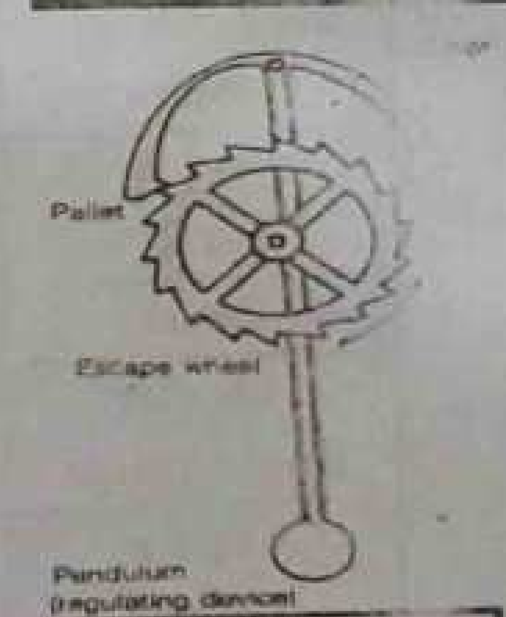
For Your Information

Dimensional variables
→ Force, velocity etc
Non - Dimensional variables
→ Plane angle, solid angle etc
Dimensional constants
→ g, G, k etc
Non – Dimensional constants
→ Reflective index, dielectric constant etc

For Your Information

Dimensional analysis deals with the physical quantities in their qualitative meaning

Do You Know



DIMENSIONS OF PHYSICAL QUANTITIES

Physical Quantity	Symbol or Formula	Unit	Dimension
Mass	m	kilogram	$[ML^0T^0] = [M]$
Length	l	Meter	$[M^0LT^0] = [L]$
Time	t	second	$[M^0L^0T] = [T]$
Velocity, speed	$v = d/t$	m/s	$[M^0LT^{-1}] = [LT^{-1}]$
Acceleration	$a = \Delta v/\Delta t$	m/s^2	$[M^0LT^{-2}] = [LT^{-2}]$
Area	A	m^2	$[M^0L^2T^0] = [L^2]$
Volume	V	m^3	$[M^0L^3T^0] = [L^3]$
Density	$\rho = m/V$	Kg/m^3	$[ML^{-3}T^0] = [ML^{-3}]$
Force	$F = ma$	$N = kgms^{-2}$	$[MLT^{-2}]$
Pressure	$P = F/A$	$Pa = N/m^2$	$[ML^{-1}T^{-2}]$
Momentum, Impulse	$P = mv$ $I = \Delta p$	$Ns = kgms^{-1}$	$[MLT^{-1}]$
Torque, Work, Energy	$\tau = lF$ $W = Fd$	Nm $J = Nm = kgm^2s^{-2}$	$[ML^2T^{-2}]$
Power	$P = W/t$	$W = J/s$	$[ML^2T^{-3}]$
Angular displacement	θ	radian	none
Angular velocity	$\omega = \Delta\theta/\Delta t$	rad/s	$[T^{-1}]$
Angular acceleration	$\alpha = \Delta\omega/\Delta t$	rad/s^2	$[T^{-2}]$
Angular momentum	$L = mvr$	$Js = kg\ m^2\ s^{-1}$	$[ML^2T^{-1}]$
Moment of inertia	$I = mr^2$	$Kg\ m^2$	$[ML^2]$
Gravitational constant	$G = Fr^2/m_1m_2$	Nm^2/kg^2	$[M^{-1}L^3T^{-2}]$
Acceleration due to gravity	$g = W/m$	m/s^2	$[LT^{-2}]$
Time period	T	sec	[T]
Frequency, angular .frequency	$f = 1/T$ $\omega = 2\pi f$	Hz = cycles/sec	$[T^{-1}]$
Coefficient of viscosity	$\eta = F/6\pi rv$	$Ns/m^2 = kg\ m^{-1}\ s^{-1}$	$[ML^{-1}T^{-1}]$

Wavelength	$\lambda = v/f$	m	[L]
Stress	$\sigma = F/A$	$Pa = N/m^2$	$[ML^{-1}T^{-2}]$
Strain	$\varepsilon = \Delta V/V$	none	none
Elastic modulus	$E = \sigma/\varepsilon$	$Pa = N/m^2$	$[ML^{-1}T^{-2}]$
Focal length	F	m	[L]
Refractive index	N	none	none

Multiple Choice Questions

♦ *Four possible answers to each statement are given below. Tick (✓) the correct answer:*

1. The main frontiers of fundamental sciences are
   (a) One (b) Two
   (c) Three (d) Four
2. The branch of physics which deals with the ultimate particles of which the matter is composed is called _____ physics
   (a) Plasma (b) Nuclear
   (c) Particle (d) Solid state
3. Which one of the following is not a unit of energy?
   (a) Kilowatt (b) Erg
   (c) Joule (d) Kilowatt hour
4. Steradian is the unit of
   (a) Plane angle (b) Solid angle
   (c) Temperature (d) Intensity of light
5. Which one of the following is a base quantity?
   (a) Area (b) Charge
   (c) Current (d) Force
6. If p is the momentum of an object of mass m, the expression $\left(\frac{p^2}{2m}\right)$ has base units identical to
   (a) Energy (b) Force
   (c) Power (d) Velocity
7. By increasing the number of observations in an experiment, the error in measurement:
   (a) Increases (b) Decreases
   (c) Remains same (d) None of these
8. How many significant zeros are there in 0.0005010?
   (a) 3 (b) 4
   (c) 2 (d) 6
9. Error in the measurement of radius of sphere is 1%. The error in the calculated value of its volume is:
   (a) 7% (b) 5%

(c) 3% (d) 1%

10. The percentage errors in the measurement of mass and speed and 2% and 3%, respectively. Th maximum error in the calculation of K.E. will be
 (a) 1% (b) 5%
 (c) 8% (d) 11%

11. How many significant zeros are there in 5.00×10^{-3}?
 (a) 3 (b) 4
 (c) 2 (d) 6

12. Zero error is the example of
 (a) Personal error (b) Random error
 (c) Systematic error (d) All of these

13. Poor calibration is the example of
 (a) Personal error (b) Random error
 (c) Systematic error (d) None of these

14. $\sqrt{\frac{F \times \ell}{m}}$ has the dimensions of
 (a) velocity (b) Acceleration
 (c) Momentum (d) None of these

15. The unit of energy is _______ and its unit symbol is:
 (a) Joule, j (b) joule, J
 (c) Joule, J (d) joule, j

16. The dimension of light year are
 (a) $[LT^{-1}]$ (b) $[M^0L^0T]$
 (c) $[ML^2T^{-2}]$ (d) $[M^0LT^0]$

17. Which one of the following is not dimensionally correct equation
 (a) $E = mc^2$ (b) $v_f = v_i + at$
 (c) $S = vt^2$ (d) $S = \frac{1}{2}at^2$

18. The gravitational constant G has dimensions
 (a) $[ML^{-1}T^{-1}]$ (b) $[M^0L^0T]$
 (c) $[ML^2T^{-2}]$ (d) $[ML^{-1}T^0]$

19. Which of the following has the same dimensions as that of momentum
 (a) Force (b) Energy
 (c) stress (d) Impulse

20. SI unit of light year is
 (a) candela (b) Meter
 (c) Second (d) None of these

ANSWERS

1. c	2. c	3. a	4. b	5. c	6.	7. b	8. c	9. c	10. c
11. a	12. c	13. c	14. a	15.	16. d	17. c	18. a	19.	20. b

Short Questions of Exercise

Q.1.1 Name the several repetitive phenomenon occurring in nature which serve as reasonable time standard? لازمی، قابل اعتبار (Lhr 2005, Bwp 2006-2008, Mtn 2009, Lhr 2010, Lhr 2011)

Ans. The phenomenon that repeats itself after certain intervals of time is called ***repetitive phenomenon***, which can serve as reasonable time standard.

Examples:
(i) lattice vibrations in a crystal.
(ii) the rotation moon around the earth.
(iii) the rotation of Earth about its own axis.
(iv) sunrise and sunset.
(v) change of weathers. , (vi) shadow of building.

Q.1.2 Give the drawbacks to use the period of pendulum as time standards?
(Grw 2003-2008, Rwp 2005, Mtn 2006, Bwp 2008, Lhr 2009,Grw 2010-11)

Ans. As the time period of the simple pendulum can be expressed as;

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

but this time period can not served as reasonable time slandered due to several reasons.

Reasons:
(i) time period of the simple pendulum ***varies with g*** and g varies with altitude. اونچائی
(ii) The length of pendulum may change with temperature (in winter or summer)

Q.1.3 Why do we find it useful to have two units for the amount of substance, the kilogram and mole?
(Mtn 2003, Mir Pur 2006, D.G.Khan 2006, Mir Pur 2009

Ans. Reason:
When we are concerned with number of particles then we use mole as the unit of amount o substance because one mole of a substance contains the equal number of particles (i.e., $N_A = 6.022 \times 10^{23}$).
But in case when we are concerned quantity of matter we use the unit kilogram.

Q.1.4 Three students measured a length of needle with a scale on which the minimum division is 1 mm an recorded as
(i) 0.2145m (ii) 0.21m (iii) 0.214m which record is correct and why? (Grw 2008)

Ans. 0.214 is ***correct*** record.
Reason:
The correct record is 0.214, because the least count of the scale is ***1 mm or 0.001m***. So length can b precisely measured up to three decimals. Hence 0.214 is correct record.

Q.1.5 An old saying is that "a chain is only as strong as its weakest link." What analogous statement c you make regarding to experimental data used in computation?

Ans. Analogous statement:
Its analogous statement is
"A result of experimental data is only as much accurate as its least accurate reading in experimen data."

Q.1.6 **The period of simple pendulum is measured by a stop watch. What type of errors is possible in time period?** (Mir Pur 2004, Lhr 2006, Fsd 2008)

Ans. **Possible Errors**

There are two types of possible errors:

(i) **Systematic error:** Stop watch may be faulty (*zero error may present or the calibration is incorrect*).

(ii) **Random error:** Negligence and inexperience of person (*at the time to start or stop the stopwatch*).

> ***Note:*** Systematic error remains fixed for an instrument while random error changes for every observation.

Q.1.7 **Does a dimensional analysis give any information on constant of proportionality that may appears in algebraic expression? Explain.** (Federal 2003-2005, Grw 2005-2008)

Ans. Dimensional analysis does not provide any sort of information about the value of constant of proportionality, it can be determined by experiment. But dimensional analysis can provide the information about the units of the dimensional constants like g (gravitational acceleration), G (universal gravitational constant), k (spring constant) etc.

For example: For time period of simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where dimensional analysis provides no information about the constant 2π.

Q.1.8 **What are the dimensions of pressure and density?** (Rwp 2003, Bwp 2004, Mtn 2004-2005, D.G.Khan 2005, Grw 2005-2006-2009, Federal 2006, Lhr 2009, Lhr 2010-2011)

Ans. ***(i) Dimensions of pressure:***

$$[\text{Pressure}] = [P] = \frac{[F]}{[A]} = \frac{[ma]}{[A]} = \frac{[M][LT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$$

(ii) Dimensions of density:

$$[\text{Density}] = [\rho] = \frac{[m]}{[V]} = \frac{[M]}{[L^3]} = [ML^{-3}]$$

Q.1.9 **The wave length depends on speed 'v' of wave and its frequency 'f'. knowing that $[\lambda] = [L]$, $[v] = [LT^{-1}]$ and $[f] = [T^{-1}]$. Decide which one is correct (i) $f = v\lambda$ (ii) $f = \frac{v}{\lambda}$** (Mtn 2005, Grw 2009)

Ans. (i) **For $f = v\lambda$**

Dimensions of L.H.S. $= [f] = [T^{-1}]$ ...(1)

Dimensions of R.H.S. $= [v\lambda] = [LT^{-1}][L] = [L^2T^{-1}]$ ...(2)

As from (1) and (2)

$$\text{L.H.S.} \neq \text{R.H.S.}$$

So, the equation is *not* dimensionally correct.

(ii) **For $f = \frac{v}{\lambda}$**

Dimensions of L.H.S. $= [f] = [T^{-1}]$ ...(1)

Dimensions of R.H.S. $= \frac{[v]}{[\lambda]} = \frac{[LT^{-1}]}{[L]} = [T^{-1}]$ ...(2)

From equations (1) and (2),

$$\text{L.H.S.} = \text{R.H.S.}$$

So, the equation is dimensionally *correct*.

Solved Examples

Example 1.1:

The length breadth and thickness of a sheet are 3.233m, 2.105m and 1.05 cm respectively. Calculate the volume of the sheet correct upto the appropriate significant digits.

Given Data:

Length of sheet $= \ell = 3.233\text{m}$

Breadth of sheet $= b = 2.105\text{m}$

Thickness of sheet $= h = 1.05\text{cm}$ $= 1.05 \times 10^{-2}\text{m}$

To Find:

Volume of the sheet $= V = ?$

Calculations:

Volume = (length) × (breadth) × (thickness)

$$V = \ell \times b \times h$$

$$V = 3.233 \times 2.105 \times 1.05 \times 10^{-2}\text{m}^3$$

$$V = 7.14573825 \times 10^{-2}\text{m}^3$$

As the factor 1.05 cm has the minimum number of significant figures equal to three. Therefore volume recorded upto 3 significant figures.

$$V = 7.15 \times 10^{-2}\text{m}^3$$

As the factor 1.05 cm has minimum number of significant figures equal to three, therefore, volume is recorded upto 3 significant figures, hence, $V = 7.15\ \text{m}^3$.

Example 1.2

The mass of a metal box measured by a lever balance is 2.2 kg. Two silver coins of masses 10.01 g and 10.02 g measured by a beam balance are added to it. What is now the total mass of the box correct upto the appropriate precision?

Given Data:

Mass of the metal box $= m_1 = 2.2$ kg

Mass of first silver coin $= m_2 = 10.01\text{ g} = 0.01001$ kg

Mass of second silver coin $= m_3 = 10.02\text{ g} = 0.01002$ kg

To Find:

Total mass of the box $= m = ?$

Calculation:

Total mass when silver coins are added to box

$$m = m_1 + m_2 + m_3$$

$$m = 2.2 + 0.01001 + 0.01002$$

$$m = 2.22003\text{ kg}$$

Since least precise is 2.2 kg, having one decimal place, hence total mass should be to one decimal place which is the appropriate precision. Thus

Total mass $= m = 2.2$ kg

Example 1.3

The diameter and length of a metal cylinder measured with the help of vernier calipers of least count 0.01 cm are 1.22 cm and 5.35 cm. Calculate the volume V of the cylinder and uncertainty in it.

Given Data:

Least count of vernier caliper $= 0.01$ cm

Diameter of metal cylinder $= d = 1.22$ cm

Length of metal cylinder $= \ell = 5.35$ cm

To Find:

Volume of the cylinder = V = ?

Uncertainty in the volume = ?

Calculations:

Absolute uncertainty in length = 0.01 cm

% age uncertainty in length $= \frac{0.01\text{cm}}{5.53\text{cm}} \times \frac{100}{100} = 0.2\%$

Absolute uncertainty in diameter $= \frac{0.01\text{cm}}{1.22\text{cm}} \times \frac{100}{100} = 0.8\%$

As volume $V = \frac{\pi d^2 \ell}{4}$ ______ (1)

Total uncertainty in volume = 2(% age uncertainty in diameter) + (% age uncertainty in length)

$= 2\,(0.8\%) + 0.2\%$

$= 1.8\%$

Putting these values in equ. (1)

$$V = \frac{3.14 \times (1.22)^2 \times 5.35}{4} = 6.2509079\ (\text{cm})^3 \text{ with } 1.8\% \text{ uncertainty}$$

(as 1.8% of 6.2509079 $= \frac{1.8}{100} \times 6.2509079 = 0.1$)

Thus $V = (6.2 \pm 0.1)\ (\text{cm})^3$

Where 6.2 $(\text{cm})^3$ is calculated volume and 0.1 m^3 is the uncertainty in it.

Example 1.4

Check the correctness of the relation $v = \sqrt{\frac{F \times \ell}{m}}$ where v is he speed of transverse wave on stretched string of tension F, length ℓ and mass in m.

Given Data:

Equation $v = \sqrt{\frac{F \times \ell}{m}}$

To Find:

To check the correctness of that equation.

Calculations:

As $v = \sqrt{\frac{F \times \ell}{m}}$

Dimensions of the L.H.S. of the equation $= v$

$= [LT^{-1}]$

Dimensions of the R.H.S. of the equation $= \sqrt{\frac{F \times \ell}{m}}$

$= \sqrt{\frac{[MLT^{-2}][L]}{[M]}}$

$= \left(\frac{[MLT^{-2}][L]}{[M]}\right)^{\frac{1}{2}}$

$= [L^2T^{-2}]^{\frac{1}{2}}$

$= [LT^{-1}]$

As dimension of L.H.S. = dimensions of R.H.S.

So that equation is dimensionally correct.

Example 1.5

Derive a relation for the time period of a simple pendulum (Fig. 1.2) using dimensional analysis. The various possible factors on which the time period T may depend are:

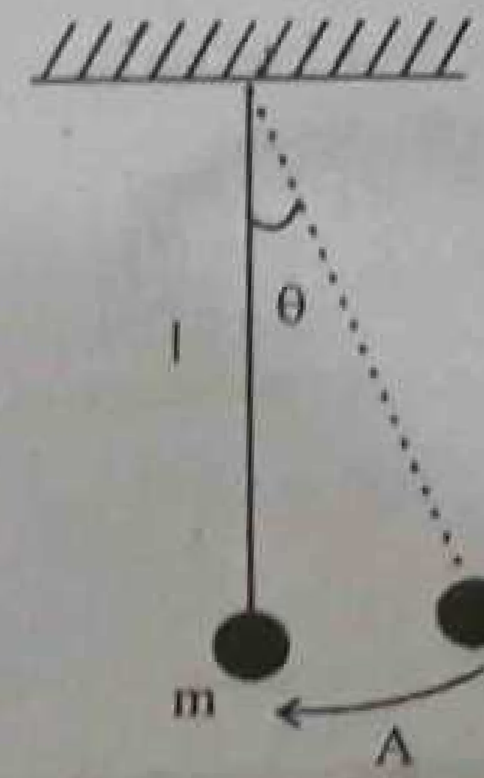
Given Data:

(i) Length of the pendulum (l).

(ii) Mass of the bob (m).

(iii) Angle θ which the thread makes with the vertical.

(iv) Acceleration due to gravity (g).



To Find:

Relation for the time period of simple pendulum = T = ?

Calculations:

The relation for the time period T will be of the form.

$T \propto m^a \times l^b \times \theta^c \times g^d$

or $T = \text{constant}\ m^a\, l^b\, \theta^c\, g^d$ ______ (1)

Taking dimensions on both sides, we get

As $S = r\theta$

Or $\theta = \frac{S}{r}$

Thus $\theta = [LL^{-1}]$

So equation 1 becomes,

$[T] = \text{constant}\ [M]^a [L]^b [LL^{-1}]^c [LT^{-2}]^d$

Comparing the dimensions on both sides

$[T]^1\ ]\ [T]^{-2d}$

$[M]^0 = [M]^a$

$[L]^0 = [L]^{b+c-c+d}$

Equating powers on both sides, we get

$-2d = 1$ or $d = -\frac{1}{2}$

$a = 1$

and $b + d = 0$ or $b = -d$ or $b = \frac{1}{2}$

and $\theta = [LL^{-1}]^c = [L^0]^c = 1$

Substituting the values of a, b, c and d in equ.(1)

$T = \text{constant} \times m^0 \times \ell^{\frac{1}{2}} \times 1 \times g^{-\frac{1}{2}}$

or $T = \text{constant} \times \ell^{\frac{1}{2}} \times g^{-\frac{1}{2}}$

or $T = \text{constant}\sqrt{\frac{\ell}{g}}$

The numerical value of the constant cannot be determined by dimensional analysis, however it can be found by experiments.

Example 1.6

Find the dimensions and hence, the SI units of coefficient of viscosity η in the relation of Stokes law for the drag forces F for a spherical object of radius r moving with velocity v given as $F = 6\pi\eta rv$.

Given Data:

Stokes law $F = 6\pi\,\eta rv$

Where F = drag force,

r = radius,

And η = co-efficient of viscosity.

To Find:

(i) Dimension of co-efficient of viscosity = ?

(ii) SI unit of co-efficient of viscosity = ?

Calculations:

(i) As $F = 6\pi\,\eta rv$

6π is a number having no dimension, so it is not taken in dimensional analysis.

Thus

$$[F] = [\eta rv]$$

or $$[\eta] = \frac{[F]}{[r][v]}$$

Putting the dimensions of F, r and v in R.H.S., we get

$$[\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]}$$

$$[\eta] = [ML^{-1}T^{-1}]$$

(ii) Thus SI unit of co-efficient of viscosity is $kg\,m^{-1}s^{-1}$

Exercise Problems

1.1 A light year is the distance light travels in one year. How many meters are there?

Given Data:

Time = $t = 1$ year

t = 365 days

t = 365×24 hours

t = $365 \times 24 \times 60$ minutes

t = $365 \times 24 \times 60 \times 60$ seconds

t = 31536000 seconds

t = 3.1536×10^7 seconds

Speed of light = $v = c = 3.0 \times 10^8\,ms^{-1}$

To find:

Distance = S = ?

Calculations:

As $S = vt$ $(v = c)$

Or $S = ct$

putting values, we get

$$S = 3.0\times10^8\times3.1536\times10^7$$

$$S = 9.5\times10^{15}\ m$$

1.2 (a) How many seconds are there in 1 year?

(b) How many nano seconds in 1 year?

(c) How many year in 1 second?

Given Data:

1 year = 365 days

To find:

(a) Seconds in 1 year = ?

(b) Nano seconds in 1 year = ?

(c) Years in 1 second = ?

Calculations:

(a) As 1 year = 365days

1 year = 365×24 hours

1 year = $365 \times 24 \times 60$ minutes

1 year = $365 \times 24 \times 60 \times 60$ sec

1 year = 31536000 sec

1 year = 3.1536×10^7 sec

(b) As 1 year = 3.1536×10^7 sec

and 1 second = 10^9 nano second

So 1 year = $3.1536 \times 10^7 \times 10^9$ ns

1 year = 3.1536×10^{16} ns

(c) As 1 year = 3.1536×10^7 sec

or 1 second = $\frac{1}{3.1536\times10^7}$ year

Or 1 second = 3.17×10^{-8} years

1.3 The length and width of rectangular plate are measured to be 15.3 cm and 12.80 cm, respectively. Find the area of the plate.

Given Data:

Length of rectangular plate = ℓ = 15.3 cm

Width of rectangular plate = W = 12.80 cm

To find:

Area of rectangular plate = A = ?

Calculations:

As Area = length × width

$A = \ell \times W$

putting values, we get

$A = 15.3 \times 12.80$

$A = 195.84\ (cm)^2$

Or $A = 196\ (cm)^2$

1.4 Add the following masses given in kg upto appropriate precision. 2.189, 0.089, 11.8 and 5.32.

Given Data:

Let $m_1 = 2.189kg$

$m_2 = 0.089kg$

$m_3 = 11.8kg$
$m_4 = 5.32kg$

To find:

Add these masses up to appropriate precision

Calculations:

Total mass $= m = m_1 + m_2 + m_3 + m_4$

$m = 2.189+0.089+11.8+5.32$

$m = 19.398kg$

As in the given masses, 11.8kg is the mass of least precision, having one decimal place, which is th appropriate precision.

Hence Total mass = 19.4kg

1.5 Find the value of g and its uncertainly using $T = 2\pi\sqrt{\frac{\ell}{g}}$ from the following

Measurements made during an experiment :
Length of simple pendulum = ℓ = 100 cm
Time for 20 vibrations = t = 40.2 s
Length was measured by a metre scale of accuracy upto 1 mm and time by stop watch of accurac upto 0.1 s.

Given Data:

Length of simple pendulum $= \ell = 100\ cm = 1m$
Time for 20 vibration s $= t = 42.2\ s$
Time period $= T = \frac{t}{20} = \frac{42.2}{20} = 2.01s$
Least count of meter rod = 1 mm = 0.001m
Least count of stop watch = 0.1s

To find:

Acceleration due to gravity = g = ?

Calculations:

As time period of simple pendulum is given by

$$T = 2\pi\sqrt{\left(\frac{\ell}{g}\right)}$$

Squaring both sides , we get

$$T^2 = 4\pi^2\left(\frac{\ell}{g}\right)$$

Or $$g = 4\pi^2\left(\frac{\ell}{T^2}\right)$$

Putting values , we have

$$g = \frac{4\times(3.14)^2\times 1}{(2.01)^2}$$

$$g = 9.76\ ms^{-2}$$

Calculation for uncertainty:

Absolute uncertainty in length $= \frac{0.001}{1} = 0.001$

Percentage uncertainty in length $= 0.001\times\frac{100}{100} = .01\%$

Uncertainty in time measurement $= \frac{\text{least count}}{\text{no. of vibrations}} = \frac{0.1}{20} = 0.005\,s$

Percentage uncertainty in time $= \frac{0.005}{2.01}\times\frac{100}{100} = 0.25\%$

Total uncertainty in g = % uncertainty in length + 2(%age uncertainty in time period)
= 0.1%+2x0.25%
= 0.1%+0.5%
= 0.6%

So $g = 9.76\ cm^2$ with 0.6% uncertainty

(As 0.6% of 9.76 $= 9.76\times\frac{0.6}{100} = 0.06$)

Hence $g = (9.76 \pm 0.06)\ ms^{-2}$

1.6 What are the dimensions and units of gravitational constant G in the formula $F = G\frac{m_1m_2}{r^2}$?

Given Data:

Gravitational force $= F = G\frac{m_1m_2}{r^2}$

To find:

Dimension of G = ?
SI unit of G = ?

Calculation:

As $F = G\frac{m_1m_2}{r_2}$

Or $G = \frac{Fr^2}{m_1m_2}$

Dimension of G $= \frac{(\text{Dimensions of force})\times(\text{Dimension of length})^2}{(\text{Dimension of mass})\times(\text{Dimension of mass})}$

$$= \frac{[MLT^{-2}][L^2]}{[M][M]}$$

$$= \frac{ML^3T^{-2}}{[M^2]}$$

$$= [M^{-1}L^3T^{-2}]$$

As SI units of force, length and mass are Newton, meter and kilogram respectively. So

SI units of G $= \frac{Nm^2}{kg^2}$ or $Nm^2\ kg^{-2}$

1.7 Show that the expression $v_f = v_i + at$ is dimensionally correct, where v_i is the velocity at $t = 0$, a i acceleration and v_f is the velocity at time t

Given Data:

First equation of nation is $v_f = v_i + at$

To show:

The equation $v_f = v_i + at$ is dimensionally correct.

Proof:

Dimensions of L.H.S of the equation $[v_f]=[LT^{-1}]$

Dimension of R.H.S of the equation = $[v_i+at]$

$$R.H.S=[v_i]+[a][t]$$
$$R.H.S=[LT^{-1}]+[LT^{-2}][T]$$
$$R.H.S=[LT^{-1}]+[LT^{-1}]$$
$$R.H.S=2[LT^{-1}]$$

As 2 has no dimension being a number, so

$$R.H.S.=[LT^{-1}]$$

Thus R.H.S. = L.H.S

Hence the equation is dimensionally correct.

1.8 The speed v of sound waves through a medium may be assumed to depend on (a) the density ρ of the medium and (b) its modulus of elasticity E which is the ratio of stress to stain. Deduce by the method of dimensions, the formula for the speed of sound.

Given Data:

Speed of sound depends on

Density of medium = ρ

Modulus of elasticity = E

To find:

Formula for speed of sound v dimensionally

Calculations:

As speed of sound depends on the following factors

$v \propto \rho^a$ and $v \propto E^b$

On combining, we get

$v \propto \rho^a E^b$

Or $v = \text{constant}\ \rho^a E^b$ ....(1)

Where we have to find the values of powers a and b

As the dimensions of

Velocity = v = $[LT^{-1}]$

Density = $\rho = \dfrac{\text{mass}}{\text{volume}} = [ML^{-3}]$

Elastic modulus = $E = \dfrac{\text{stress}}{\text{strain}} = [ML^{-1}T^{-2}]$

Because,

dimensions of stress = $[ML^{-1}T^{-2}]$ (As stress = $\dfrac{F}{A}$)

Strain is dimension less, because it is ratio

Now, writing the dimensions of both sides of equation (1)

$$[LT^{-1}] = \text{Constant}\ [ML^{-3}]^a[ML^{-1}T^{-2}]^b$$

Or $$[L][T^{-1}]=[M^{a+b}][L^{-3a-b}][T^{-2b}]$$

comparing the dimensions on both sides, we get

$$[M]^0=[[M]^{a+b}]$$
$$[L]^1=[L^{-3a-b}]$$
$$[T]^{-1}=[T]^{-2b}$$

Equation powers on both sides, we get

$$a+b=0 \text{ or } a=-b \quad ...(2)$$

and $$-3a-b=1 \quad ...(3)$$

and $$-2b=-1 \text{ or } b=\frac{1}{2} \quad ...(4)$$

substituting value of b in equation (2) we get

$$a=-\frac{1}{2}$$

Now putting values of 'a' and 'b' in equation (1), we get

$$v=\text{constant}\ \rho^{-\frac{1}{2}}E^{\frac{1}{2}}$$
$$v=\text{constant}\left(\frac{E}{\rho}\right)^{\frac{1}{2}}$$
$$v=\text{constant}\sqrt{\frac{E}{\rho}}$$

1.9 Show that the famous "Einstein equation" $E=mc^2$ is dimensionally consistent.

Given Data:

Einstein equation $E=mc^2$

To prove:

Einstein equation $E=mc^2$ is dimensionally consistent

Calculations:

As $E=mc^2$

Where E is the energy in joules

Dimensions of L.H.S of equation = E = $[ML^2T^{-2}]$ ...(1)

(As E = W = Fd) ...(1)

Dimensions of R.H.S of equation = $mc^2=[M][LT^{-1}]^2$

$[ML^2T^{-2}]$ ...(2)

Since the dimensions of both sides of equations are same

Thus L.H.S.=R.H.S.

Hence, the equation is dimensionally consistent or correct.

1.10 Suppose, we are told that the acceleration of a particle moving in a circle of radius *r* with uniform speed v is proportional to some power of *r*, say r^n, and some power of v, say v^m, determine the powers of *r* and v ?

Given Data:

Acceleration of a particle moving in circle depends on

Radius of the circle = r^n

The uniform speed = v^m

To find:

powers of r and v i.e., n = ?, m = ?

Calculations:

As given

$a \propto r^n$

$a \propto v^m$

on combining, we get

$a \propto r^n \ v^m$

$a = \text{constant} \ r^n \ v^m$

writing the dimension of both sides, we get

$[LT^{-2}] = \text{Constant} \times [L]^n \times [LT^{-1}]^m$

$[L]^1[T]^{-2} = \text{Constant} \times [L]^{n+m}[T]^{-m}$

comparing the dimensions on both sides, we get

$[L]^1 = [L]^{n+m}$

$[T]^{-2} = [T]^{-m}$

Equation powers on both sides , we get

$n + m = 1 \quad \ldots(1)$

$-m = -2$

$m = 2 \quad \ldots(2)$

putting value of 'm' in equation (1) , we get

$n + 2 = 1$

Or $n = -1$

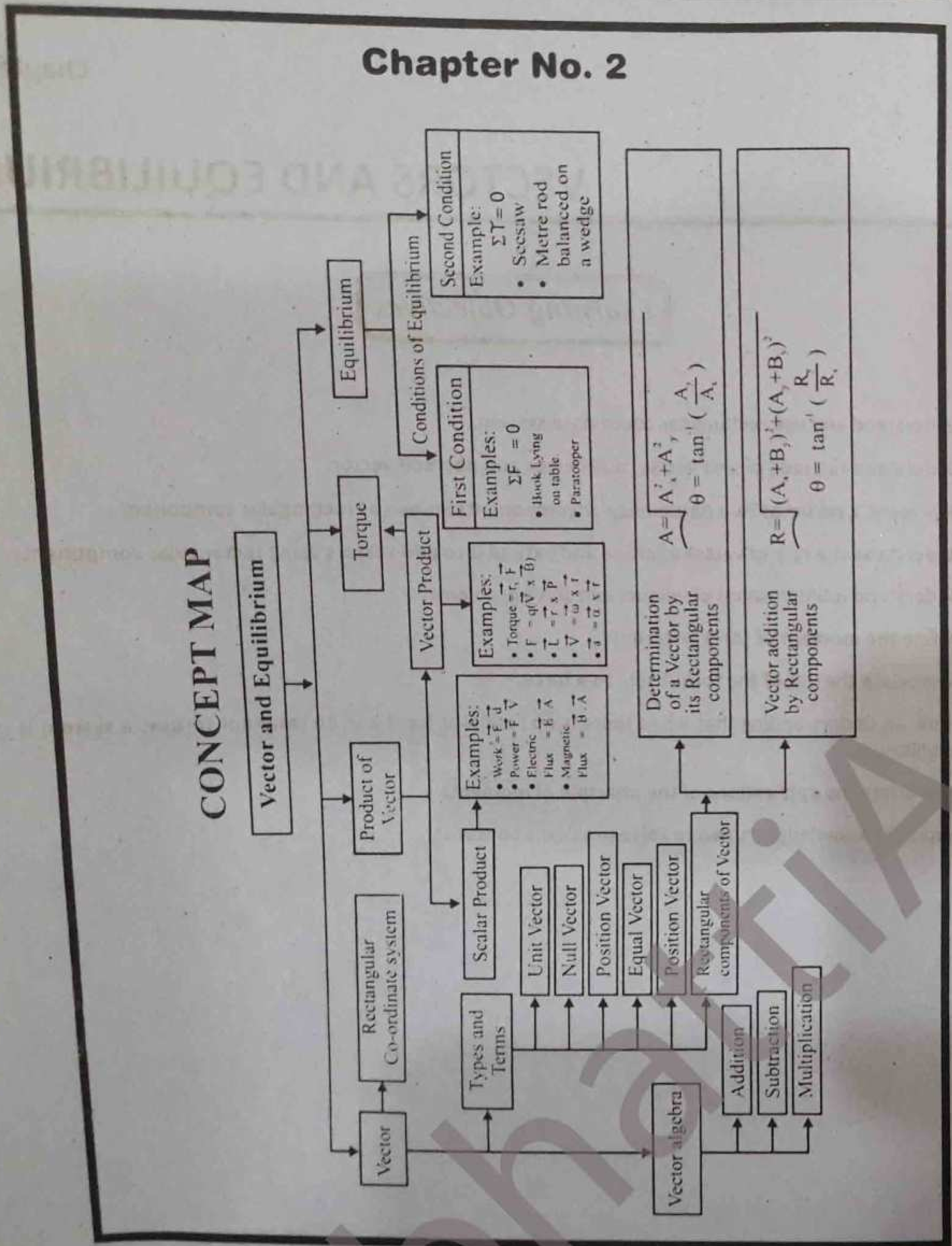
****

Chapter **2**

VECTORS AND EQUILIBRIUM

Learning Objectives

1. Understand and use rectangular coordinate system.
2. Understand the idea of unit vector, null vector and position vector.
3. Represent a vector as two perpendicular components known as rectangular components.
4. Understand the rule of vector addition and extend it to add vectors using rectangular components.
5. Understand multiplication of vectors and solve problems.
6. Define the moment of force or torque.
7. Appreciate the use of the torque due to a force.
8. Show an understanding that when there is no resultant force and no resultant torque, a system is in equilibrium.
9. Appreciate the applications of the principle of moments.
10. Apply the knowledge gained to solve problems on statics.



Physical Quantities

Quantities which can be ***observed*** as well as ***measured*** are called physical quantities.

OR

Quantities in terms of which all the ***laws of physics*** can be expressed are called physical quantities.

Types of Physical quantities

(i) Scalars (ii) Vectors

(i)Scalar

A physical quantity which has ***magnitude only*** is called scalar quantity.

Examples

Time, distance, mass, temperature, speed, energy, work, volume, area, electric charge etc.

(ii)Vector

A physical quantity which has both ***magnitude*** and ***direction*** is called vector quantity.

Examples

Force, velocity, displacement, torque, momentum, acceleration, weight, angular velocity, electric intensity etc.

Q.1 How is a vector represented?

Ans.

Vector Representation

A vector is represented in two ways.

(i) Symbolic representation (ii) Graphical representation

Symbolic Representation

It is represented by ***bold face*** letter such as **A, d, r** and **v** etc. It can be also be represented by a ***letter with an arrow*** placed above or below the letter such as $\vec{A}$ or $\underset{\rightarrow}{A}$

Graphical Representation

It is represented by a ***straight line*** with an ***arrow head*** at its one end. The length of line represents magnitude of vector (according to suitable scale).Arrow head represents the direction of vector.

Note:

Representation of magnitude of vector

The magnitude of vector is represented by ***light face*** letter such as A, d, r and v etc or by the modulus of a vector such as $|\vec{A}|, |\vec{v}|$ etc.

Q.2 What is rectangular coordinate system?

Ans.

باہمی تعلق

Rectangular Coordinate System/(Cartesian Co-ordinate System)

The set of two or three mutually perpendicular lines intersecting at a point is called rectangular coordinate system.

The lines are called *coordinate axes*. One of these lines is called *x-axis (or horizontal axis)* The other is called *y-axis or vertical axis*. The line perpendicular to both x and y –axes is called *z – axis*. The point of intersection is called *origin*.

Two dimensional coordinate system (Plane)

If the system consists of *two* perpendicular lines then it is called two dimensional coordinate system.

Direction of a vector in plane

It is represented by the angle which the vector makes with *positive x-axis* in *anti-clock* wise direction.

> **For Your Informatio**
> In plane only one angle required to represent a vec while three angles required to represent a vec in space.

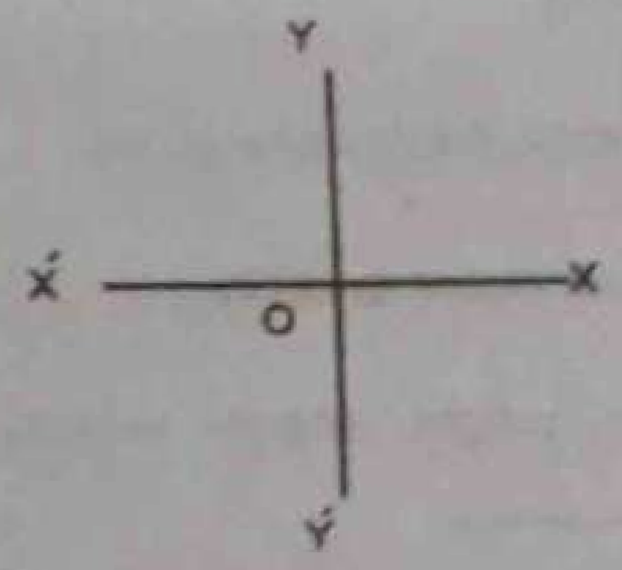


Fig. 2.1(a)

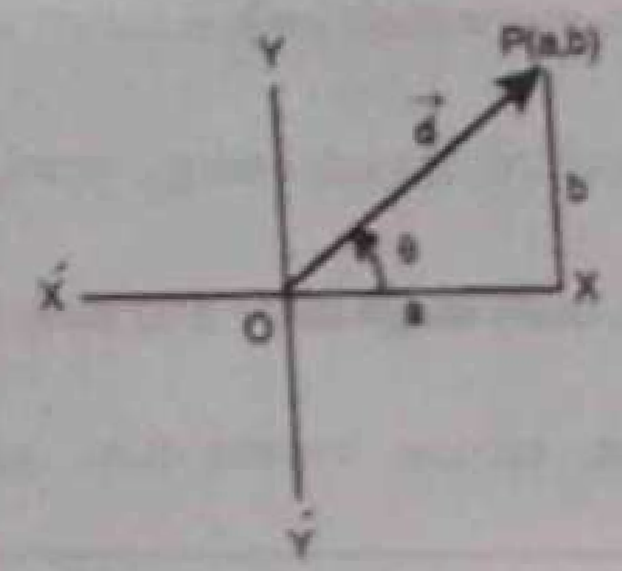


Fig. 2.1(b)

Three dimensional co-ordinate systems (Space)

If the system consists of *three* perpendicular lines, then it is called three dimensional co-ordinate systems.

Direction of a vector in space

It is represented by *three* angles which the vector makes with *x*, *y* and *z* – *axes*.

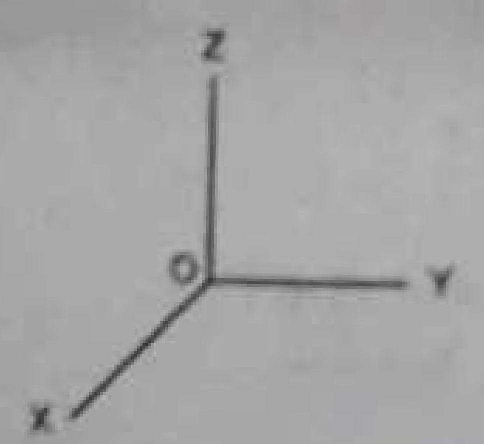


Fig. 2.2(a)

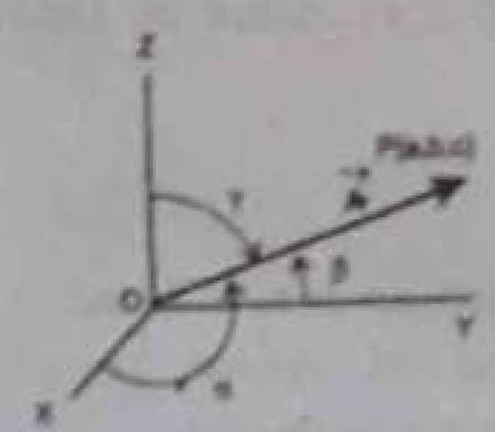


Fig. 2.2(b)

Q.3 Describe the addition of vectors by head to tail rule.

> **For Your Information**
> The reverse process of vector addition is called resolution of vector.

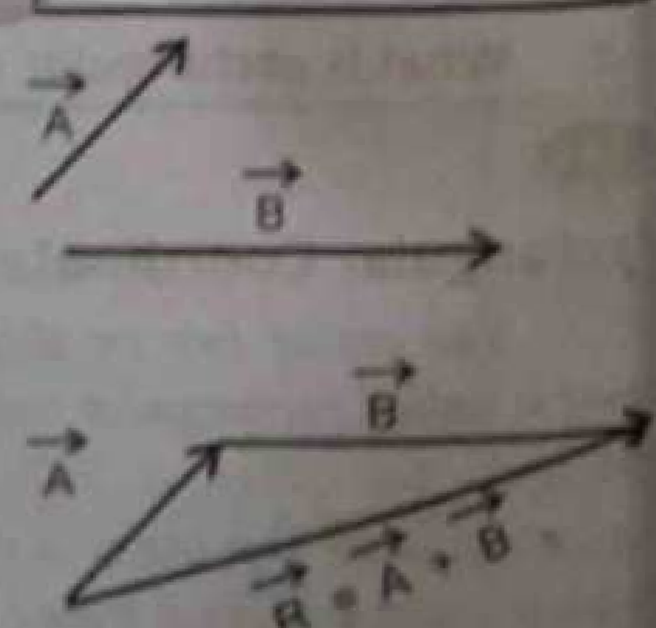
Ans.

Head to tail rule

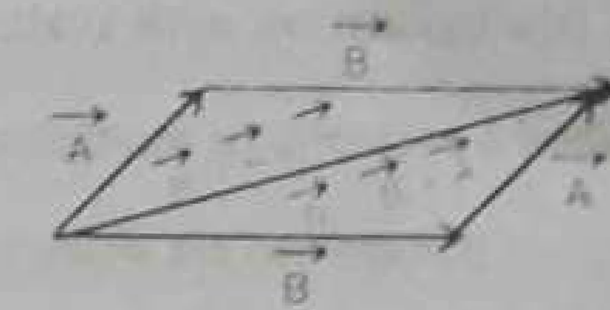
It is a graphical method to add two or more vectors.

Explanation

Draw the representative lines of vectors $\vec{A}$ and $\vec{B}$. Join the tail of vector $\vec{B}$, with the head of vector $\vec{A}$. Join the tail of vector $\vec{A}$ with head of vector $\vec{B}$.



This line gives the magnitude of resultant vector R. The direction of resultant vector is from tail of vector A to the head of vector B.



Commutative Property

From figure, it is clear that either we add A to B or B to A, the resultant is same. ie,

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

It means that when vectors are added, the result is the same for any order of addition. Hence, vector addition is commutative.

Q.4 Explain the following terms:

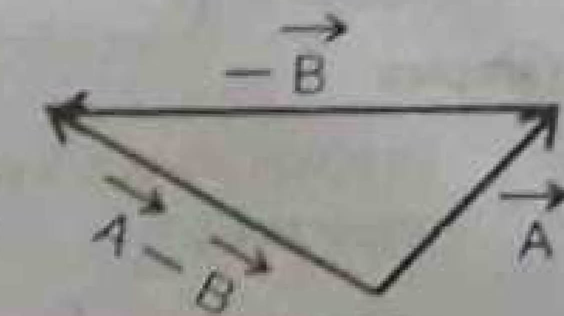
(i) Resultant vector (ii) Vector subtraction
(iii) Multiplication of vector by a scalar
(iv) Unit vector (v) Null vector
(vi) Equal vectors

> **For Your Information**
> Resultant of two vectors will be maximum when they are along the same direction and will be minimum when they are opposite in direction.

Ans.

(i) Resultant Vector

A vector which has the same effect as the combined effect of all the vectors to be added is called resultant vector.



(ii) Vectors Subtraction

The subtraction of a vector is equivalent to the *addition of same vector with its direction reversed.*

Explanation

Consider two vectors $\vec{A}$ and $\vec{B}$. To subtract $\vec{B}$ from vector $\vec{A}$. First, take the negative of vector $\vec{B}$. Add $(-\vec{B})$ into vector $\vec{A}$ graphically as shown in fig. So

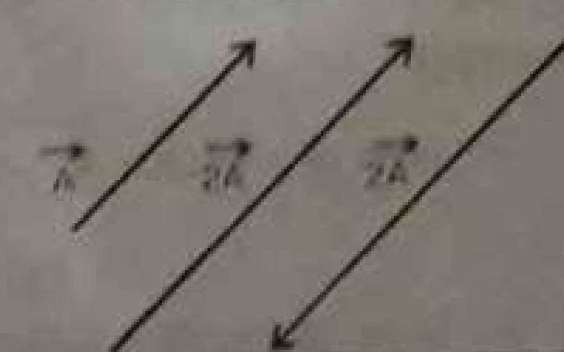
$$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$$

> **For Your Information**
> Vectors subtraction does not obey the commutative law.

(iii) Multiplication of a vector by a scalar

A vector can be multiplied by

1. a positive number
2. a negative number
3. a scalar with dimension



1. Multiplication with positive number

When a vector $\vec{A}$ is multiplied by a positive number n (i.e. n > 0) Then the product vector will have magnitude equal to nA and *same* direction as that of $\vec{A}$

> **For Your Information**
> If vector A is multiplied by a number 'n' either positive or negative, the magnitude of new vector be nA

2. Multiplication with negative number

When a vector $\vec{A}$ is multiplied by a negative number n (i.e. n < 0), then the product vector will have magnitude equal to nA but direction *opposite* to that of $\vec{A}$.

3. Multiplication with scalar quantity

When a vector $\vec{A}$ is multiplied by a scalar quantity n, then the product vector will be a *new physical quanti* whose dimensions are equal to the *product of dimensions of n and* $\vec{A}$

Examples

- Product of mass m and velocity $\vec{v}$ is momentum[$\vec{p}=m\vec{v}$]
- Product of mass m and acceleration $\vec{a}$ is force[$\vec{F}=m\vec{a}$]
- Product of force $\vec{F}$ time t impulse [$\vec{I}=\vec{F}\times t$]

For Your Information
Resultant of unit vectors $\hat{i}$ and $\hat{j}$ is $\sqrt{2}$

(iv) Unit vector

A vector whose magnitude is equal to one with no units in a given direction is called unit vector.

It is represented by a letter with a cap or hat on it.

Mathematical Form

If $\vec{A}$ is a vector with magnitude A, then $\vec{A} = A\hat{A}$

OR $\boxed{\hat{A}=\dfrac{\vec{A}}{A}}$

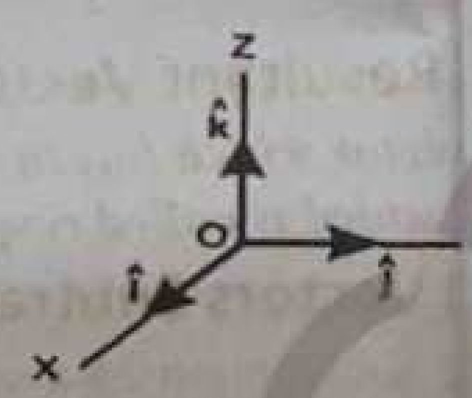
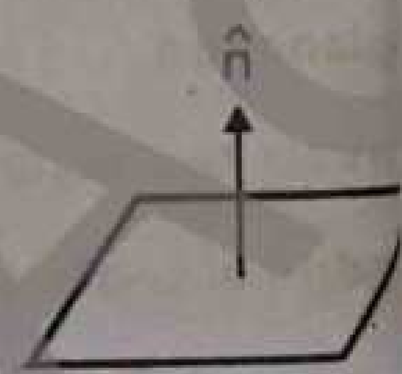


Fig. 2.5(a)

Examples

- Direction along x, y and z-axes are represented by unit vectors $\hat{i},\hat{j},\hat{k}$ respectively.
- unit vector $\hat{r}$ represents the direction of $\vec{r}$.
- unit vector $\hat{n}$ represents the direction of normal drawn on a certain surface



(v) Null or Zero Vector

A vector whose magnitude is zero and direction arbitrary is called a null vector.

It is represented by $\vec{O}$

Example

- Sum of a vector $\vec{A}$ and its negative vector (- $\vec{A}$) is a null vector. i.e.

$$\vec{A} + (-\vec{A}) = \vec{O}$$

- Sum of vectors taken head to tail along the *sides of a closed polygon* is null vector

For Your Information
Position vector of origin is null vector.
The acceleration of a body moving with uniform velocity is null vector.
Vertical velocity of projectile at the highest point is null vector

(vi) Equal Vectors

The vectors are said to be equal vectors if they have *same magnitude and same direction* regardless of the position of their initial points.

Example of their origin

Two parallel vector $\vec{A}$ and $\vec{B}$ of same magnitude and direction are equal vectors.

For Your Information
Two like parallel vectors of equal magnitudes are equal vectors.

Q.5 What is meant by component of vector and what are rectangular components?
(i) Find rectangular components of vector.
(ii) How can we determine a vector from its rectangular components?

Ans.

Components of a vector

The effective value of a vector in a given direction is called component of a vector.

A vector may split up into two or more than two parts these parts are known as components of vector.

Rectangular Components of Vector

The components of a vector which are perpendicular to each other are called rectangular components.

Explanation

Consider a vector $\vec{A}$ which makes an angle θ with x-axis as shown in figure.

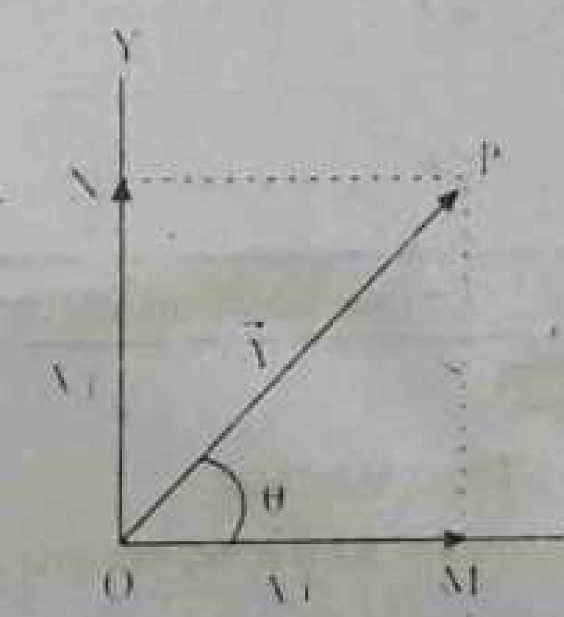


Fig 2.6

OM is projection of vector $\vec{A}$ on x – axis and **ON** is projection of vector $\vec{A}$ on y – axis

By head to tail rule

$$\mathbf{OP = OM + MP} \qquad \text{[As ON=MP]}$$

OR $$\vec{A} = A_x\hat{i} + A_y\hat{j} \qquad \text{\_\_\_\_\_\_\_\_(1)}$$

Thus $A_x\hat{i}$ and $A_y\hat{j}$ are the components of vector $\vec{A}$. Since $A_x\hat{i}$ and $A_y\hat{j}$ are at *right angle* to each other, so they are called rectangular components.

X – Component of $\vec{A}$

In right angled triangle OPM,

$$\frac{OM}{OP} = \cos\theta$$

OR $$\frac{A_x}{A} = \cos\theta$$

OR $$\boxed{A_x = A\cos\theta} \qquad \text{\_\_\_\_\_\_\_\_(2)}$$

Y – Component of $\vec{A}$

In right angled triangle OPM,

$$\frac{PM}{OP} = \sin\theta$$

OR $$\frac{A_y}{A} = \sin\theta$$

$$A_y = A\sin\theta$$

$$\boxed{A_y = A\sin\theta} \qquad \text{\_\_\_\_\_\_\_\_(3)}$$

(ii) Vector $\vec{A}$ from its rectangular components

Magnitude of $\vec{A}$

In right angled triangle OPM,

$OP^2 = OM^2 + MP^2$ (by Pythagorean Theorem)

OR $A^2 = A_x^2 + A_y^2$

$$\boxed{A = \sqrt{A_x^2 + A_y^2}} \quad \text{\_\_\_\_\_\_\_\_ (4)}$$

Direction of $\vec{A}$

Again in right angled triangle OPM,

$\tan\theta = \dfrac{MP}{OM}$ = per pardicular / Base

OR $\tan\theta = \dfrac{A_y}{A_x}$

OR $$\boxed{\theta = \tan^{-1}\frac{A_y}{A_x}} \quad \text{\_\_\_\_\_\_\_\_ (5)}$$

Do You Know?
The process by which a vector ca[n] be obtained from its rectangula[r] components is called compositio[n] of a vector.

Q. 6 Define position vector.

Ans

Position Vector

The vector which represents the position of a point or a particle with respect to fixed origin is called position vector. It is denoted by $\vec{r}$

Explanation

In *two dimensional coordinate system* (plane), the position of a point P (a, b) is represented by

$$\boxed{\vec{r} = a\hat{i} + b\hat{j}}$$

The magnitude of this position vector is

$$r = \sqrt{a^2 + b^2}$$

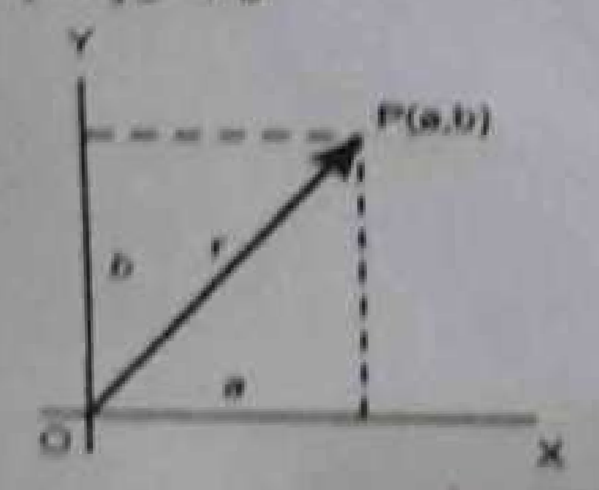


Fig. 2.7(a)

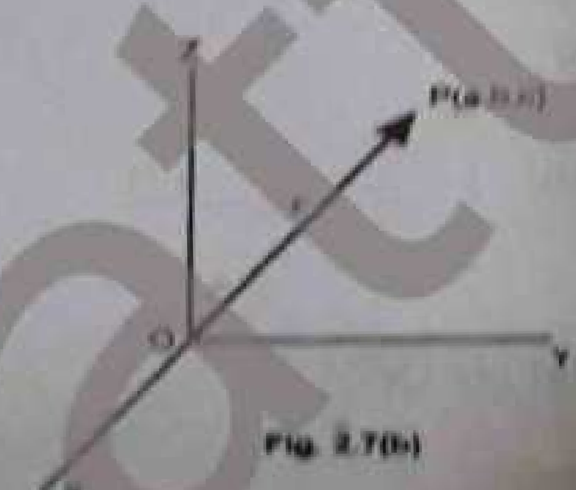


Fig. 2.7(b)

In *three dimensional coordinate system* (space), the position of a point P (a, b, c) is represented by

$$\boxed{\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}}$$

and its magnitude is

$$r = \sqrt{a^2 + b^2 + c^2}$$

Q.7 Describe the method of addition of vectors by rectangular components.

Ans

Vector Addition by Rectangular Components

Consider two vectors $\vec{A}$ and $\vec{B}$ represented by the lines OM and ON as shown in figure. By head to tail rule the resultant of these two vectors is $\vec{R} = \vec{A} + \vec{B}$

Resolve the vectors $\vec{A}$, $\vec{B}$ and $\vec{R}$ into their rectangular components.

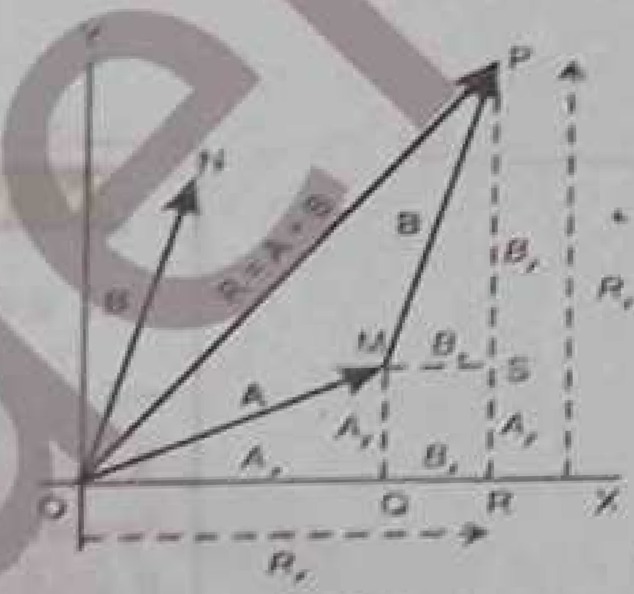


Fig. 2.9

The Chinese acrobats in this incredible balancing act are in equilibrium.
EXPLANATION:
The Chinese acrobats are in stable equilibrium because the line through the centre of mass passes through the base area. If the line falls in or outside the base area then the acrobats will not be in equilibrium.

X- Component of Resultant

OQ is the magnitude of x-component of vector $\vec{A}$, MS is the magnitude of x-component of vector $\vec{B}$ and OR is the magnitude of x-component of vector $\vec{R}$

From figure

$OR = OQ + QR$

Or $OR = OQ + MS$ [since QR=MS]

$$\boxed{R_x = A_x + B_x} \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

This shows that the sum of magnitudes of x-components of $\vec{A}$ and $\vec{B}$ is equal to the magnitude of x-component of resultant vector $\vec{R}$.

Y- Component of Resultant

QM is the magnitude of y-component of vector $\vec{A}$,SP is the magnitude of y-component of vector $\vec{B}$ and RP is the magnitude of y-component of vector $\vec{R}$

From figure

$RP = RS + SP$

$RP = QM + SP$ [since RS = QM]

$$\boxed{R_y = A_y + B_y} \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

This shows that the sum of magnitudes of y-components of $\vec{A}$ and $\vec{B}$ is equal to the magnitude of y-component of resultant vector $\vec{R}$.

Since $R_x\hat{i}$ and $R_y\hat{j}$ are the rectangular components of resultant vector $\vec{R}$, hence

$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

Putting values of R_x and R_y from equation (1) and (2) we get,

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

Magnitude of Resultant Vector $\vec{R}$

The magnitude of resultant vector $\vec{R}$ is

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\boxed{R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}} \quad \text{------------ (3)}$$

Direction of Resultant Vector $\vec{R}$

The direction of the resultant vector $\vec{R}$ is given by

$$\tan\theta = \left(\frac{R_y}{R_x}\right)$$

Or

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\boxed{\theta = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)} \quad \text{------------ (4)}$$

In General

For any number of coplanar vectors $\vec{A}, \vec{B}, \vec{C}, \vec{D}$, ... we can write

$$R = \sqrt{(A_x + B_x + C_x + ...)^2 + (A_y + B_y + C_y + ...)^2}$$

And

$$\theta = \tan^{-1}\left(\frac{A_y + B_y + C_y + ...}{A_x + B_x + C_x + ...}\right)$$

For Your Information

If $\vec{R}$ is the resultant of vectors $\vec{A}$ and $\vec{B}$ then its magnitude is

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Special Cases:

If $\theta = 0°$

$R_{max} = A + B$

If $\theta = 180°$

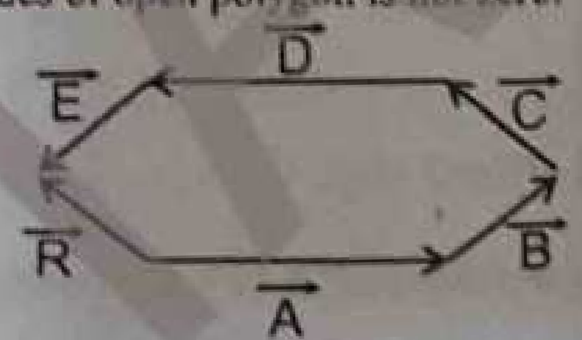
$R_{max} = A - B$

If $\theta = 90°$

$R = \sqrt{A^2 + B^2}$

Do You Know?

The sum of vector which forms the sides of open polygon is not zero.



Summary

(i) Find the x and y-components of all given vectors.

(ii) Add x-components of all the vectors to find the x-component R_x of the resultant vector.

(iii) Add y-components of all the vectors to find the y-component R_y of the resultant vector.

(iv) Find the magnitude of resultant vector $\vec{R}$ by using

$$R = \sqrt{R_x^2 + R_y^2}$$

(v) Find the direction of resultant vector $\vec{R}$ by using

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Q.8 How can you determine the angle θ of the vector $\vec{R}$ by its rectangular components

Ans.

Determination of Angle (θ)

1) First find ϕ by the following relation.

$$\phi = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Where ϕ = the angle which $\vec{R}$ makes with nearest x-axis

2) By the signs of R_x and R_y, find the quadrant in which $\vec{R}$ lies as follows:

- If both R_x and R_y are positive, the resultant lies in the 1st quadrant and its direction is

$$\theta = \phi$$

- If R_x is – ve and R_y is +ve, the resultant lies in 2nd quadrant and its direction is

$$\theta = 180° - \phi.$$

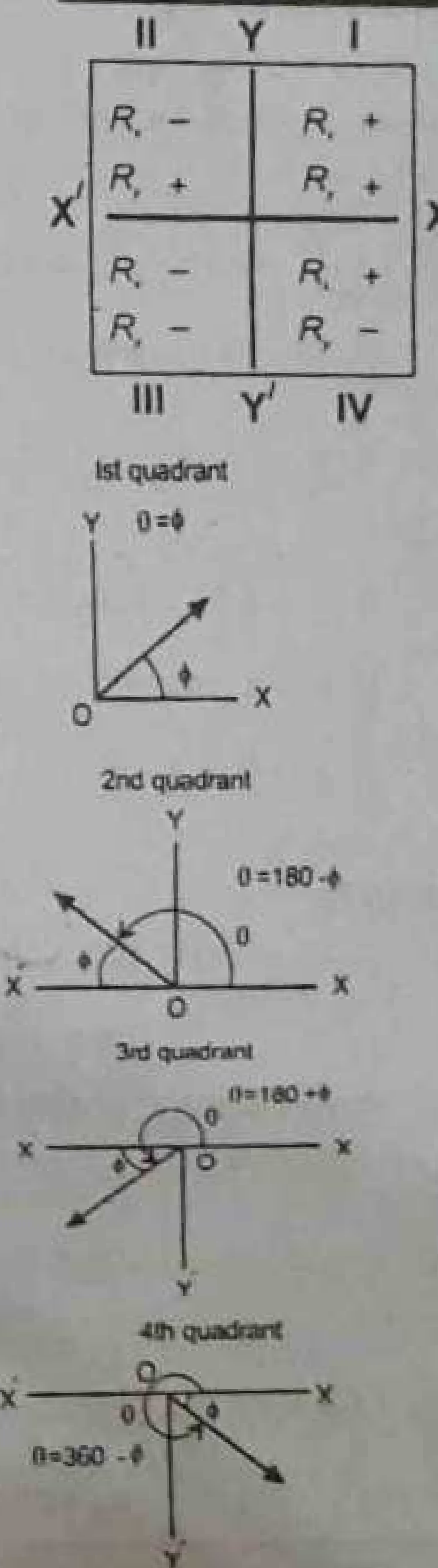
- If both R_x and R_y are –ve , the resultant lies in 3rd quadrant and its direction is

$$\theta = 180° + \phi.$$

- If R_x is + ve and R_y is –ve, the resultant lies in 4th quadrant and its direction is

$$\theta = 360° - \phi$$

Table 2.1



MULTIPLICATION OF TWO VECTORS

Vectors can be multiplied in two ways:

(i) Scalar Product

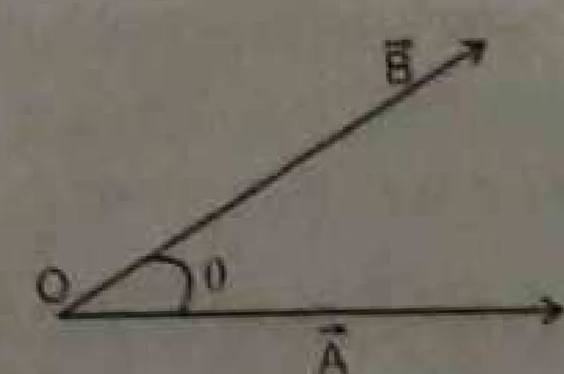
(ii) Vector Product

Q.9 Define scalar product of two vectors. Give examples.

Ans.

Scalar Product (Dot Product)

If the product of two vectors is a scalar quantity then the product is called scalar product



Scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\boxed{\vec{A}.\vec{B} = AB\cos\theta}$$

Where A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$ and θ is the angle between them.

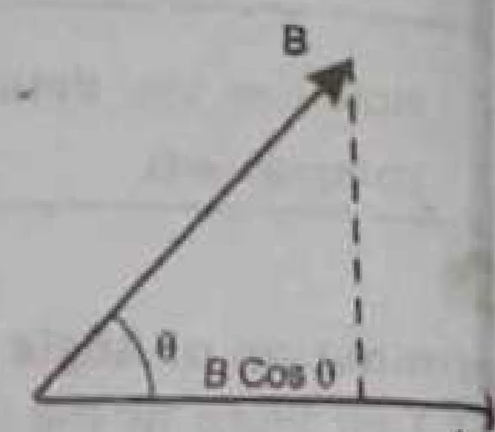


Fig. 2.10 (a)

Physical meaning

Dot product of two vectors is equals to product of magnitude of one vector and the component of the second vector in the direction of first vector.

From figure

$\vec{A}.\vec{B}$ = A (projection of $\vec{B}$ on $\vec{A}$)

Or $\vec{A}.\vec{B}$ = A (magnitude of component of $\vec{B}$ along $\vec{A}$)

$\vec{A}.\vec{B}$ = A (B cosθ)

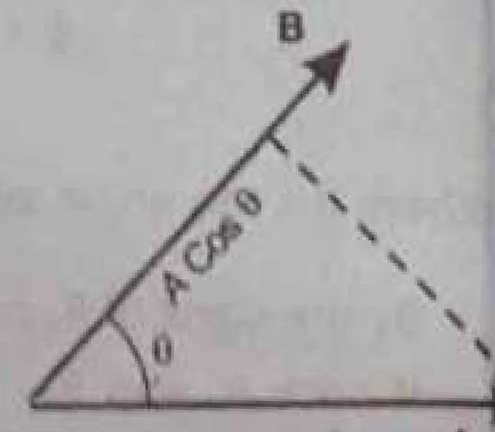


Fig. 2.10 (b)

Similarly

$\vec{B}.\vec{A}$ = B (Projection of $\vec{A}$ on $\vec{B}$)

$\vec{B}.\vec{A}$ = B (magnitude of component of $\vec{A}$ along $\vec{B}$)

$\vec{B}.\vec{A}$ = B (A cosθ)

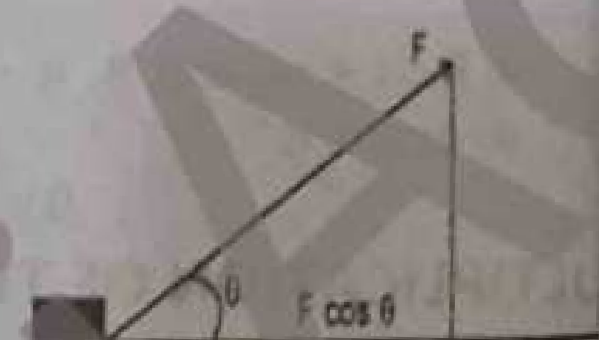
Examples

1) Work is scalar product of force and displacement

$[W = \vec{F}\cdot\vec{d}]$

2) Power is scalar product of force and velocity

$[P = \vec{F}\cdot\vec{v}]$



3) Electric flux is scalar product of electric intensity and vector area

$[\phi_E = \vec{E}\cdot\vec{A}]$

4) Magnetic flux is scalar product of magnetic field strength and vector area

$[\phi_B = \vec{B}\cdot\vec{A}]$

Q.10 Write down the characteristics of scalar product of two vectors.

Ans.

Characteristics of Scalar Product

(i) Commutative property

Scalar product of two vectors is *commutative*

If $\vec{A}$ and $\vec{B}$ be two vectors and θ is the angle between them. Then

and $\vec{A}.\vec{B} = AB\cos\theta$ ....(1)

OR $\vec{B}.\vec{A} = AB\,A\cos\theta$

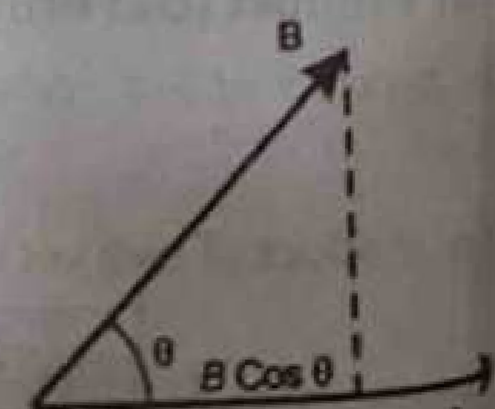


Fig. 2.10 (a)

$\vec{B}.\vec{A} = AB\cos\theta$ ....(2) [since AB = BA]

From equations (1) and (2)

$$\boxed{\vec{A}.\vec{B} = \vec{B}.\vec{A}}$$

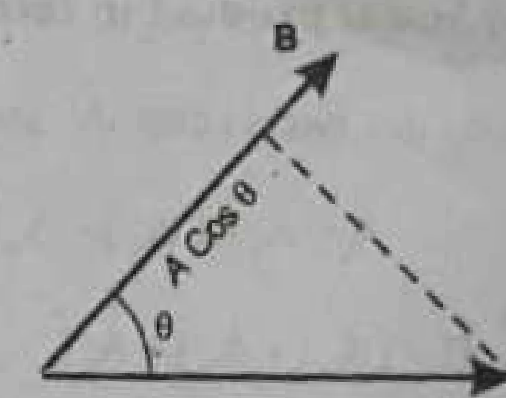


Fig. 2.10 (b)

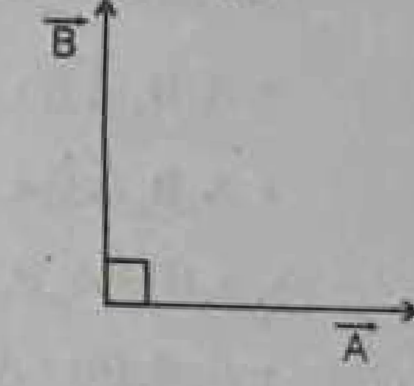
(ii) Perpendicular vectors

If two vectors are mutually perpendicular ($\theta = 90^\circ$) to each other then, their scalar product is *zero*. i.e.

$\vec{A}.\vec{B} = AB\cos 90^\circ$

$\vec{A}.\vec{B}$ = AB (0)

$\vec{A}.\vec{B}$ = 0

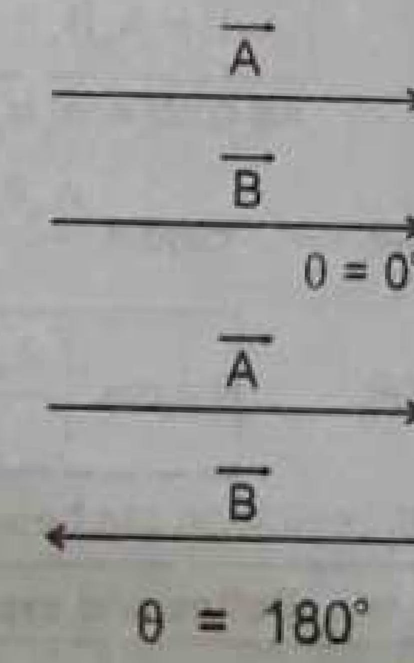


In case of units vectors,

$\hat{i}.\hat{j} = (1)(1)\cos 90^\circ = (1)(1)(0) = 0$

Similarly $\hat{j}.\hat{k} = 0$ and $\hat{k}\cdot\hat{i} = 0$

Thus $\boxed{\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0}$



(iii) Parallel and anti-vectors

If two vectors are parallel ($\theta = 0^\circ$) to each other then their scalar product is equal to the product of their magnitudes. i.e.

$\vec{A}.\vec{B} = AB\cos 0^\circ = AB(1) = AB$

This is the positive maximum value of scalar product.

If two vectors are anti-parallel ($\theta = 180^\circ$) then their scalar product is negative.

$\vec{A}.\vec{B} = AB\cos 180^\circ = AB(-1) = -AB$

This is the negative maximum value of scalar product.

> **Do You Know?**
> Magnitude of a vector $\vec{A}$ is $\sqrt{\vec{A}.\vec{A}}$

(iv) Self Scalar product

The self product of a vector is equal to square of its magnitude i.e.,

$\vec{A}.\vec{A} = AA\cos 0^\circ = A^2(1) = A^2$

In case of unit vectors,

$\hat{i}.\hat{i} = (1)(1)\cos 0^\circ = (1)(1)(1) = 1$

Similarly

$\hat{j}.\hat{j} = 1$ and $\hat{k}.\hat{k} = 1$

Thus $\boxed{\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1}$

> **For Your Information**
> Why K.E is scalar quantity even through velocity is a vector.
> $K.E = \frac{1}{2}mv^2 = \frac{1}{2}m(\vec{v}.\vec{v})$

(v) Scalar product in terms rectangular components

Consider two vector $\vec{A}$ and $\vec{B}$ in space, then

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \quad \text{and} \quad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A}.\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}).(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_xB_x(\hat{i}.\hat{i}) + A_xB_y(\hat{i}.\hat{j}) + A_xB_z(\hat{i}.\hat{k})$$
$$+ A_yB_x(\hat{j}.\hat{i}) + A_yB_y(\hat{j}.\hat{j}) + A_yB_z(\hat{j}.\hat{k})$$
$$+ A_zB_x(\hat{k}.\hat{i}) + A_zB_y(\hat{k}.\hat{j}) + A_zB_z(\hat{k}.\hat{k})$$
$$= A_xB_x(1) + A_xB_y(0) + A_xB_z(0)$$
$$+ A_yB_x(0) + A_yB_y(1) + A_yB_z(0)$$
$$+ A_zB_x(0) + A_zB_y(0) + A_zB_z(1)$$

$$\boxed{\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

$$AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

OR $$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

OR $$\boxed{\theta = \cos^{-1}\left(\frac{A_xB_x + A_yB_y + A_zB_z}{AB}\right)}$$

This equation can be used to find angle between two vectors.

Q.11 Define and explain vector product of two vectors? Give examples

Ans

Vector Product (Cross Product)

If the product of two vectors is a vector quantity then the product is called vector product.

The vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\boxed{\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}}$$

Where angle between $\vec{A}$ and $\vec{B}$ is θ and $\hat{n}$ is the unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$

Direction of vector product

The direction of vector product $\vec{A}\times\vec{B}$ can be found by right hand rule.

Right hand Rule

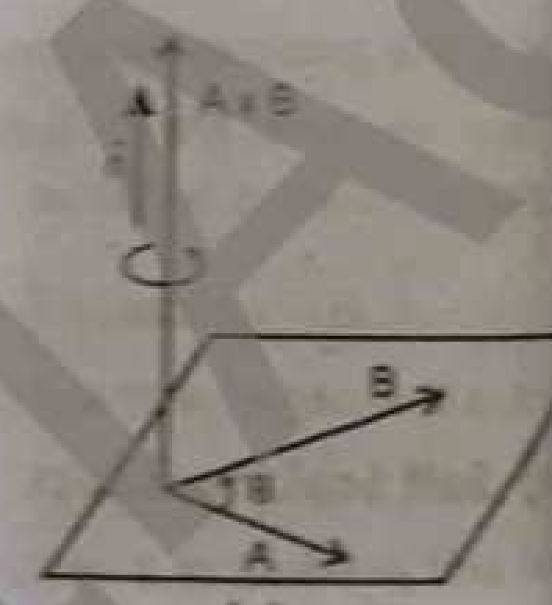
Join the tails of the two vectors to define a plane of vectors. Rotate vector $\vec{A}$ into $\vec{B}$ through smaller of the two possible angles. Curl the

What should You do?

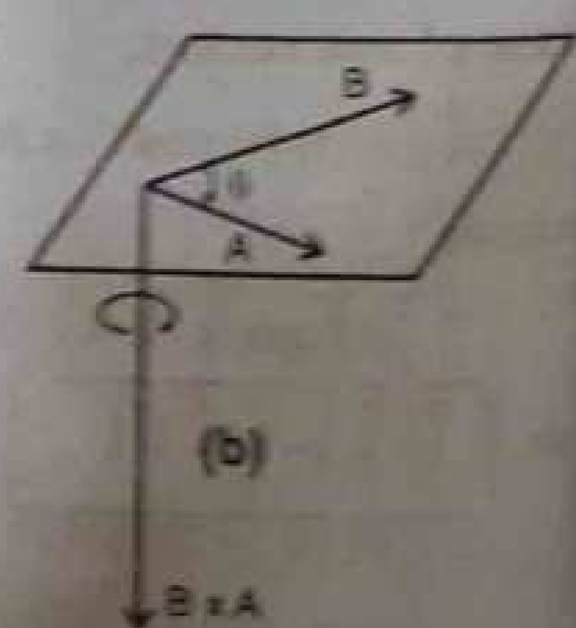
You are falling off the edge. What should you do to avoid falling?

EXPLANATION:

The boy is holding a heavy book as shown in the figure. To avoid falling, he is required to change the position of his body so that the line from his C.G may pass within the base area. Therefore, he must lean in back direction as shown in the figure.



(a)



(b)

fingers of the right hand in the direction of rotation. Erect thumb represents the direction of $\vec{A}\times\vec{B}$.

Examples

1) Torque is the vector product of position vector $\vec{r}$ and force $\vec{F}$ i.e. $[\vec{\tau} = \vec{r}\times\vec{F}]$
2) Force on a moving charged particle in magnetic field is vector product of velocity of charged particle and magnetic field $\vec{B}$ i.e. $[\vec{F} = q(\vec{v}\times\vec{B})]$
3) Angular momentum is vector product of position vector $\vec{r}$ and linear momentum $\vec{p}$ i.e. $[\vec{L} = \vec{r}\times\vec{p}]$

Q.12 Write down the characteristics of vector product of two vectors.

Ans

Characteristics of Vector Product

(i) Violation of Commutative law

The cross product of two vector $\vec{A}$ and $\vec{B}$ is not commutative.

If $\vec{A}$ and $\vec{B}$ be two vectors and θ is the angle between them. Then

$$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n} \quad \text{\_\_\_\_(1)}$$

and $$\vec{B}\times\vec{A} = BA\sin\theta(-\hat{n})$$

$$\vec{B}\times\vec{A} = AB\sin\theta(-\hat{n}) \quad \text{\_\_\_\_(2)} \quad [\text{since } AB = BA]$$

From equations (1) and (2)

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

OR $$\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$$

(ii) Perpendicular Vectors

The cross product of two perpendicular ($\theta = 90°$) vectors $\vec{A}$ and $\vec{B}$ has ***maximum*** magnitude. i.e.

$$\vec{A}\times\vec{B} = AB\sin 90°\,\hat{n}$$
$$\vec{A}\times\vec{B} = AB\,(1)\,\hat{n}$$
$$\vec{A}\times\vec{B} = AB\,\hat{n}$$

In case of units vectors

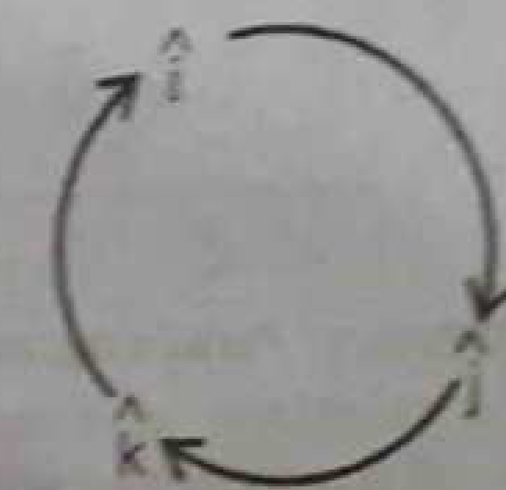
$$\hat{i}\times\hat{j} = (1)(1)\sin 90°\,\hat{k} = (1)(1)(1)\,\hat{k} = \hat{k}$$

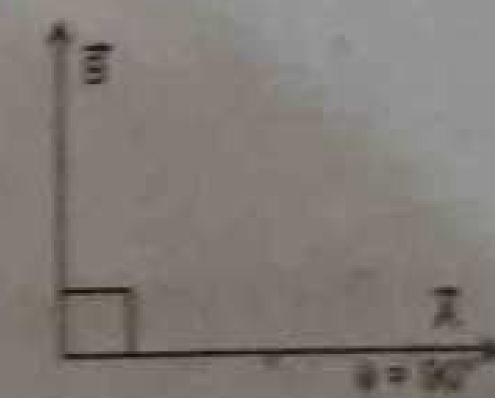
Similarly,

$$\hat{j}\times\hat{k} = \hat{i} \text{ and } \hat{k}\times\hat{i} = \hat{j}$$

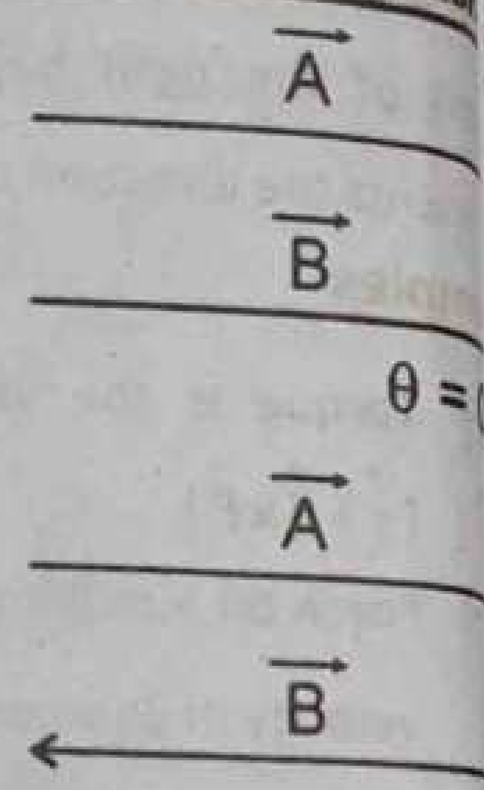
For Your Information

$\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ can be equal only when $\vec{A}$ and $\vec{B}$ are parallel or anti-parallel.





Hence



Note:-

$\hat{i}\times\hat{j}=\hat{k}$	$\hat{j}\times\hat{k}=\hat{i}$	$\hat{k}\times\hat{i}=\hat{j}$
$\hat{j}\times\hat{i}=-\hat{k}$	$\hat{k}\times\hat{j}=-\hat{i}$	$\hat{i}\times\hat{k}=-\hat{j}$

(iii) Parallel and Anti-parallel Vectors

The cross product of two parallel (θ = 0°) or two anti parallel (θ = 180°) vectors is a ***null*** vector. i.e.

In case of parallel vectors

$$\vec{A}\times\vec{B} = AB\sin 0^\circ\,\hat{n} = AB\,(0)\,\hat{n} = 0\,\hat{n} = \vec{O}$$

In case of anti – parallel vectors

$$\vec{A}\times\vec{B} = AB\sin 180^\circ\,\hat{n} = AB\,(0)\,\hat{n} = 0\,\hat{n} = \vec{O}$$

Why do you keep your legs far apart when you have to stand in the aisle of a bumpy-riding bus?

EXPLANATION:

When you stand in the aisle of a bumpy-riding bus, you are in unstable position and you may fall. To make you stable, you keep your legs far apart, so that your base area may increase and the line from C.G may pass within the base area, so no torque is produced.

(iv) Self Vector product

The self product of a vector $\vec{A}$ is ***null*** vector.

$$\vec{A}\times\vec{A} = AA\sin 0^\circ\,\hat{n} = AA\,(0)\,\hat{n} = 0\,\hat{n} = \vec{O}$$

In case of unit vectors,

$$\hat{i}\times\hat{i} = (1)\,(1)\sin 0^\circ\,\hat{n} = (1)\,(1)\,(0)\,\hat{n} = \vec{O}$$

Similarly

$$\hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{O}$$

So,

$\hat{i}\times\hat{i}=\vec{O}$	$\hat{j}\times\hat{j}=\vec{O}$	$\hat{k}\times\hat{k}=\vec{O}$

(v) Area of Parallelogram

The magnitude of cross product of two vectors represents the ***area of parallelogram*** formed with these vectors taken as its two adjacent sides.

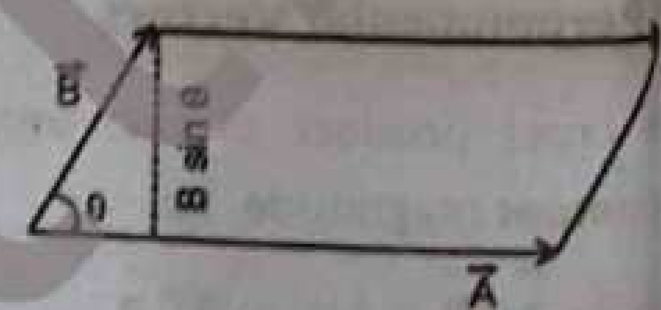
Area of parallelogram = (length) (height)

= (A) (B sinθ)

= AB sin θ

= magnitude of $(\vec{A}\times\vec{B})$

Area of parallelogram = $|\vec{A}\times\vec{B}|$



(vi) Vector product in Rectangular component Form

Consider two vector $\vec{A}$ and $\vec{B}$ in space, then

$$\vec{A} = A_x\hat{i}+A_y\hat{j}+A_z\hat{k} \quad\text{and}\quad \vec{B} = B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$$

$$\vec{A}\times\vec{B} = (A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\times(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$= A_xB_x(\hat{i}\times\hat{i})+A_xB_y(\hat{i}\times\hat{j})+A_xB_z(\hat{i}\times\hat{k})$$

$$+ A_yB_x(\hat{j}\times\hat{i})+A_yB_y(\hat{j}\times\hat{j})+A_yB_z(\hat{j}\times\hat{k})$$

$$+ A_zB_x(\hat{k}\times\hat{i})+A_zB_y(\hat{k}\times\hat{j})+A_zB_z(\hat{k}\times\hat{k})$$

$$= A_xB_x(\vec{O})+A_xB_y(\hat{k})+A_xB_z(-\hat{j})$$

$$+ A_yB_x(-\hat{k})+A_yB_y(\vec{O})+A_yB_z(\hat{i})$$

$$+ A_zB_x(\hat{j})+A_zB_y(-\hat{i})+A_zB_z(\vec{O})$$

$$= (A_yB_z - A_zB_y)\hat{i}+(A_zB_x - A_xB_z)\hat{j}+(A_xB_y - A_yB_x)\hat{k}$$

$$\boxed{\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i}+(A_zB_x - A_xB_z)\hat{j}+(A_xB_y - A_yB_x)\hat{k}}$$

This result can be written in determinant form as

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Q.13 Define and explain the term torque or moment of force.

Ans

For Your Information

Pivot Point or Axis of Rotation is the point or axis about which an object rotates. ***Torque*** is the tendency of force to produce a rotation

Torque

Definition

The turning effect of force produced in a body about an axis is called torque.

OR

The product of magnitude of force and the perpendicular distance from axis of rotation to line of action of force is called torque.

OR

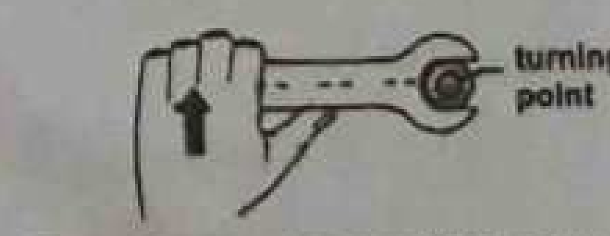
The moment of a force can also be defined as *the vector product of the radius vector from the axis of rotation to the point of application of the force and the force vector.*

Mathematical Form

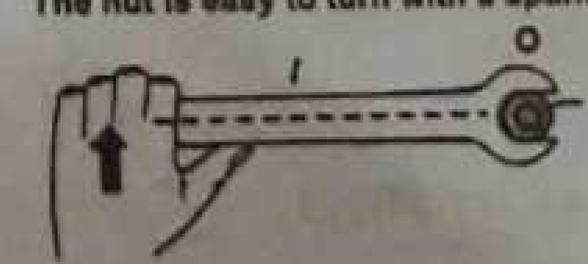
$$\boxed{\tau = \ell F}$$

Where ℓ = perpendicular distance.

And F = Magnitude of applied force



The nut is easy to turn with a spanner.



It is easier still if the spanner has a long handle.

Dependence of torque

Torque depends on the following factors;

1) Magnitude of *force*
2) Perpendicular distance from axis of rotation to line of action of force called *moment arm.*

Unit

The SI -unit of torque is newton meter (Nm) and its dimension is $\left[ML^2T^{-2}\right]$

For Your Information
Torque is important in the operation of ***electric motor*** which is used in vacuum cleaners, dishwashers, computers, printers, videocassette recorders, water pumping stations etc.

Examples of torque

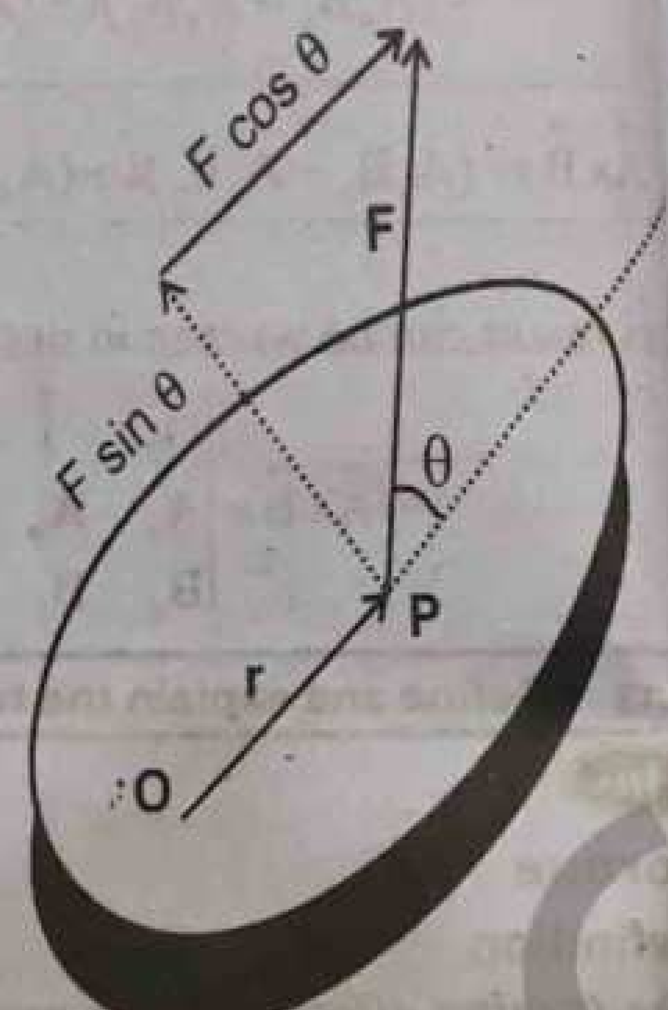
- Tightening of a nut with a spanner (wrench).
- A seesaw rotates on and off the ground due to torque imbalance

Q.14 Calculate the torque due force acting on a rigid body.

Ans.

Torque in a rigid body

Consider a rigid body as shown in figure. Let $\vec{F}$ is the force acting on the body at point P, $\vec{r}$ is position vector of P with respect to pivot O and θ is the angle between $\vec{F}$ and $\vec{r}$.



Resolution of force $\vec{F}$

Resolving the force into its rectangular components we have

Fsinθ = component of force perpendicular to $\vec{r}$

Fcosθ = component of force along the direction of $\vec{r}$.

Torque due to force about O

As the line of action of Fcosθ passes through point O, so the torque due to this component is zero. Hence torque due to force F is equal to the torque produced due to Fsinθ and is given by

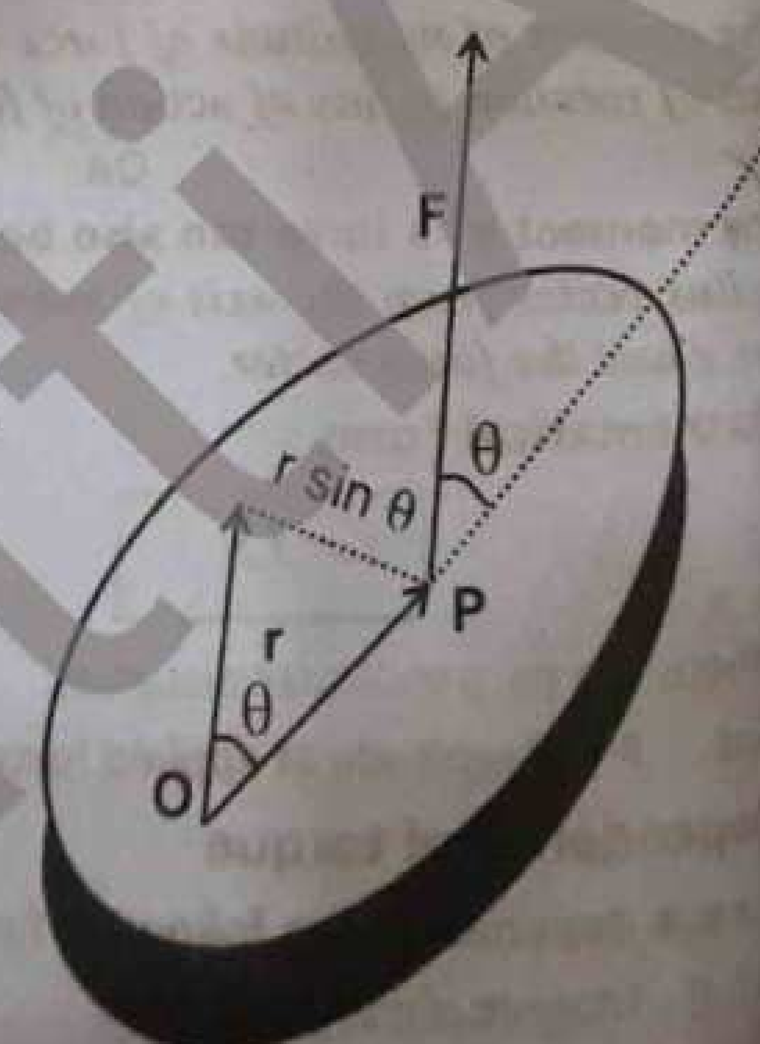
$$\tau = r\,(F\sin\theta)$$

OR $\tau = rF\sin\theta$

In vector form,

$$\vec{\tau} = rF\sin\theta\,\hat{n}$$

OR $\boxed{\vec{\tau} = \vec{r}\times\vec{F}}$



Alternatively

Resolving the position vector $\vec{r}$ in to its components we have

rsinθ = component of $\vec{r}$ perpendicular to $\vec{F}$

r cosθ = component of $\vec{r}$ along the direction of $\vec{F}$.

Torque due to force $\vec{F}$

In this case the torque is given by

$$\tau = \ell F$$

Where ℓ = r sinθ = moment arm

Thus

$$\tau = (r\sin\theta)F$$

$$\tau = rF\sin\theta$$

In vector form,

Or $\vec{\tau} = rF\sin\theta\,\hat{n}$

$$\boxed{\vec{\tau} = \vec{r}\times\vec{F}}$$

Direction of torque

Where $\hat{n}$ represents the direction of $\vec{\tau}$ and is perpendicular to the plane containing $\vec{r}$ and $\vec{F}$. It can be found by right hand rule.

Important Note

- Torque is the ***counter part*** of force for rotational motion.
- Torque plays the same role in angular motion as force plays in linear motion.
- Torque determines angular acceleration as force determines linear acceleration.
- Anti-clock wise torque is taken positive and clockwise torque is taken negative.

Can You Do ?

Stand with one arm and the side of one foot pressed against a wall. Can you raise the other leg side ways? If not, then why not?

EXPLANATION:
You cannot raise the other leg side ways, because in doing so, your base area will decrease and you will be in unstable equilibrium and you may fall down, because the line from C.G will fall outside the base area.

Q.15 What is value of torque if the body is at rest or rotating with uniform angular velocity?

Ans.

Torque acting on the body will be ***zero***,

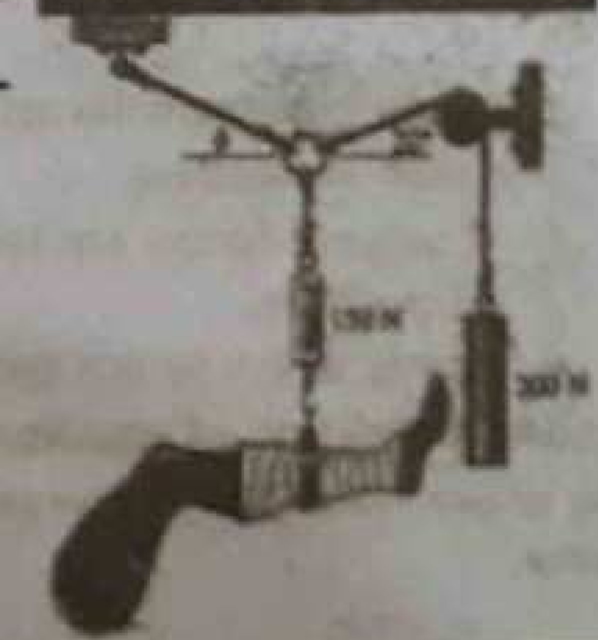
Reason

In this case angular acceleration is zero, so torque will be

$$\tau = I\alpha \qquad [2^{nd} \text{ law for rotational motion}]$$

$$\tau = I(0) = 0$$

Interesting Application



A concurrent force system is in equilibrium. The tension applied can be adjusted as desired.

EXPLANATION:
$T = T_{1y} + T_{2y}$
$T = T_1\sin\theta_1 + T_2\sin\theta_2$
The tension T depends on θ_1 and θ_2
The values of θ_1 and θ_2 can be adjusted by adjusting the leg.
Therefore, the tension T can be adjusted as desired.

Q.16 What is equilibrium? Give its types.

Ans.

Equilibrium

A body is said to be in equilibrium if it is at rest or moving with uniform velocity under the action of a number of forces.

Types of equilibrium

There are two types of equilibrium.

1. Static equilibrium

If a body is at ***rest***, it is said to be in static equilibrium.

Examples

Book lying on a table

2. Dynamic equilibrium

If a body is moving with ***uniform velocity***, it is said to be in dynamic equilibrium.

Examples

- A car moving with uniform linear velocity
- A body is rotating with uniform angular velocity
- Motion of a paratrooper

Q.17 State the two conditions of equilibrium.

Ans.

First condition of equilibrium (equilibrium of forces)

The vector sum of all the forces acting on a body must be null vector.

i.e. $\sum \vec{F} = \vec{O}$ ______ (1)

In case of coplanar force, 1st condition can be expressed as:

$\sum \vec{F}_x = \vec{O}$ ______ (2)

And $\sum \vec{F}_y = \vec{O}$ ______ (3)

Where

$\sum \vec{F}_x$ = sum of x-directed forces

$\sum \vec{F}_y$ = sum of y-directed forces

Note

- If the rightward forces are taken as positive then leftward forces are taken as negative.
- If upward forces are taken as positive, then downward forces are taken as negative.
- Forces which lie in a common plane are said to be coplanar.

Second Condition of equilibrium (equilibrium of torques)

The vector sum of all the torques acting on the body about an axis must be null vector.

i.e. $\sum \vec{\tau} = \vec{O}$

Q.18 Under what conditions the body is said to be in complete equilibrium?

Ans.

Translational equilibrium

When first condition is satisfied, the linear acceleration of body is zero and the body is said to be in translational equilibrium

Rotational equilibrium

When second condition is satisfied, angular acceleration of body is zero and the body is said to be in rotational equilibrium.

Thus for a body to be in complete equilibrium, both conditions must be satisfied. i.e. both ***linear*** acceleration and ***angular*** acceleration must be zero.

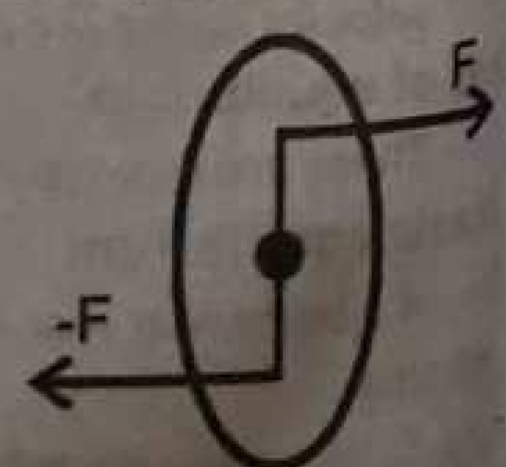
Can You Do?

With your nose touching the end of the door, put your feet astride the door and try to rise up on your toes.

EXPLANATION:

When you try to rise up on your toes, the line of C.G will fall outside the base area and you may fall.

Also in this case, line of action of our weight passes through the axis of rotation. Moment arm is zero, torque will also be zero. So we can not rise.



Note

1. We will apply the conditions of equilibrium to situations in which all the forces are coplanar.
2. To calculate to torque we choose an axis. The position of axis is arbitrary.
3. A most suitable place is one through which line of action of many forces pass.

Point to ponder

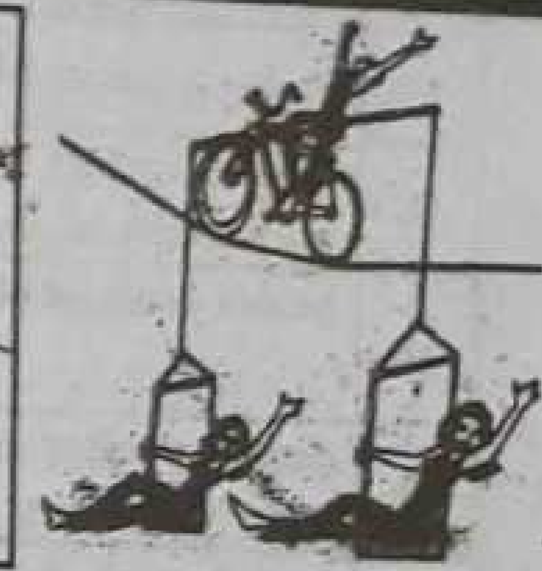
Do you think the rider in the figure is really in danger? What if persons below were removed?

EXPLANATION:

The rider in the above figure is not in danger because the line through the C.G passes through the base area. The rider will be in danger when the people below are removed because, now, the line from C.G may fall out of the base area.

Alternate:

If the persons are removed then effective center of gravity will raised so the raider is in more unstable condition.



FORMULAE

1	Commutative law for vector addition	$\vec{A}+\vec{B}=\vec{B}+\vec{A}$	
2	Subtraction of vectors	$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$	
3	Vector representation	$\vec{A}=A\hat{A}$	
4	Unit vector	$\hat{A}=\frac{\vec{A}}{A}$	
5	Null vector	$\vec{A}+(-\vec{A})=\vec{O}$	
6	Vector in terms of rectangular components	$\vec{A}=A_x\hat{i}+A_y\hat{j}$	
7	x-component of a vector $\vec{A}$	$A_x = A\cos\theta$	
8	y-component of a vector $\vec{A}$	$A_y = A\sin\theta$	
9	Magnitude of vector $\vec{A}$	$A^2 = A_x^2 + A_y^2$	$A=\sqrt{A_x^2+A_y^2}$
10	Direction of vector $\vec{A}$	$\theta=\tan^{-1}(\frac{A_y}{A_x})$	
11	Position vector of a point P(a, b) in plane	$\vec{r}=a\hat{i}+b\hat{j}$	
12	Position vector of a point P(a, b, c) in space	$\vec{r}=a\hat{i}+b\hat{j}+c\hat{k}$	

13	Magnitude of resultant $(\vec{R})$ of vectors $\vec{A}$ and $\vec{B}$	$R=\sqrt{(A_x+B_x)^2+(A_y+B_y)^2}$		
14	Direction of resultant $(\vec{R})$ of vectors $\vec{A}$ and $\vec{B}$	$\theta=\tan^{-1}(\frac{A_y+B_y}{A_x+B_x})$		
15	Scalar product of two vectors	$\vec{A}\cdot\vec{B}=AB\cos\theta$		
16	Scalar product of two perpendicular vectors	$\vec{A}\cdot\vec{B}=0$		
17	Scalar product of unit vectors $\hat{i},\hat{j}$ and $\hat{k}$	$\hat{i}\cdot\hat{j}=0$	$\hat{j}\cdot\hat{k}=0$	$\hat{k}\cdot\hat{i}=0$
18	Scalar product of two parallel vectors	$\vec{A}\cdot\vec{B}=AB$		
19	Scalar product of two anti-parallel vectors	$\vec{A}\cdot\vec{B}=-AB$		
20	Self dot product of vector $\vec{A}$	$\vec{A}\cdot\vec{A}=A^2$	$A=\sqrt{\vec{A}\cdot\vec{A}}$	
21	Self scalar product of unit vectors $\hat{i},\hat{j}$ and $\hat{k}$	$\hat{i}\cdot\hat{i}=1$	$\hat{j}\cdot\hat{j}=1$	$\hat{k}\cdot\hat{k}=1$
22	Scalar product in terms of rectangular components	$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$		
23	Angle between two vectors $\vec{A}$ and $\vec{B}$	$\theta=\cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$	$\theta=\cos^{-1}\left(\frac{A_xB_x+A_yB_y+A_zB_z}{AB}\right)$	
24	Projection of vector $\vec{A}$ on $\vec{B}$	$A\cos\theta=\frac{\vec{A}\cdot\vec{B}}{B}$	$A\cos\theta=\vec{A}\cdot\hat{B}$	
25	Projection of vector $\vec{B}$ on $\vec{A}$	$B\cos\theta=\frac{\vec{A}\cdot\vec{B}}{A}$	$B\cos\theta=\hat{A}\cdot\vec{B}$	
26	Vector product of two vectors	$\vec{A}\times\vec{B}=AB\sin\theta\cdot\hat{n}$		
27	Vector product of two perpendicular vectors	$\vec{A}\times\vec{B}=AB\hat{n}$		
28	Vector product of unit vectors $\hat{i},\hat{j}$ and $\hat{k}$	$\hat{i}\times\hat{j}=\hat{k}$	$\hat{j}\times\hat{k}=\hat{i}$	$\hat{k}\times\hat{i}=\hat{j}$
29	Vector product of two parallel or anti-parallel vectors	$\vec{A}\times\vec{B}=\vec{O}$		
30	Self cross product of vector $\vec{A}$	$\vec{A}\times\vec{A}=\vec{O}$		

31	Self Vector product of unit vectors $\hat{i},\hat{j}$ and $\hat{k}$	$\hat{i}\times\hat{i}=\vec{O}$	$\hat{j}\times\hat{j}=\vec{O}$	$\hat{k}\times\hat{k}=\vec{O}$
32	Vector product in terms of rectangular components	$\vec{A}\times\vec{B}=(A_yB_z-A_zB_y)\hat{i}+(A_zB_x-A_xB_z)\hat{j}+(A_xB_y-A_yB_x)\hat{k}$		
33	Angle between two vectors $\vec{A}$ and $\vec{B}$	$\theta=\sin^{-1}\left(\frac{\lvert\vec{A}\times\vec{B}\rvert}{AB}\right)$		
34	Area of a parallelogram and vector product	Area of a parallelogram $=\lvert\vec{A}\times\vec{B}\rvert=AB\sin\theta$		
35	Torque	$\tau=\ell F$	$\vec{\tau}=\vec{r}\times\vec{F}$	$\vec{\tau}=rF\sin\theta\,\hat{n}$
36	1st condition of equilibrium	$\sum\vec{F}=0$	$\sum\vec{F}_x=0$, $\sum\vec{F}_y=0$	
37	2nd condition of equilibrium	$\sum\vec{\tau}=0$		

Multiple Choice Questions

- *Four possible answers to each statement are given below. Tick (✓) the correct answer:*

1. Pick out the scalar quantity
   (a) power (b) torque
   (c) momentum (d) impulse
2. Addition of vectors obeys
   (a) commutative law (b) distributive law
   (c) associative law (d) all of these
3. Maximum number of components of a vector may be:
   (a) one (b) two
   (c) three (d) infinite
4. Two forces act together on an object. The magnitude of their resultant force is minimum when they act at
   (a) 180° (b) 90°
   (c) 45° (d) 0°
5. Magnitude of resultant vector of 6N and 8N which are perpendicular to each other is:
   (a) 14 N (b) 10 N
   (c) 20 N (d) 2 N
6. Two vectors are $\vec{A} = 3\hat{i} + 2\hat{j} - \hat{k}$, $\vec{B} = -3\hat{i} - 2\hat{j} + \hat{k}$, then
   (a) $\vec{B}$ is anti-parallel to $\vec{A}$ (b) $\vec{B}$ is negative vector of $\vec{A}$
   (c) Both a & b (d) $\vec{B}$ is perpendicular to $\vec{A}$
7. If a force of 5N is applied parallel to moment arm of 5m, then torque is
   (a) 25 Nm (b) 5 Nm
   (c) 10 Nm (d) 0 Nm
8. If $\vec{A}\cdot\vec{B} = 0$ then $\vec{A}\times\vec{B}$ = ______
   (a) 0 (b) $\vec{0}$
   (c) $A\hat{n}$ (d) $AB\hat{n}$
9. If $|\vec{F_1}| = |\vec{F_2}| = |\vec{F}|$, then angle between vectors is:
   (a) 0° (b) 30°
   (c) 120° (d) 180°
10. The magnitude of product vector $\vec{C}$ i.e., $\vec{A}\times\vec{B} = \vec{C}$ is equal to:
    (a) sum of adjacent sides of parallelogram (b) area of parallelogram

    (c) product of four sides (d) parameters of parallelogram
11. If magnitudes of scalar and vector products of two vectors are same, then angle between them is:
    (a) 30° (b) 45°
    (c) 60° (d) 180°
12. Reverse process of vector addition is called.
    (a) negative of a vector (b) subtraction of a vector
    (c) resolution of a vector (d) multiplication of a vector
13. When a vector is multiplied by a negative number, its direction
    (a) Does not change (b) Changes by 90°
    (c) Change by 270° (d) Changes by 180°
14. The minimum number of vectors of un-equal magnitudes, whose vector sum can be zero is
    (a) 1 (b) 2
    (c) 3 (d) 4
15. Two forces act together on an object the magnitude of their resultant is maximum when the angle between the forces is
    (a) 0° (b) 90°
    (c) 180° (d) 270°
16. If $\vec{A}\times\vec{B}$ points along x-axis then the vector $\vec{A}$ and $\vec{B}$ must be in
    (a) xy plane (b) yz - plane
    (c) xz plane (d) All of these
17. A vector in space has ______ rectangular components.
    (a) one (b) two
    (c) three (d) infinite
18. What is the angle that the given vector makes with y-axis? $\vec{A} = 2\hat{i} + \sqrt{12}\hat{j}$
    (a) 30° (b) 60°
    (c) 90° (d) 120°
19. If for two non-zero vectors $\vec{A}$ and $\vec{B}$, $\vec{A}\cdot\vec{B} = 0$, the vectors will be:
    (a) parallel (b) perpendicular
    (c) anti-parallel (d) $\theta = 60°$
20. Which pair contains one vector and one scalar?
    (i) Displacement; Acceleration (ii) Force; Kinetic energy
    (iii) Momentum; Velocity (iv) Power; Speed
    (a) only (i) (b) only (ii)
    (c) both (ii) and (iii) (d) only (iv)

ANSWERS

1. a	2. d	3. d	4. a	5. b	6. a	7. d	8. d	9. c	10. b
11. b	12. c	13. d	14. c	15. a	16. b	17. c	18. a	19. b	20. b

Short Questions of Exercise

Q.2.1 Define the terms (i) Unit vector (ii) Position vector and (iii) components of a vector.
(Lhr 2006, Fsd 2006, Grw 2004-2008, Bwp 2008, Mtn 2009, Grw 2010, Lhr 2010-...)

Ans.

(i) Unit Vector

A vector whose magnitude is ***one with no units*** in a given direction is called unit vector. It is represented by a letter with a cap or hat on it.

Mathematical Form

If $\vec{A}$ is a vector with magnitude A, then $\vec{A} = A\hat{A}$

OR $\boxed{\hat{A} = \frac{\vec{A}}{A}}$

(ii) Position Vector

The vector which represents the ***position*** of point or a particle with respect to fixed origin is called position vector. It is denoted by $\vec{r}$.

In two dimensional coordinate system (plane), the position of a point P (a,b) is represented by

$$\boxed{\vec{r} = a\hat{i} + b\hat{j}}$$

In three dimensional coordinate system (space), the position of a point P (a, b, c) is represented by

$$\boxed{\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}}$$

(iii) Components of a vector

The ***effective value*** of a vector in a given direction is called component of a vector. A vector may be split up into ***two or more*** components.

Q.2.2 The vector sum of three vectors gives a zero resultant. What can be the possible orientation of the vectors?
(D.G.Khan 2006, Fsd 20...)

Ans. If the three vectors are represented by the ***sides of triangle*** joined by head to tail rule, their sum will be zero.

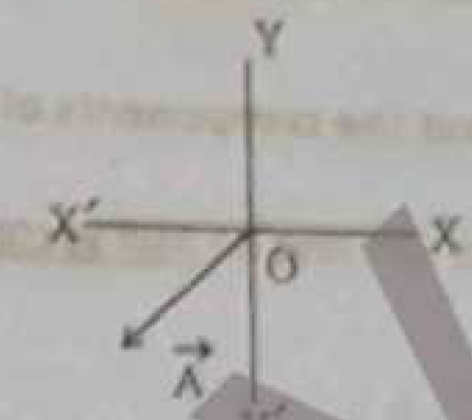
Explanation

Consider three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ as shown in figure. It is clear that sum of the vectors is zero because tail of the first vector ***coincides*** with the head of the last vector.

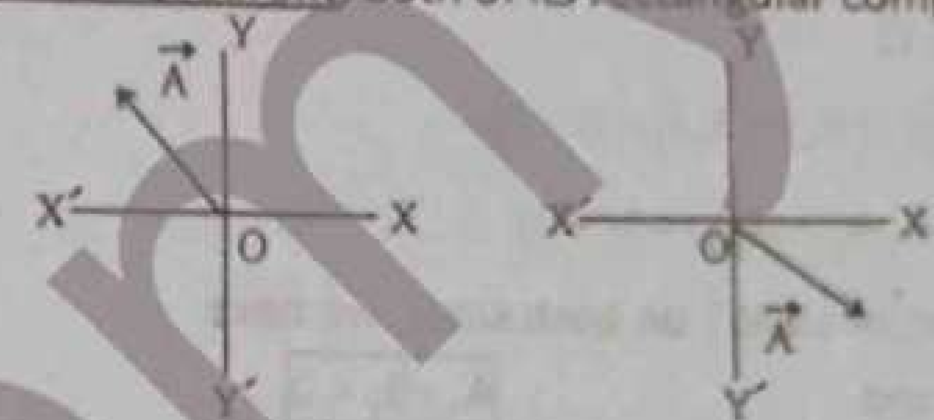
$$\vec{A} + \vec{B} + \vec{C} = \vec{O}$$

Q.2.3 Vector $\vec{A}$ lies in the xy-plane. (i) For what orientation will both the rectangular components be negative? (ii) For what orientation will its components have opposite signs?

Ans. (i) If the vector lies in ***3rd quadrant***, both of its rectangular components will be negative.



(ii) If the vector lies in ***2nd or 4th quadrant***, both of its rectangular components will have opposite signs.



Q.2.4 If one of the rectangular components of a vector is not zero, can its magnitude be zero? Explain.
(D.G.Khan 2004, Mir Pur 2004, Lhr 2009, Lhr 2010, Grw 2011)

Ans. ***No***, its magnitude can not be zero.

Reason

The magnitude of a $\vec{A}$ is given by

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

This equation shows that magnitude of the vector will be zero only when all of its rectangular components are ***zero.***

Q.2.5 Can a vector has components greater than the vector's magnitude?
(Rwp 2008, Lhr 2008, Mir Pur 2009, Grw 2009, Lhr 2011)

Ans. The rectangular component of a vector can ***never*** be greater than the vector's magnitude. It ***may equal*** to vector's magnitude. The component of a vector ***other than rectangular components*** may be greater than the magnitude of the vector.

Explanation

The magnitude of the vector $\vec{A}$ is given by

$$A = \sqrt{A_x^2 + A_y^2}$$

OR $A^2 = A_x^2 + A_y^2$

$\Rightarrow$ $A^2 \geq A_x^2$ OR $A^2 \geq A_y^2$

$\Rightarrow$ $A \geq A_x$ OR $A \geq A_y$

This shows that the magnitude of rectangular component can be equal or less than the vectors magnitude.

Q.2.6 Can the magnitude of a vector have a negative value? (Mir Pur 2004, Fsd 2008)

Ans. ***No***, it can never be negative.

Reason

By ***definition*** the magnitude of a vector quantity is a scalar quantity (a number) and is ***always*** positive (As it is measured by the length of the vector which can never be negative). The magnitude a vector $\vec{A}$ is,

$$A = \sqrt{A_x^2 + A_y^2}$$

In this case, we always take the ***positive square root.***

Q.2.7 If $\vec{A}+\vec{B}=\vec{O}$, **What can you say about the components of two vectors?** (D.G.Khan,

Ans. Sum of their respective components will also be ***null vector***.

Explanation

If $\vec{A}=A_x\hat{i}+A_y\hat{j}$ and $\vec{B}=B_x\hat{i}+B_y\hat{j}$

Then $\vec{A}+\vec{B}=\vec{O}$

$\Rightarrow \quad (A_x\hat{i}+A_y\hat{j})+(B_x\hat{i}+B_y\hat{j})=0\hat{i}+0\hat{j}$

OR $\quad (A_x+B_x)\hat{i}+(A_y+B_y)\hat{j}=0\hat{i}+0\hat{j}$

Comparing the coefficient of $\hat{i}$ and $\hat{j}$ on both sides, we have

$\boxed{A_x+B_x=0}$ and $\boxed{A_y+B_y=0}$

In vector form

$\vec{A}_x+\vec{B}_x=\vec{O}$ and $\vec{A}_y+\vec{B}_y=\vec{O}$

Q.2.8 Under what circumstances would a vector has components that are equal in magnitude? (Grw

Ans. It is possible only when the vector makes an angle of ***45° with x-axis***.

Proof

Let A_x and A_y be the rectangular components of vector $\vec{A}$

If $\quad A_y = A_x$

OR $\quad A\sin\theta = A\cos\theta$

OR $\quad \sin\theta = \cos\theta$

OR $\quad \dfrac{\sin\theta}{\cos\theta}=1$

OR $\quad \tan\theta = 1$

OR $\quad \theta=\tan^{-1}(1)$

$\Rightarrow \quad \boxed{\theta = 45°}$

Q. 2.9 Is it possible to add a vector quantity to a scalar quantity? Explain. (Sgd 2005, Bwp 2008, Mtn

Ans. ***No***, it is not possible.

Reason

Both physical quantities are ***different*** in their physical nature, scalars have magnitude only vectors have magnitudes as well as direction.

Scalars can be added by simple arithmetic rules while the vectors can be added by special rules (ve algebra).

Q.2.10 Can you add zero to a null vector? (Grw 2005, Bwp 2008, Lhr 2008, Lhr 2009, Grw

Ans. ***No***, it is not possible.

Reason

Both zero and null vector are two ***different*** physical quantities, one is scalar and the other is vecto we can not add zero into a null vector.

Q.2.11 Two vectors have unequal magnitudes. Can their sum be equal to zero? Explain.

(Sgd 2003, Mir Pur 2006-2009, Mtn 2009, Lhr 2010-2011, G

Ans. ***No***, their sum can not be zero.

Explanation

Their sum can be zero only if the two vectors have ***same*** magnitude and ***opposite*** direction.

Q.2.12 Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of the same length?

Ans. Consider two vectors $\vec{A}$ and $\vec{B}$ as shown in figure.

As $\quad$ A=B and angle between two vector is 90°. [Given]

By using head to tail rule,

$\vec{R}=\vec{A}+\vec{B}$ and $\vec{R}'=\vec{A}-\vec{B}$

Proof

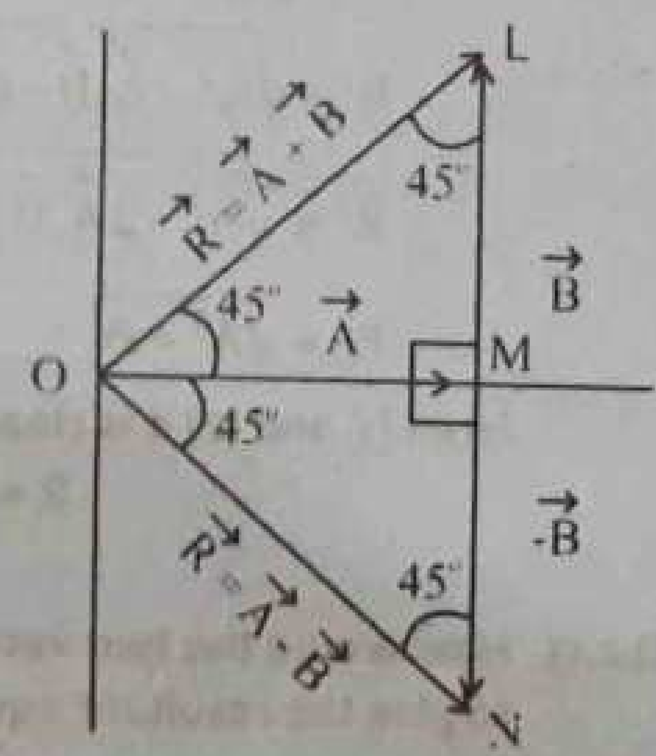
Magnitude of $\vec{R}$ and $\vec{R}'$

$R=\sqrt{(A)^2+(B)^2}=\sqrt{A^2+B^2}$ ——— (1)

$R'=\sqrt{(A)^2+(-B)^2}=\sqrt{A^2+B^2}$ ——— (2)

From equations (1) & (2), it is clear

$R=R'$ ——— (3)



Angle between $\vec{R}$ and $\vec{R}'$

Since A = B

From figure

$\angle LOM = \angle NOM = 45°$

Therefore,

$\angle LON = \angle LOM + \angle NOM = 45° + 45° = 90°$

So $\vec{R}$ and $\vec{R}'$ are perpendicular to each other.

Hence proved.

Alternate Method

Angle between $\vec{R}$ and $\vec{R}'$

(b) $(\vec{A}+\vec{B}).(\vec{A}-\vec{B})$

$=\vec{A}.\vec{A}-\vec{A}.\vec{B}+\vec{B}.\vec{A}-\vec{B}.\vec{B}$

$=A^2-\vec{A}.\vec{B}+\vec{A}.\vec{B}-B^2 \qquad [\because \vec{A}.\vec{B}=\vec{B}.\vec{A}]$

$=A^2-B^2 \qquad (\because \vec{A}.\vec{B}=0 \;\; as\, \vec{A}\perp\vec{B})$

$=A^2-A^2 \qquad [\because A=B]$

$=0$

So $\quad \vec{R}'$ is perpendicular to $\vec{R}$

Magnitude of $\vec{R}$ and $\vec{R'}$

(a)

As $R = \sqrt{\vec{R}.\vec{R}}$

$$R = \sqrt{(\vec{A}+\vec{B}).(\vec{A}+\vec{B})}$$

$$R = \sqrt{\vec{A}.\vec{A}+\vec{A}.\vec{B}+\vec{B}.\vec{A}+\vec{B}.\vec{B}}$$

$$R = \sqrt{A^2+2\vec{A}.\vec{B}+B^2}$$

$$R = \sqrt{A^2+B^2} \quad \_\_\_\_(1) \qquad (\because \vec{A}.\vec{B}=0 \quad as\ \vec{A}\perp\vec{B})$$

And $R' = \sqrt{\vec{R'}.\vec{R'}}$

$$R' = \sqrt{(\vec{A}-\vec{B}).(\vec{A}-\vec{B})}$$

$$R' = \sqrt{A^2-\vec{A}.\vec{B}-\vec{B}.\vec{A}+B^2}$$

$$R' = \sqrt{A^2-2\vec{A}.\vec{B}+B^2}$$

$$R' = \sqrt{A^2+B^2} \quad \_\_\_\_\_\_\_\_(2) \qquad (\because \vec{A}.\vec{B}=0 \quad as\ \vec{A}\perp\vec{B})$$

From (1) and (2) it is clear that

$$R=R'$$

(Federal 2003, D.G.Khan 2005, Fsd 2...)

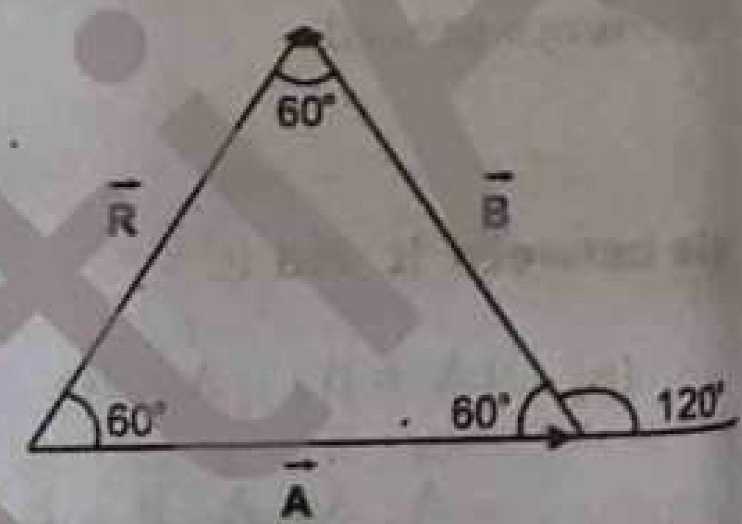
Q.2.13 How would the two vectors of the same magnitude have to be oriented, if they were to be combined to give the resultant equal to a vector of the same magnitude?

(Federal 2...)

Ans. It is possible only when the angle between two vectors is ***120°***

Explanation

If the two vectors are represented by two sides of an ***equilateral*** triangle, then the third side represents their resultant such that $A=B=R$ as shown in figure. In this case the angle between two vectors is 120°.



Q.2.14 The two vectors to be combined have magnitudes 60N and 35N. Pick the correct answer from those given below and tell why it is the only one of the three that is correct.

(i) 100N **(ii) 70N** **(iii) 20N**

Ans. The correct answer is ***70 N***

Reason

Sum of two vectors is maximum, if they are ***parallel*** to each other.

i.e 60 N + 35 N = 95N

Sum of two vectors is minimum, if they are ***opposite*** to each other.

i.e 60 N + (- 35) N = 25N

This shows that range of resultant is from 25N to 95N i.e., it must not be less than 25 and greater than 95. So the values 20N and 100N are disqualified. Thus the correct answer is 70N (which is within range).

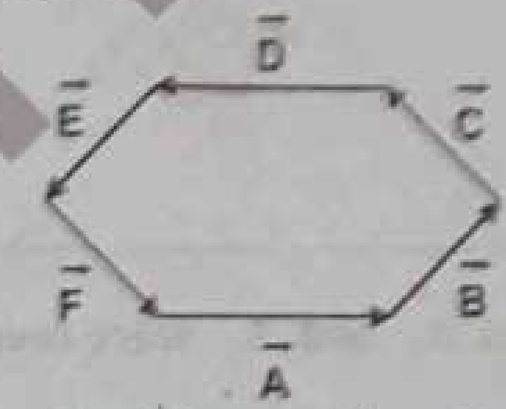
Q.2.15 Suppose the sides of closed polygon represent vector arranged head to tail. What is the sum of these vectors?

Ans. The sum of these vectors will be ***zero***

Reason

In this case, the head of the last vector ***coincides*** with the tail of the first vector as shown in figure. So, the resultant is zero.

$$\vec{A}+\vec{B}+\vec{C}+\vec{D}+\vec{E}+\vec{F}=\vec{0}$$



Q.2.16 (a) Two ships X and Y are traveling in different directions at equal speeds. The actual direction of motion of X is due north but to an observer on Y, the apparent direction of motion of X lies north-east. The actual direction of motion of Y as observed from the shore will be

(a) East (b) West (c) South-East (d) South-West.

Ans. (b) The actual direction of motion will be due ***west.***

Explanation

Let $\vec{v}_x$ = velocity of ship X

$\vec{v}_y$ = velocity of ship Y

$\vec{v}_x - \vec{v}_y$ = velocity of ship X relative to ship Y

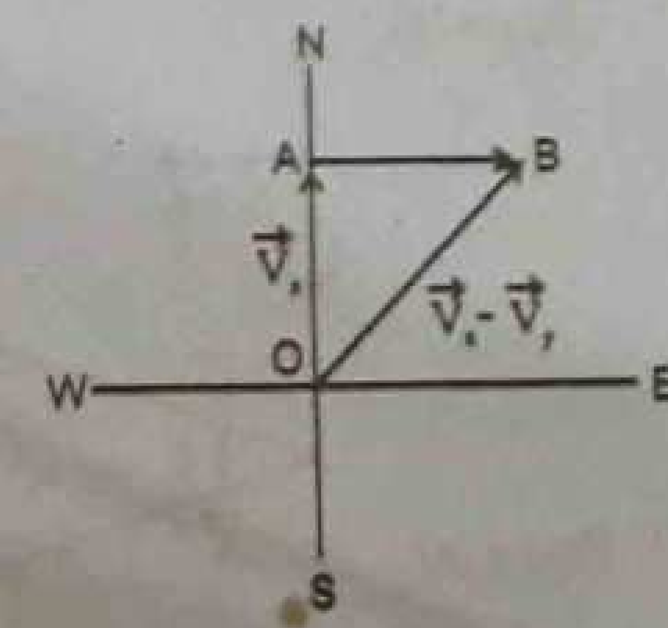
Or $\vec{OA}+\vec{AB}=\vec{OB}$

$\vec{v}_x+\vec{AB}=\vec{v}_x-\vec{v}_y$

Or $\vec{AB}=-\vec{v}_y$

This shows that $\vec{v}_y$ is directed opposite to $\vec{AB}$

i.e. due west



Q.2.16 (b)A horizontal force F is applied to a small object P of mass m at rest on the smooth plane inclined at an angle θ to the horizontal as shown in figure. The magnitude of the resultant force acting up and along the surface of the plane, on the object is

(a) F cosθ – mg sinθ

(b) F sinθ – mg cosθ

(c) F cosθ + mg cosθ

(d) F sinθ - mg sinθ

(e) mg tanθ

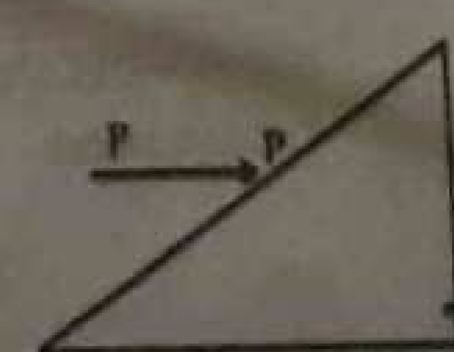


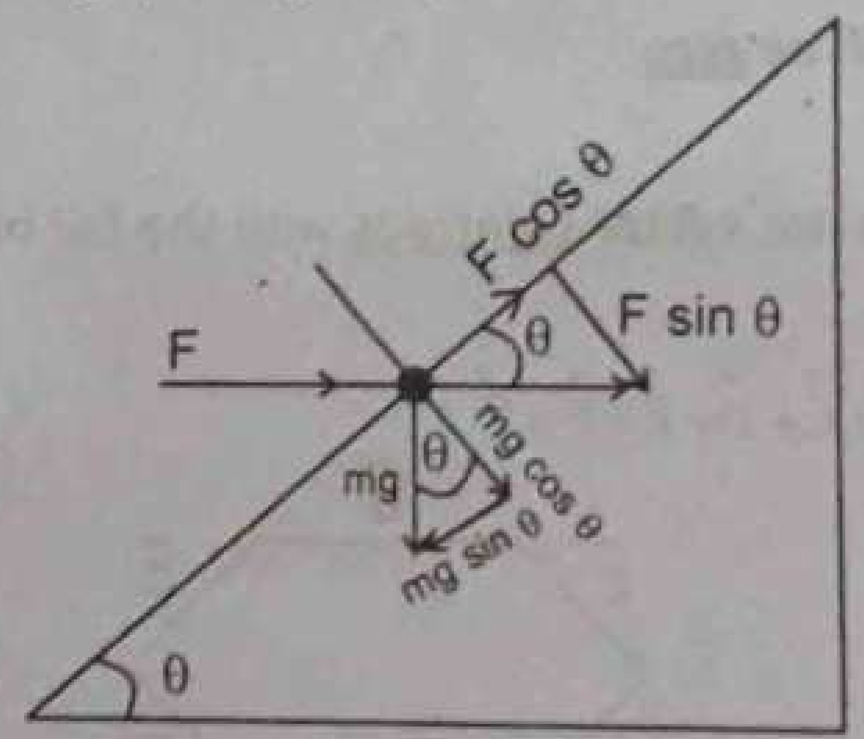
Fig. 2.22

Ans. *(a)* $F\cos\theta - mg\sin\theta$.

Explanation

Resolving $\vec{F}$ and $\vec{W}$ into rectangular components along perpendicular the inclined plane, we have

$F\cos\theta - mg\sin\theta$ = net force acting up along the plane.



Q.2.17 If all the components of the vectors $\vec{A}_1$ and $\vec{A}_2$ were reversed, how would this alter $\vec{A}_1 \times \vec{A}_2$?

(Mir Pur 2005, Lhr 2006, Bwp 2007, Grw 2…)

Ans. It would ***not change*** in this case.

Explanation

We know that direction of $\vec{A}_1 \times \vec{A}_2$ is perpendicular to the plane containing $\vec{A}_1$ and $\vec{A}_2$ as shown in figure (a)

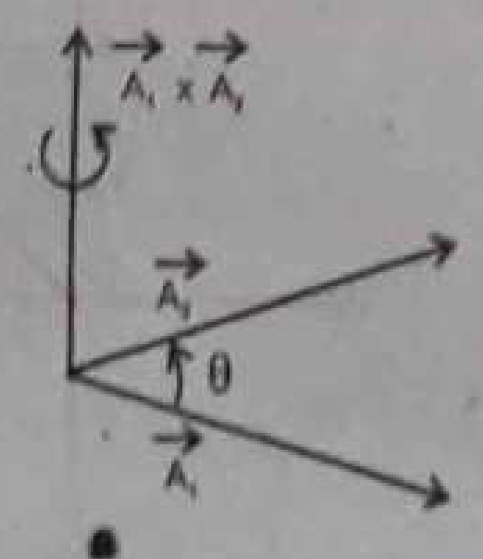


Figure (a)

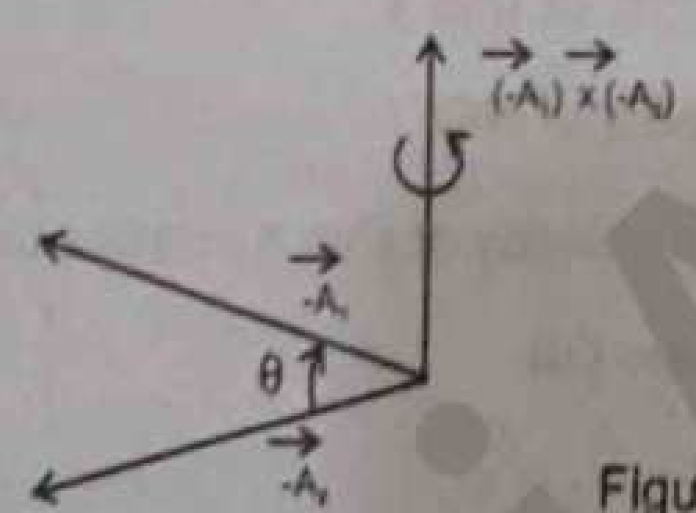


Figure (b)

Now, if all the components of vectors $\vec{A}_1$ and $\vec{A}_2$ are reversed (i.e. if we take negative of vectors $\vec{A}_1$ and $\vec{A}_2$), then again the direction of $(-\vec{A}_1) \times (-\vec{A}_2)$ remains the ***same*** as shown in figure (b).

i.e $\quad \vec{A}_1 \times \vec{A}_2 = (-\vec{A}_1) \times (-\vec{A}_2)$

Alternate Method

It would ***not change*** in this case.

Explanation

Let $\quad \vec{A}_1 = A_{1x}\hat{i} + A_{1y}\hat{j} + A_{1z}\hat{k}$

and $\quad \vec{A}_2 = A_{2x}\hat{i} + A_{2y}\hat{j} + A_{2z}\hat{k}$

Then, $\vec{A}_1 \times \vec{A}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1x & A_1y & A_1z \\ A_2x & A_2y & A_2z \end{vmatrix}$

when components are reversed i.e.,

$\vec{A}_1' = -A_{1x}\hat{i} - A_{2y}\hat{j} - A_{1z}\hat{k}$

and $\quad \vec{A}_2' = -A_{2x}\hat{i} - A_{2y}\hat{j} - A_{2z}\hat{k}$

then $\vec{A}_1' \times \vec{A}_2' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -A_{1x} & -A_{2y} & -A_{1z} \\ -A_{2x} & -A_{2y} & -A_{2z} \end{vmatrix}$

$= (-1)(-1)\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$.

So $\quad \vec{A}_1 \times \vec{A}_2 = \vec{A}_1' \times \vec{A}_2'$

Q.2.18 Name the three different conditions that could make $\vec{A}_1 \times \vec{A}_2 = \vec{O}$? **(Mtn 2006, Bwp 2006)**

Ans. If $\vec{A}_1$ and $\vec{A}_2$ are two vectors then

$\vec{A}_1 \times \vec{A}_2 = A_1 A_2 \sin\theta\,\hat{n}$.

Conditions

$\vec{A}_1 \times \vec{A}_2$ is zero if

1) $\vec{A}_1$ or $\vec{A}_2$ is a ***null*** vector.
2) $\vec{A}_1$ and $\vec{A}_2$ are ***parallel***. [i.e., $\theta = 0^\circ$]
3) $\vec{A}_1$ and $\vec{A}_2$ are ***anti-parallel***. [i.e., $\theta = 180^\circ$]

Q.2.19 Identify true or false statements and explain the reason

(a) A body in equilibrium implies that it is neither moving nor rotating.

(b) If coplanar forces acting on a body form a closed polygon, then the body is said to be in equilibrium

Ans. (a) This statement is ***false.***

Reason

Because in dynamic equilibrium body may ***move or rotate*** with uniform velocity.

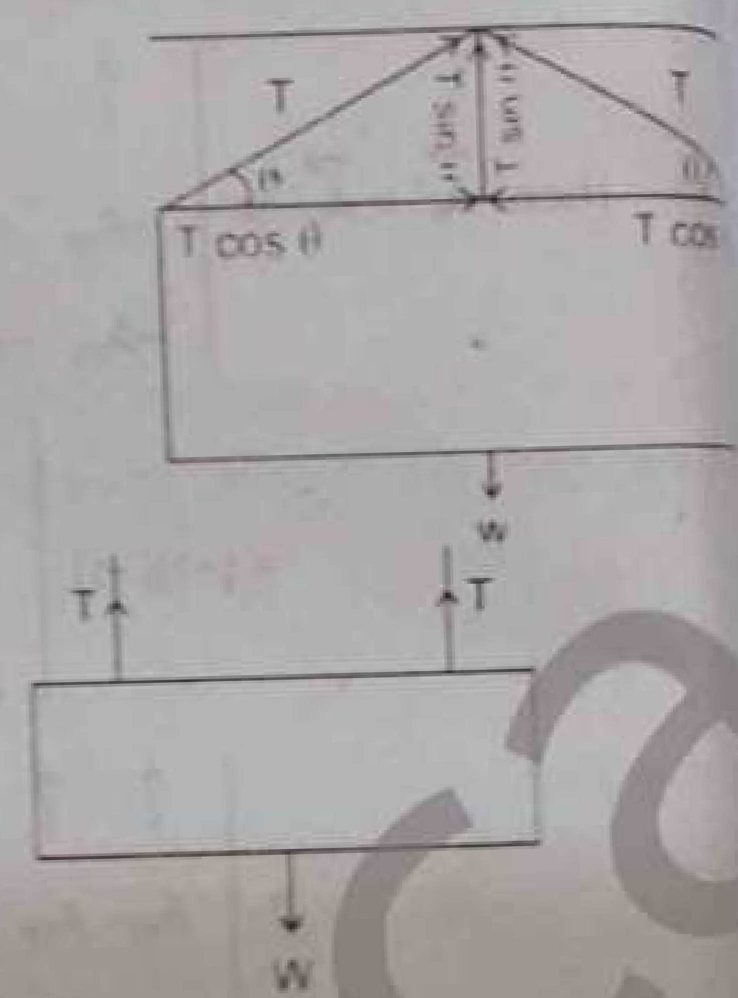
(b) The second statement is ***true***

Reason

In this case ***1st condition*** of equilibrium is satisfied and the body is said to be in ***translatory*** equilibrium.

Q.2.20 A picture is suspended from a wall by two strings. Show by diagram the configuration of the strings for which the tension in the string will be minimum.

Ans. Let the picture is suspended from wall by two strings, as shown in figure. Resolve the tension into rectangular components.



$\sum \vec{F} = 0$ gives

$T\sin\theta + T\sin\theta = W$

$2T\sin\theta = W$

$T = \dfrac{W}{2\sin\theta}$

Tension will be minimum if sinθ is maximum

That is

$\sin\theta = 1$

$\Rightarrow \quad \theta = 90^\circ$

So $\quad T = \dfrac{W}{2\sin 90^\circ}$

OR $\quad T = \dfrac{W}{2}$

Thus, the tension will be minimum if strings are ***vertical***

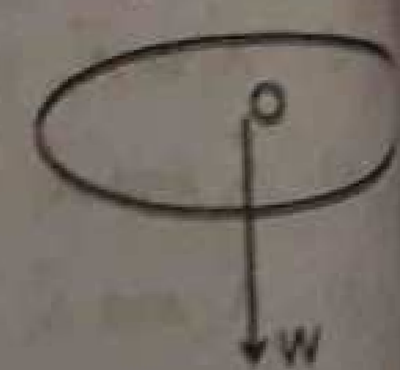
Q.2.21 Can a body rotate about its center of gravity under the action of its weight?

(Federal 2003, Mtn 2004-2005, Rwp 2005, Bwp 2006, Lhr 2006, Fsd 2008, Lhr 2009, Grw...)

Ans. ***No***, it is not possible

Reason

In this case, the line of action of force (weight) passes through ***pivot point*** (center of gravity), so moment arm becomes ***zero***.



As $\quad \tau = \ell F$

So $\quad \tau = (0)(F) \quad$ (as $\ell = 0$)

OR $\quad \tau = 0$

Hence the torque will also be zero.

Ans No a body cannot be rotated about its center of gravity under action of weight because the line of action of force passes through pivot or center of gravity that is $\ell = 0$ So, $\tau = \ell F \Rightarrow \tau = (0)F \Rightarrow \tau = 0$ So torque is zero

Solved Examples

Example 2.1

The positions of two aeroplanes at any instant are represented by two points A (2,3,4) and B (5,6,7) from an origin O in km as shown in Fig.

(i) What are their position vectors?

(ii) Calculate the distance between the two aeroplanes.

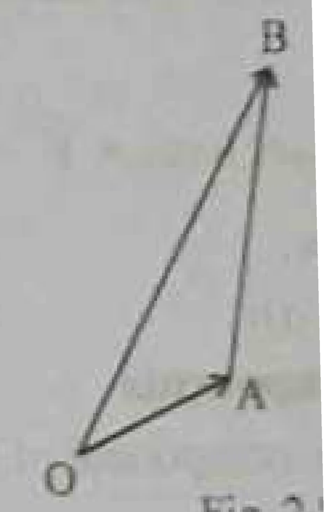


Fig. 2.1

Given Data:

A (2,3,4).

B (5,6,7).

To Find:

(i) Position vector of first aeroplane $= \vec{r}_A = ?$

Position vector of second aeroplane $= \vec{r}_B = ?$

(ii) Distance between the two aeroplane $= r = ?$

Calculation:

(i) As position vector $\vec{r}$ in three dimensional space is given by

$$\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$$

Thus position vector of first aeroplane A is

$$\overrightarrow{OA} = \vec{r}_A = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

and position vector of second aeroplane B is

$$\overrightarrow{OB} = \vec{r}_B = 5\hat{i} + 6\hat{j} + 7\hat{k}$$

(ii) According to head-to tail rule,

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

or $\quad \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

or $\quad \vec{r} = \overrightarrow{AB} = \vec{r}_B - \vec{r}_A$

$$\vec{r} = (5\hat{i} + 6\hat{j} + 7\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{r} = 5\hat{i} + 6\hat{j} + 7\hat{k} - 2\hat{i} - 3\hat{j} - 4\hat{k}$$

$$\vec{r} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

Magnitude of vector $\overrightarrow{AB}$ is the distance between the positions of two aeroplanes, therefore,

$$r = AB = \sqrt{(3)^2 + (3)^2 + (3)^2}$$

$$r = \sqrt{9+9+9} = \sqrt{27}$$

$$\boxed{r = 5.2\ \text{km}}$$

Example 2.2

Two forces of magnitude 10 N and 20 N act on a body in directions making angle is 30° ... respectively with x-axis. Find the resultant force.

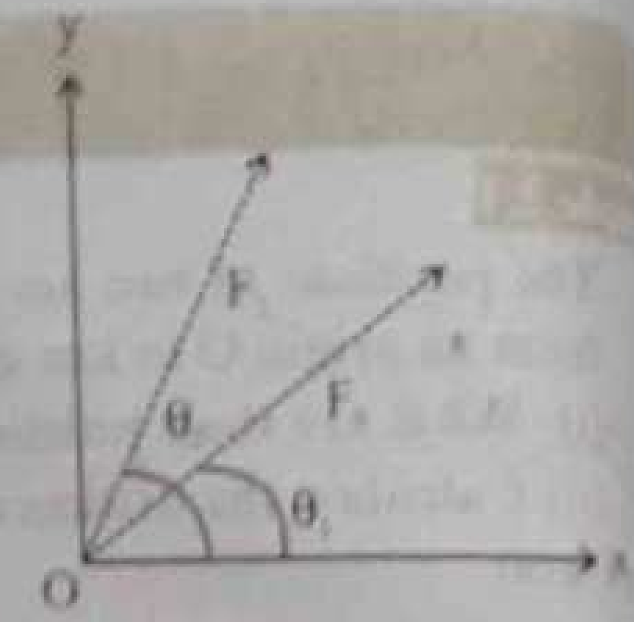
Given Data:

Magnitude of first force $= F_1 = 10N$

Magnitude of second force $= F_2 = 20N$

Angle of first force $= \theta_1 = 30^\circ$

Angle of second force $= \theta_2 = 60^\circ$



To Find:

Resultant force $= \vec{F} = ?$

Calculations:

Step (i):

x-components

The x-component of first force $= F_{1x} = F_1 \cos\theta_1$

$= 10 \cos 30^\circ = 10 \times 0.866 = 8.66$ N

The x-component of second force $= F_{2x} = F_2 \cos\theta_2$

$= 20 \times \cos 60^\circ = 20 \times 0.5 = 10$ N

y-components

The y-component of the first force $= F_{1y} = F_1 \sin\theta$

$= 10 \times \sin 30^\circ = 10 \times 0.5 = 5N$

The y-component of the second force $= F_{2y} = F_2 \sin\theta_2$

$= 20 \times \sin 60^\circ = 20 \times 0.866 = 17.32$ N

Step (ii):

The magnitude of x-component F_x of the resultant force $\vec{F}$

$$F_x = F_{1x} + F_{2x}$$
$$F_x = 8.66 + 10 = 18.66N$$

Step (iii):

The magnitude of y – component F_y of the resultant force $\vec{F}$

$$F_y = F_{1y} + F_{2y}$$
$$F_y = 5 + 17.32 = 22.32 \text{ N}$$

Step (iv):

The magnitude F of the resultant force $\vec{F}$ is given by.

$$F = \sqrt{F_x^{\,2} + F_y^{\,2}}$$
$$F = \sqrt{(18.66)^2 + (22.32)^2}$$
$$F = \sqrt{348.2 + 498.2}$$
$$F = \sqrt{846.4}$$
$$\boxed{F = 29N}$$

Step (v):

Direction of the resultant force $\vec{F}$ is given by.

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{22.32}{18.66}\right)$$
$$\theta = \tan^{-1}(1.196)$$
$$\boxed{\theta = 50^\circ}$$

Example 2.3

Find the angle between two forces of equal magnitude when the magnitude of their resultant is also equal to the magnitude of either of these forces.

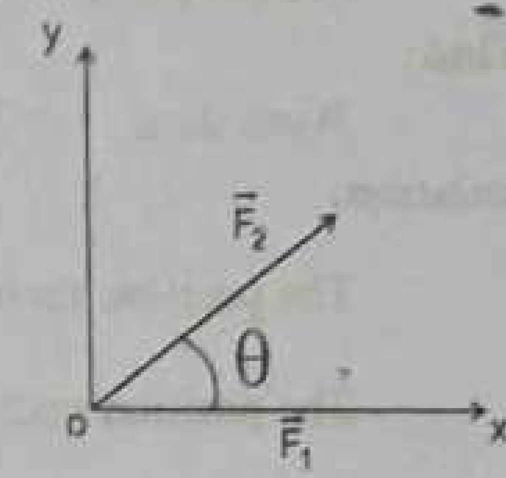
Given Data:

Let $\vec{F_1}$ and $\vec{F_2}$ are the given forces and $\vec{R}$ is their resultant.

$$|\vec{F_1}| = |\vec{F_2}| = |\vec{R}| = F \text{ (say)}$$

Let angle of 1[st] force $= \theta_1 = 0^\circ$

Angle of 2[nd] force $= \theta_2 = \theta$



To Find:

Angle between forces $= \theta = ?$

Calculation:

The x-component of the resultant is:

$$F_x = F_{1x} + F_{2x}$$
$$F_x = F_1\cos\theta_1 + F_2\cos\theta_2$$

or $F_x = F_1\cos 0^0 + F_2\cos\theta$

or $F_x = F_1 + F_2\cos\theta$

Similarly y–component of the resultant is:

$$F_y = F_{1y} + F_{2y}$$
$$F_y = F_1\sin\theta_1 + F_2\sin\theta_2$$
$$F_y = F_1\sin 0^0 + F_2\sin\theta$$

$F_y = F_2\sin\theta$ As $\sin 0^0 = 0$

Now the magnitude of the resultant is:

$$R = \sqrt{Fx^2 + Fy^2}$$

putting values

$$R = \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2}$$

Or $F^2 = (F + F\cos\theta)^2 + (F\sin\theta)^2$ As $R = F_1 = F_2 = F$

$F^2 = F^2 + F^2\cos^2\theta + 2F^2\cos\theta + F^2\sin^2\theta$

$F^2 = F^2 + 2F^2\cos\theta + F^2(\sin^2\theta + \cos^2\theta)$

Thus $F^2 = F^2 + 2F^2\cos\theta + F^2$ As $\sin^2\theta + \cos^2\theta = 1$

$0 = 2F^2\cos\theta + F^2$

Or $2F^2\cos\theta = -F^2$

Or $\cos\theta = -\frac{1}{2}$

Or $\cos\theta = (-0.5)$

Or $\theta = \cos^{-1}(-0.5)$

$$\boxed{\theta = 120^\circ}$$

Thus the angle between two equal forces is 120°.

Example 2.4

A force $\vec{F} = 2\hat{i} + 3\hat{j}$ units, has its point of application moved from point A(1, 3) to the point B(5, 7). Find the work done.

Given Data:

Force $= \vec{F} = 2\hat{i} + 3\hat{j}$ units

First point $= A(1,3)$

Second point $= B(5,7)$

To Find:

Work done $= W = ?$

Calculation:

The position vector for point A is: $\vec{r}_A = \hat{i} + 3\hat{j}$

The position vector for point B is: $\vec{r}_B = 5\hat{i} + 7\hat{j}$

Now,
$$\vec{d} = \vec{r}_B - \vec{r}_A$$
$$\vec{d} = (5\hat{i} + 7\hat{j}) - (\hat{i} + 3\hat{j})$$
$$\vec{d} = 5\hat{i} + 7\hat{j} - \hat{i} - 3\hat{j}$$
$$\vec{d} = 4\hat{i} + 4\hat{j}$$

Thus,

Work done $= W = \vec{F} \cdot \vec{d}$
$$W = (2\hat{i} + 3\hat{j}) \cdot (4\hat{i} + 4\hat{j})$$
$$W = 8 + 12$$
$$\boxed{W = 20 \text{ units}}$$

Example 2.5

Find the projection of vector $\vec{A} = 2\hat{i} - 8\hat{j} + \hat{k}$ in the direction of the vector $\vec{B} = 3\hat{i} - 4\hat{j} - 12\hat{k}$.

Given Data:

$\vec{A} = 2\hat{i} - 8\hat{j} + \hat{k}$

$\vec{B} = 3\hat{i} - 4\hat{j} - 12\hat{k}$

To Find:

Projection of $\vec{A}$ on $\vec{B} = A\cos\theta = ?$

Calculation:

Let θ is the angle between $\vec{A}$ and $\vec{B}$ then,
$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

Or
$$A\cos\theta = \frac{\vec{A} \cdot \vec{B}}{B} \quad \ldots\ldots(1)$$

Now
$$\vec{A} \cdot \vec{B} = (2\hat{i} - 8\hat{j} + \hat{k}) \cdot (3\hat{i} - 4\hat{j} - 12\hat{k})$$
$$\vec{A} \cdot \vec{B} = 6 + 32 - 12$$

$$\vec{A} \cdot \vec{B} = 26$$

Magnitude of vector $\vec{B}$ is:
$$B = \sqrt{(3)^2 + (-4)^2 + (-12)^2}$$
$$B = \sqrt{9 + 16 + 144}$$
$$B = \sqrt{169}$$
$$B = 13$$

Putting values of $\vec{A} \cdot \vec{B}$ and B is equ. (1), we get,

Projection of $\vec{A}$ on $\vec{B} = A\cos\theta = \frac{26}{13}$
$$\boxed{A\cos\theta = 2}$$

Example 2.6:

The line of action of a force $\vec{F}$ passes through a point P of a body whose position vector in meters is $\hat{i} - 2\hat{j} + \hat{k}$. If $\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ (newton), determine the torque about the point 'A' whose position vector (meter) is $2\hat{i} + \hat{j} + \hat{k}$.

Given Data:

Force $= \vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

The position vector of point A $= \vec{r}_1 = 2\hat{i} + \hat{j} + \hat{k}$

The position vector of point P $= \vec{r}_2 = \hat{i} - 2\hat{j} + \hat{k}$

To Find:

Torque about point A $= \vec{\tau} = ?$

Calculation:

The position vector of P relative to A is
$$\vec{AP} = \vec{r} = \vec{r}_2 - \vec{r}_1$$
$$\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + \hat{j} + \hat{k})$$
$$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} - 2\hat{i} - \hat{j} - \hat{k}$$
$$\vec{r} = -\hat{i} - 3\hat{j}$$

Now,

The torque about point A is:
$$\vec{\tau} = \vec{r} \times \vec{F}$$

putting values, we get
$$\vec{\tau} = (-\hat{i} - 3\hat{j}) \times (2\hat{i} - 3\hat{j} + 4\hat{k})$$
$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 0 \\ 2 & -3 & 4 \end{vmatrix}$$
$$\vec{\tau} = (-12 - 0)\hat{i} + (0 + 4)\hat{j} + (3 + 6)\hat{k}$$
$$\boxed{\vec{\tau} = (-12\hat{i} + 4\hat{j} + 9\hat{k})\text{ Nm}}$$

Example 2.7

A load is suspended by two cords as shown in Fig. Determine the maximum load that can be suspended at P, if maximum breaking stress of the cord used is 50N.

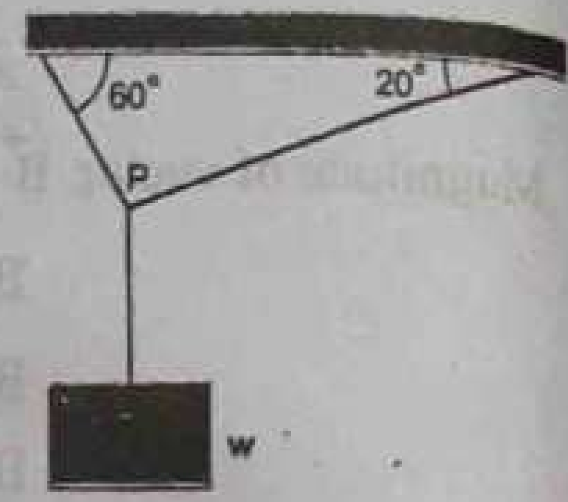


Fig. 2.15

Given Data:

Maximum breaking stress of the cord = $T_1 = 50N$

Angles made by the cord are

$\theta_1 = 60°$ and $\theta_2 = 20°$

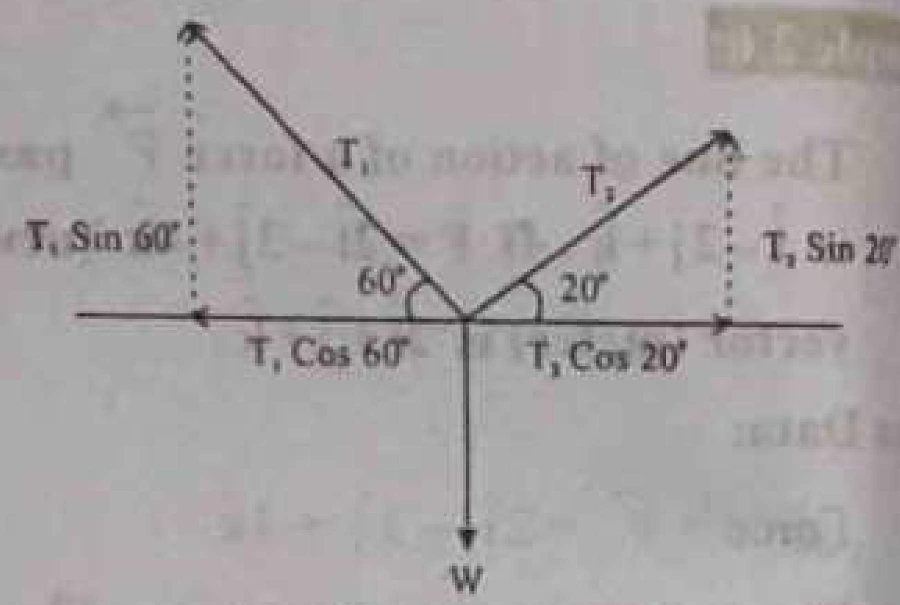
To Find:

Maximum load at P = W = ?

Calculation:

Resolve T_1 and T_2 into rectangular components.

Applying the first condition of equilibrium for x-components.



$\Sigma F_x = 0$ gives

$T_2 \cos 20° - T_1 \cos 60° = 0$

Putting values, we get

$T_2 \times 0.94 - T_1 \times 0.5 = 0$

$0.5\, T_1 = 0.94\, T_2$

$$T_1 = \frac{0.94\, T_2}{0.5}$$

$T_1 = 1.88\, T_2$ ......(1)

As $T_1 = 50$ N, then from equation (1)

$$T_2 = \frac{T_1}{1.88}$$

$$T_2 = \frac{50}{1.88}$$

$T_2 = 26.6$ N.

Now applying First condition of equilibrium for y–component.

$\sum F_y = 0$ gives,

$T_1 \sin 60° + T_2 \sin 20° - W = 0$

Putting values, we get

$50 \times 0.866 + 26.6 \times 0.34 = W$

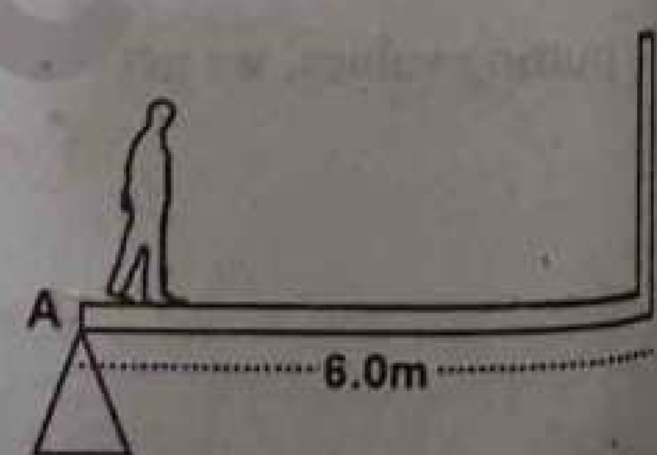
Or $43.3 + 9.04 = W$

Or $W = 52.34$ N

Or $\boxed{W = 52\text{ N}}$

Example 2.8

A uniform beam of 200N is supported horizontally as shown. If the breaking tension of the rope is 400N, how far can the man of weight 400N walk from point A on the beam as shown in Fig.?



Given Data:

Weight of the man = $W_1 = 400$ N

Weight f the beam = $W_2 = 200$ N

Breaking stress of the rope = 400 N

Length of the beam = $\ell = 6$ m

To Find:

Distance covered by the man = d = ?

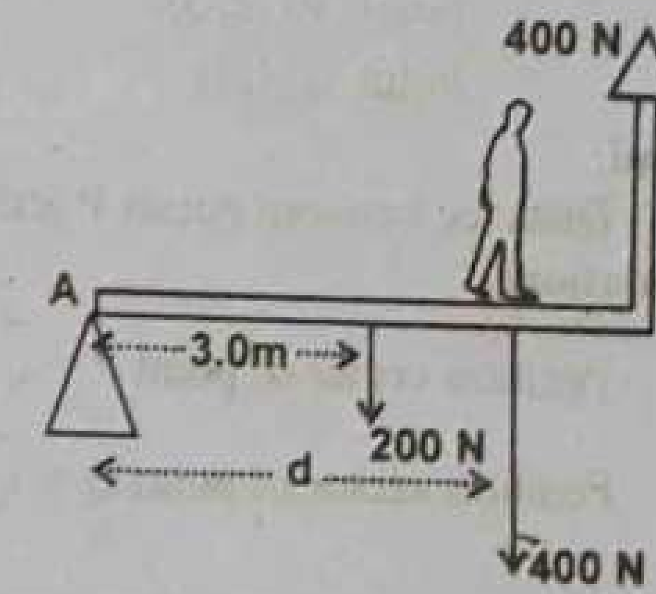
Calculation:

Let beam can rotate about point A.

Let breaking point is at a distance d from the pivot point A.

By applying second condition of equilibrium about point A.

i.e. $\Sigma \tau = 0$



$6 \times 400 - 400 \times d - 200 \times 3 = 0$

$2400 = 400\, d + 600$

$400\, d = 2400 - 600$

$400\, d = 1800$

$$d = \frac{1800}{400}$$

$\boxed{d = 4.5\text{ m}}$

The man can walk a distance 4.5 m from point A.

Example 2.9

A boy weighing 300N is standing at the edge of a uniform diving board 4m in length. The weight of the board is 200N (Fig). Find the forces exerted by pedestals on the board.

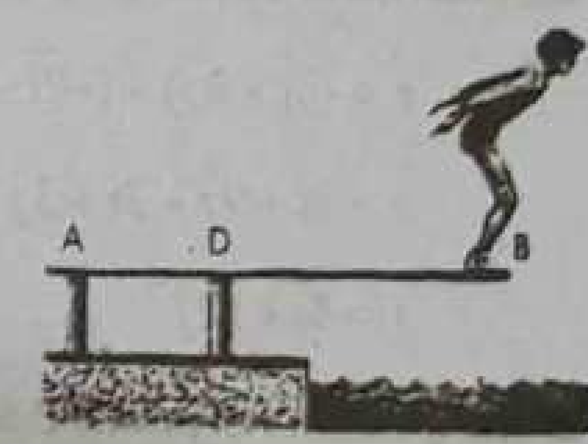


Fig. 2.20(a)

Given Data:

Weight of the boy = 300 N

Weight of the board = 200 N

Length of the diving board = 4 m

To Find:

Force exerted by the pedestals on the board = ?

Calculations:

Let R_1 and R_2 are the reaction forces exerted by the pedestals.

C is the centre of gravity at which the weight of the board acts.

Applying first condition of equilibrium

As $\Sigma F_x = 0$ gives no information because there is no force acting along x-axis

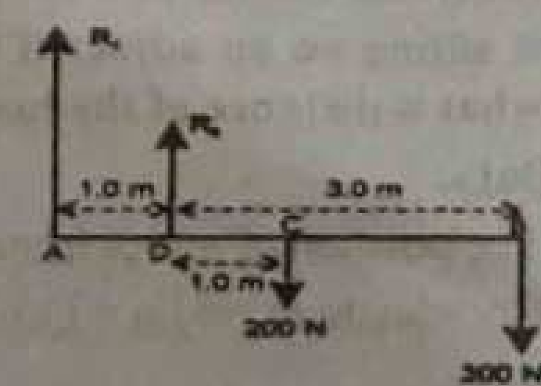


Fig. 2.20(b)

Now $\Sigma F_y = 0$ gives

$R_1 + R_2 - 200 - 300 = 0$

or $R_1 + R_2 = 500$ N .... (1)

Applying second condition of equilibrium consider the point D as an axis of rotation. $\sum \tau = 0$

$-R_1 \times AD - 300 \times DB - 200 \times DC = 0$

$-R_1 \times 1 - 300 \times 3 - 200 \times 1 = 0$

$-R_1 - 900 - 200 = 0$

$-R_1 - 1100 = 0$

$R_1 = -1100$ N

Putting the value of R_1 in equ. (1)

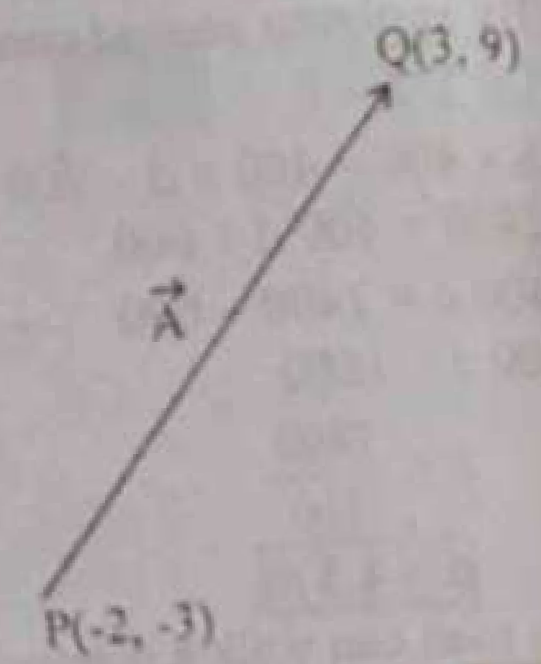
$-1100 + R_2 = 500$

$\boxed{R_2 = 1600\text{ N}}$

Negative sign of R_1 shows that it is directed downward.

Exercise Problems

2.1 Suppose, in a rectangular coordinate system, a vector $\vec{A}$ has its tail at the point P(−2, −3) and its tip at Q(3, 9). Determine the distance between these two points.



Given Data:

Point P(-2,-3)

Point Q(3,9)

To find:

Distance between points P and Q = r = ?

Calculation:

Position vector of point P = $\vec{r_1} = -2\hat{i} - 3\hat{j}$

Position vector of point Q = $\vec{r_2} = 3\hat{i} + 9\hat{j}$

Displacement vector from point P to point Q = $\vec{r} = \vec{r_2} - \vec{r_1}$

$$\vec{r} = (3\hat{i} + 9\hat{j}) - (-2\hat{i} - 3\hat{j})$$

$$\vec{r} = 3\hat{i} + 9\hat{j} + 2\hat{i} + 3\hat{j}$$

$$\vec{r} = 5\hat{i} + 12\hat{j}$$

Distance between points P and Q = Magnitude of $\vec{r} = r = \sqrt{x^2 + y^2}$

So, $r = \sqrt{(5)^2 + (12)^2}$

$r = \sqrt{25 + 144}$

$r = \sqrt{169}$

$\boxed{r = 13 \text{ units}}$

2.2 A certain corner of a room is selected as the origin of a rectangular coordinate system. If an insect is sitting on an adjacent wall at a point having coordinates (2 ,1), where the units are in meters, what is distance of the insect from this corner of the room?

Given Data:

The position of insect from corner = P(2, 1)

Coordinate of origin = O(0, 0)

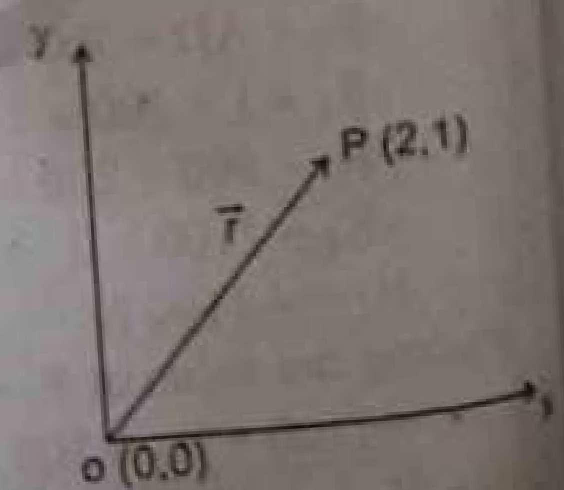
To find:

Distance of insect from corner = r = ?

Calculations:

As the position vector of insect with respect to origin is

$$\vec{r} = 2\hat{i} + \hat{j}$$



Magnitude of $\vec{r} = r = \sqrt{x^2 + y^2}$

$r = \sqrt{(2)^2 + (1)^2}$

$= \sqrt{4+1}$

$= \sqrt{5}$

Hence $\boxed{r = 2.24m}$

2.3 What is the unit vector in the direction of the vector $\vec{A} = 4\hat{i} + 3\hat{j}$?

Given Data:

$$\vec{A} = 4\hat{i} + 3\hat{j}$$

To find:

Unit vector = $\hat{A}$ = ?

Calculations:

By definition

$$\hat{A} = \frac{\vec{A}}{A}$$

Where A is the magnitude of vector $\vec{A}$, which is given by

$$A = \sqrt{A_x^2 + A_y^2}$$

putting value, we get

$$A = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Thus

$$\boxed{\hat{A} = \frac{4\hat{i} + 3\hat{j}}{5}}$$

2.4 Two particles are located at $\vec{r_1} = 3\hat{i} + 7\hat{j}$ and $\vec{r_2} = -2\hat{i} + 3\hat{j}$ respectively. Find both the magnitude of the vector $\vec{r_2} - \vec{r_1}$ and its orientation with respect to the x-axis.

Given data:

Location of first particle = $\vec{r_1} = 3\hat{i} + 7\hat{j}$

Location of second particle = $\vec{r_2} = -2\hat{i} + 3\hat{j}$

To find:

Magnitude of $(\vec{r_2} - \vec{r_1})$ = ?

and orientation (direction) = θ = ?

Calculations:

As relative position vector

$$\vec{r} = \vec{r_2} - \vec{r_1}$$

putting value, we get

$$\vec{r} = (-2\hat{i} + 3\hat{j}) - (3\hat{i} + 7\hat{j})$$

$$\vec{r} = -2\hat{i} + 3\hat{j} - 3\hat{i} - 7\hat{j}$$

$$\vec{r} = -5\hat{i} - 4\hat{j}$$

Magnitudes

$$r = \sqrt{x^2 + y^2}$$

$r=\sqrt{(-5)^2+(-4)^2}$

$r=\sqrt{25+16}$

$r=\sqrt{41}$

$\boxed{r=6.4}$

Orientation (direction) is

$\varphi=\tan^{-1}\left(\frac{y}{x}\right)$

$\varphi=\tan^{-1}\left(\frac{4}{5}\right)$

$\varphi=\tan^{-1}(0.8)$

$\varphi=38.6^\circ$

As the resultant lies in 3rd quadrant so direction is

$\theta=180^\circ+\varphi$

$\theta=180^\circ+38.6^\circ$

$\theta=218.6^\circ$

$\boxed{\theta=219^\circ \text{ Approx}}$

2.5 If a vector $\vec{B}$ is added to vector $\vec{A}$, the result is $6\hat{i}+\hat{j}$. If $\vec{B}$ is subtracted from $\vec{A}$ the result is $-4\hat{i}+7\hat{j}$. What is magnitude of vector $\vec{A}$?

Given Data:

$\vec{A}+\vec{B}=6\hat{i}+\hat{j}$ ....(1)

$\vec{A}-\vec{B}=-4\hat{i}+7\hat{j}$ ....(2)

To find:

Magnitude of $\vec{A}=A=?$

Calculations:

Adding equation (1) and (2)

$(\vec{A}+\vec{B})+(\vec{A}-\vec{B})=6\hat{i}+\hat{j}-4\hat{i}+7\hat{j}$

$2\vec{A}=2\hat{i}+8\hat{j}$

$\vec{A}=\hat{i}+4\hat{j}$

Magnitude of $\vec{A}=A=\sqrt{(1)^2+(4)^2}$

$=\sqrt{1+16}$

$=\sqrt{17}$

$\boxed{A=4.1\text{ units}}$

2.6 Given that $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=3\hat{i}-4\hat{j}$, find the magnitude and angle of (a) $\vec{C}=\vec{A}+\vec{B}$ and (b) $\vec{D}=3\vec{A}-2\vec{B}$.

Given Data:

$\vec{A}=2\hat{i}+3\hat{j}$

$\vec{B}=3\hat{i}-4\hat{j}$

To find:

Magnitude and direction of (a) $\vec{C}=\vec{A}+\vec{B}$

(b) $\vec{D}=3\vec{A}-2\vec{B}$

Calculations:

(a) $\vec{C}=\vec{A}+\vec{B}$

Putting the value, we get

$\vec{C}=(2\hat{i}+3\hat{j})+(3\hat{i}-4\hat{j})$

$\vec{C}=5\hat{i}-\hat{j}$

Now magnitude of $\vec{C}$ is $C=\sqrt{(5)^2+(-1)^2}$

$=\sqrt{25+1}$

$=\sqrt{26}$

$=5.09$

$\boxed{C=5.1\text{ units}}$

Direction of $\vec{C}$

$\varphi=\tan^{-1}\left(\frac{1}{5}\right)$

$\varphi=11^\circ$

As x-component is +ive and y-component is –ive, so vector lies in 4th quadrant, so

$\theta=360^\circ-\varphi$

$\theta=360^\circ-11^\circ$

$\theta=349^\circ$

Hence $\boxed{\theta=349^\circ}$

(b) $\vec{D}=3\vec{A}-2\vec{B}$

putting the value, we get

$\vec{D}=3(2\hat{i}+3\hat{j})-2(3\hat{i}-4\hat{j})$

$\vec{D}=6\hat{i}+9\hat{j}-6\hat{i}+8\hat{j}$

$\vec{D}=17\hat{j}$

Now magnitude of $\vec{D}$ is $D=\sqrt{(0)^2+(17)^2}=\sqrt{(17)^2}=17$

$\boxed{D=17}$

As x-component is zero so the vector lies along y-axis

Direction of $\vec{D}$ is along Y- axis i.e. $\theta=90^\circ$ w.r.t. x-axis.

Hence $\boxed{\theta=90^\circ}$

2.7 Find the angle between the two vectors, $\vec{A} = 5\hat{i} + \hat{j}$ and $\vec{B} = 2\hat{i} + 4\hat{j}$.

Given Data:

$$\vec{A} = 5\hat{i} + \hat{j}$$

$$\vec{B} = 2\hat{i} + 4\hat{j}$$

Calculation:

As $\vec{A}.\vec{B} = AB\cos\theta$

Thus $\cos\theta = \frac{\vec{A}.\vec{B}}{AB}$..........(1)

Now

$$\vec{A}.\vec{B} = (5\hat{i} + \hat{j}).(2\hat{i} + 4\hat{j})$$

$$\vec{A}.\vec{B} = (5)(2) + (1)(4)$$

$$\vec{A}.\vec{B} = 10 + 4$$

$$\vec{A}.\vec{B} = 14$$

The magnitude of vector $\vec{A}$ is

$$A = \sqrt{(5)^2 + (1)^2} = \sqrt{25+1} = \sqrt{26} = 5.09$$

$$B = \sqrt{(2)^2 + (4)^2} = \sqrt{4+16} = \sqrt{20} = 4.47$$

Putting value is 'equ'….(1)

$$\cos\theta = \left(\frac{14}{5.09 \times 4.47}\right)$$

$$\cos\theta = 0.164$$

$$\theta = \cos^{-1}(0.614)$$

$$\theta = 52^\circ$$

Hence $\boxed{\theta = 52^\circ}$

2.8 Find the work done when the point of application of the force $3\hat{i} + 2\hat{j}$ moves in a straight line the point (2,-1) to the point (6, 4).

Given Data:

Force $\vec{F} = 3\hat{i} + 2\hat{j}$

Point A (2,-1)

and Point B (6,4)

To find:

work done = W = ?

Calculations:

The position vector of point A(2,-1) = $\vec{r_A} = 2\hat{i} - \hat{j}$

The position vector of point B (6,4) = $\vec{r_B} = 6\hat{i} + 4\hat{j}$

If displacement between points A and B is $\vec{d}$, them

$$\vec{d} = \vec{r_B} - \vec{r_A}$$

$$\vec{d} = (6\hat{i} + 4\hat{j}) - (2\hat{i} - \hat{j})$$

$$\vec{d} = 6\hat{i} + 4\hat{j} - 2\hat{i} + \hat{j}$$

$$\vec{d} = 4\hat{i} + 5\hat{j}$$

Now work done = W = $\vec{F}.\vec{d}$

Putting value, we get

$$W = (3\hat{i} + 2\hat{j}).(4\hat{i} + 5\hat{j})$$

$$W = (3)(4) + (2)(5)$$

$$W = 12 + 10$$

$$\boxed{W = 22\text{units}}$$

2.9 Show that the three vector $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} - 3\hat{j} + \hat{k}$ and $4\hat{i} + \hat{j} - 5\hat{k}$ are mutually perpendicular.

Given Data:

$$\vec{A} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}$$

$$\vec{C} = 4\hat{i} + \hat{j} - 5\hat{k}$$

To find:

To prove, these three vector are mutually perpendicular.

Criteria:

These three vectors will be mutually perpendicular if $\vec{A}.\vec{B} = 0, \vec{B}.\vec{C} = 0$ and $\vec{C}.\vec{A} = 0$

Calculations:

$$\vec{A}.\vec{B} = (\hat{i} + \hat{j} + \hat{k}).(2\hat{i} - 3\hat{j} + \hat{k})$$

$$\vec{A}.\vec{B} = (1)(2) + (1)(-3) + (1)(1)$$

$$\vec{A}.\vec{B} = 2 - 3 + 1$$

$$\boxed{\vec{A}.\vec{B} = 0}$$

This shows that $\vec{A}$ and $\vec{B}$ are mutually perpendicular

$$\vec{B}.\vec{C} = (2\hat{i} - 3\hat{j} + \hat{k}).(4\hat{i} + \hat{j} - 5\hat{k})$$

$$\vec{B}.\vec{C} = (2)(4) + (-3)(1) + (1)(-5)$$

$$\vec{B}.\vec{C} = 8 - 3 - 5$$

$$\boxed{\vec{B}.\vec{C} = 0}$$

This shows that $\vec{B}$ and $\vec{C}$ are mutually perpendicular

$$\vec{C}.\vec{A}=(4\hat{i}+\hat{j}-5\hat{k}).(\hat{i}+\hat{j}+\hat{k})$$

$$\vec{C}.\vec{A}=(4)(1)+(1)(1)+(-5)(1)$$

$$\vec{C}.\vec{A}=4+1-5$$

$$\vec{C}.\vec{A}=0$$

This shows that $\vec{C}$ and $\vec{A}$ are mutually perpendicular

Result:

Thus all these three vector are mutually perpendicular, because their mutual scalar products are zero.

2.10 Given that $\vec{A}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{B}=3\hat{i}-4\hat{k}$, find the projection of $\vec{A}$ on $\vec{B}$.

Given Data:

$$\vec{A}=\hat{i}-2\hat{j}+3\hat{k}$$

$$\vec{B}=3\hat{i}-4\hat{k}$$

To find:

Projection of $\vec{A}$ on $\vec{B}=A\cos\theta=?$

Calculation:

Let θ is the angle between $\vec{A}$ and $\vec{B}$, then,

$$\vec{A}.\vec{B}=AB\cos\theta$$

or

$$A\cos\theta=\frac{\vec{A}.\vec{B}}{B} \quad \ldots\ldots(1)$$

Now

$$\vec{A}.\vec{B}=(\hat{i}-2\hat{j}+3\hat{k}).(3\hat{i}-0\hat{j}-4\hat{k})$$

$$\vec{A}.\vec{B}=(1)(3)+(-2)(0)+(3)(-4)$$

$$\vec{A}.\vec{B}=3+0-12$$

$$\vec{A}.\vec{B}=-9$$

And $B=\sqrt{(3)^2+(4)^2}=\sqrt{9+16}=\sqrt{25}=5$

Putting values in eq (1), we get

$\boxed{A\cos\theta=\frac{-9}{5}}$ This is projection of $\vec{A}$ on $\vec{B}$.

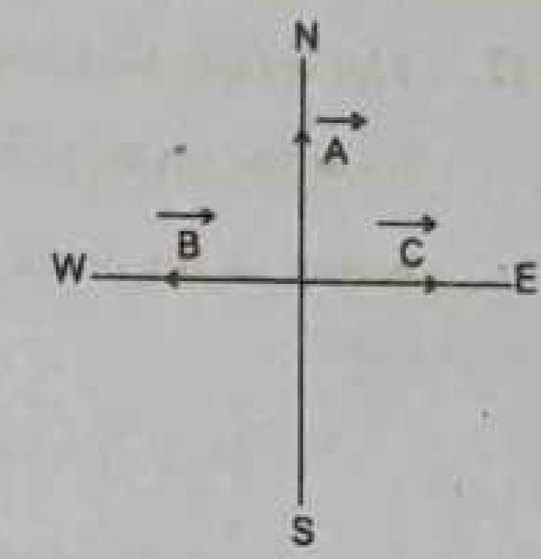
2.11 Vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 4 units north, 3 units west and 8 units east, respectively. Describe carefully (a) $\vec{A}\times\vec{B}$ (b) $\vec{A}\times\vec{C}$ (c) $\vec{B}\times\vec{C}$

Given Data:

$\vec{A}=4$ units north

$\vec{B}=3$ units west

$\vec{C}=8$ units east



To find:

(a) $\vec{A}\times\vec{B}$ (b) $\vec{A}\times\vec{C}$ (c) $\vec{B}\times\vec{C}$

Calculation:

(a) $\vec{A}\times\vec{B}=AB\sin\theta\hat{n}$

As angle between $\vec{A}$ and $\vec{B}$ (i.e north and west) is 90°

$$\vec{A}\times\vec{B}=AB\sin 90°\hat{n}$$

putting values, we get

$$\vec{A}\times\vec{B}=4\times 3\,(1)\,\hat{n} \qquad \sin 90°=1$$

$$\vec{A}\times\vec{B}=12\,\hat{n}$$

Direction:

According to right hand rule, direction of $\vec{A}\times\vec{B}$ (i.e. $\hat{n}$) is vertically upward.

$\boxed{\vec{A}\times\vec{B}=12\text{ units upward}}$

(b) $\vec{A}\times\vec{C}=AC\sin\theta\hat{n}$

As angle between $\vec{A}$ and $\vec{C}$ is 90°

$$\vec{A}\times\vec{C}=4\times 8\sin 90°\hat{n}$$

$$\vec{A}\times\vec{C}=32(1)\hat{n}$$

$$\vec{A}\times\vec{C}=32\hat{n}$$

Direction:

According to right hand rule, direction of $\vec{A}\times\vec{C}$ (i.e. $\hat{n}$) is vertically downward.

$\boxed{\vec{A}\times\vec{C}=32\text{ units downward}}$

(c) $\vec{B}\times\vec{C}=BC\sin\theta\hat{n}$

As angle between $\vec{B}$ and $\vec{C}$ is 180°

$$\vec{B}\times\vec{C}=3\times 8\sin 180°\hat{n}$$

$$\vec{B}\times\vec{C}=3\times 8(0)\hat{n}$$

$$\vec{B}\times\vec{C}=0\hat{n}$$

$\boxed{\vec{B}\times\vec{C}=\vec{O}}$

Direction:

As $\vec{B}\times\vec{C}$ results into a null vector so it has arbitrary direction.

2.12 The torque or turning effect of force about a given point is given by $\vec{r}\times\vec{F}$ where $\vec{r}$ is the v... from the given point to the point of application of $\vec{F}$. Consider a force $\vec{F} = -3\hat{i}+\hat{j}+5\hat{k}$ (ne... acting on the point $7\hat{i}+3\hat{j}+\hat{k}$ (m). What is the torque in N m about the origin ?

Given Data:

$$\vec{F}=(-3\hat{i}+\hat{j}+5\hat{k})\ \text{(newton)}$$

$$\vec{r} = (7\hat{i}+3\hat{j}+\hat{k})\ \text{(meter)}$$

To find:

$$\vec{\tau}=\vec{r}\times\vec{F}=?$$

Calculations:

As $\vec{\tau}=\vec{r}\times\vec{F}$

putting values, we get

$$\vec{\tau}=(7\hat{i}+3\hat{j}+\hat{k})\times(-3\hat{i}+\hat{j}+5\hat{k})$$

In determinant form we can write the cross product as,

$$\vec{\tau}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & 3 & 1\\ -3 & 1 & 5\end{vmatrix}$$

$$\vec{\tau}=\hat{i}(15-1)-\hat{j}(35+3)+\hat{k}(7+9)$$

$$\boxed{\vec{\tau}=14\hat{i}-38\hat{j}+16\hat{k}\ \text{Nm}}$$

2.13 The line of action of force, $\vec{F}=\hat{i}-2\hat{j}$, passes through the point whose position vect... $(-\hat{j}+\hat{k})$. Find (a) the moment of $\vec{F}$ about the origin, (b) the moment of $\vec{F}$ about the poi... which the position vector is $\hat{i}+\hat{k}$

Given data:

$$\vec{F}=\hat{i}-2\hat{j}$$

Position vector $=\vec{r}=-\hat{j}+\hat{k}$

To find:

(a) moment of $\vec{F}$ about the origin $=\vec{\tau}=?$

(b) moment of $\vec{F}$ about the point of which the position vector is $\hat{i}+\hat{k}=\vec{\tau}'=?$

Calculation:

(a) torque (i.e., moment of $\vec{F}$) about origin

$$\vec{\tau}=\vec{r}\times\vec{F}$$

$$\vec{\tau}=(-\hat{j}+\hat{k})\times(\hat{i}-2\hat{j})$$

$$\vec{\tau}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & -1 & 1\\ 1 & -2 & 0\end{vmatrix}$$

$$\vec{\tau}=\hat{i}(0+2)-\hat{j}(0-1)+\hat{k}(0+1)$$

$$\boxed{\vec{\tau}=2\hat{i}+\hat{j}+\hat{k}}$$

(b) in this case

$$\vec{r}_1=\hat{i}+\hat{k}$$

$$\vec{r}_2=-\hat{j}+\hat{k}$$

$$\vec{r}_2-\vec{r}_1=(-\hat{j}+\hat{k})-(\hat{i}+\hat{k})$$

$$\vec{r}_2-\vec{r}_1=-\hat{j}+\hat{k}-\hat{i}-\hat{k}$$

$$\vec{r}=\vec{r}_2-\vec{r}_1=-\hat{i}-\hat{j}$$

Now using the formula

$$\vec{\tau}'=\vec{r}\times\vec{F}$$

$$\vec{\tau}'=(-\hat{i}-\hat{j})\times(\hat{i}-2\hat{j})$$

$$\vec{\tau}'=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & -1 & 0\\ 1 & -2 & 0\end{vmatrix}$$

$$\vec{\tau}'=\hat{i}(0-0)-\hat{j}(0-0)+\hat{k}(2+1)$$

$$\boxed{\vec{\tau}'=3\hat{k}}$$

2.14 **The magnitude of dot and cross products of two vector are $6\sqrt{3}$ and 6 respectively. Find the angle between the vectors.**

Given data:

Let $\vec{A}$ and $\vec{B}$ be two given vectors.

Magnitude of dot product of two vectors $=|\vec{A}.\vec{B}|=6\sqrt{3}$

Magnitude of cross product of two vectors $=|\vec{A}\times\vec{B}|=6$

To find:

The angle between two vectors $=\theta=?$

Calculation:

$$|\vec{A}.\vec{B}|=AB\cos\theta=6\sqrt{3} \quad \text{———(1)}$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta = 6 \qquad \text{------------(2)}$$

Dividing equ. (2) by equation (1) we get.

$$\frac{AB\sin\theta}{AB\cos\theta} = \frac{6}{6\sqrt{3}}$$

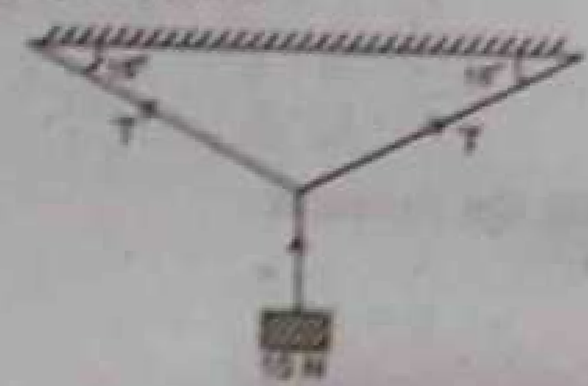
$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\boxed{\theta = 30^0}$$

2.15 A load of 10.0N is suspended from a clothes line. This distorts the line so that it makes an an... 15^0 with the horizontal at each end. Find the tension in the clothes line.



Given data:

Load = W = 10N

$\theta = 15^0$

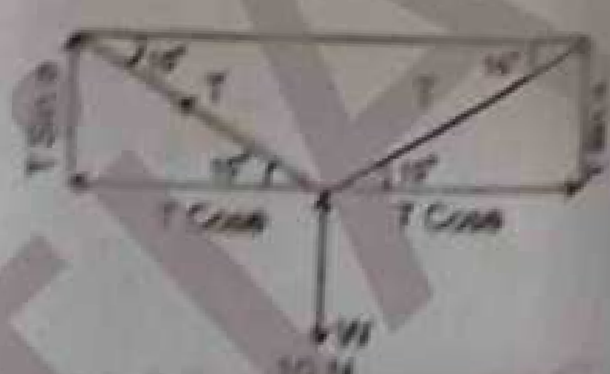
To find:

Tension in cloths line = T = ?

Calculation:

Resolving tension into its rectangular components

Applying the first condition of equilibrium $\sum F_x = 0$



$T\cos15^0 - T\cos15^0 = 0$

Or $T\cos15^0 = T\cos15^0$

Now applying $\sum Fy = 0$

$T\sin15^0 + T\sin15^0 - W = 0$

Or $2T\sin15^0 = W$

Or $T = \dfrac{W}{2\sin15^0}$

$T = \dfrac{10}{2\times0.26}$

$T = \dfrac{10}{0.52}$

$\boxed{T = 19.3\,N}$

2.16 A tractor of weight 15,000 N crosses a single span bridge of weight 8000N and of length 21... The bridge span is supported half a meter from either end. The tractor's front wheels take 1... the total weight of tractor, and the rear wheels are 3m behind the front wheels. Calculate the ... on the bridge supports when the rear wheels at the middle of the bridge span.

Given data:

Weight of the tractor = W = 15000 N

Weight of the Bridge = W_B = 8000 N

Length of Bridge = 21.0 m

Length of Bridge span between Supporting ends = 20 m

Weight, front wheels $= W_f = \frac{1}{3}\times W = \frac{1}{3}\times 15000 = 5000$ N

Weight on the rear wheels $= W_R = W - W_f = 15000 - 5000 = 10{,}000$ N

Distance between front wheel and rear wheel = 3 m

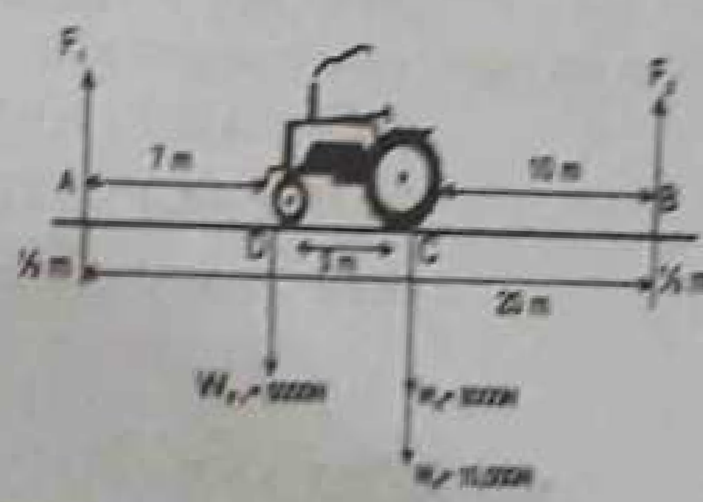
To find:

Forces on the Bridge Supports

F_1 = ? and F_2 = ?

Fig. 2.22

Calculations:



Applying the first condition of equilibrium

$\sum F_x = 0$, but there is no force along x-axis

Now applying $\sum F_y = 0$

$F_1 + F_2 - W_B - W_R - W_F = 0$

$F_1 + F_2 - 8000 - 10000 - 5000 = 0$

$F_1 + F_2 - 23000 = 0$

$F_1 + F_2 = 23000 \quad ......(1)$

Now applying second condition of equilibrium $\sum\tau = 0$

Let A is the axis of rotation.

So $F_2(AB) - W_B(AC) - W_R(AC) - W_F(AD) + F_1(0) = 0$

$F_2\times20 - 8000\times10 - 10000\times10 - 5000\times7 + F_1\times0 = 0$

$F_2\times20 - 80000 - 100000 - 35000 + 0 = 0$

$F_2\times20 - 215000$

$F_2 = \dfrac{215000}{20}$

$F_2 = 10750$ N

or $F_2 = 10.750\times10^3$ N

$F_2\,\boxed{= 10.75\text{ kN}}$

Putting value of F_2 in equation (1)

$F_1 + 10750 = 23000$

$F_1 = 23000 - 10750$

$F_1 = 12250$ N

$F_1 = 12.250\times10^3$ N

$\boxed{F_1 = 12.250\text{ kN}}$

2.17 A spherical ball of weight 50N is to be lifted over the step as shown in the Fig. 2.23. Calculate the minimum force needed just to lift it above the floor.

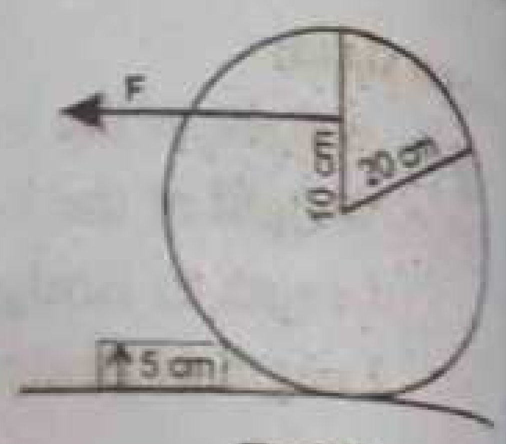


Fig. 2.23

Given data:

Weight of spherical ball = W = 50 N

Height of the spherical = h = 5 cm

Radius of the spherical ball = r = 20 cm

To find:

Minimum force required to lift the balls F=?

Calculation:

From diagram

$DE = 5cm$

$CO = CE = 20$ cm

and $CD = CE - DE$

$CD = 20 - 5 = 15$ cm

From the right angle triangle OCD

$(OC)^2 = (CD)^2 + (OD)^2$ (Pythagorean Theorem)

$(OD)^2 = (OC)^2 - (CD)^2$

$OD = \sqrt{(OC)^2 - (CD)^2}$

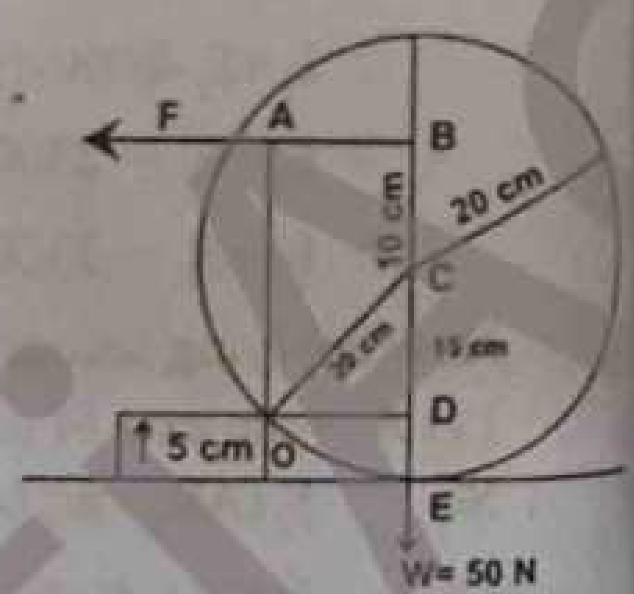
$OD = \sqrt{(20)^2 - (15)^2}$

$OD = \sqrt{400 - 225}$

$OD = \sqrt{175}$

$OD = 13.2$ cm

or $OD = 13$ cm (approx)



Take point 'O' as axis of rotation and apply second condition of equilibrium i.e., $\sum \tau = 0$

$F \times AO - W \times OD = 0$

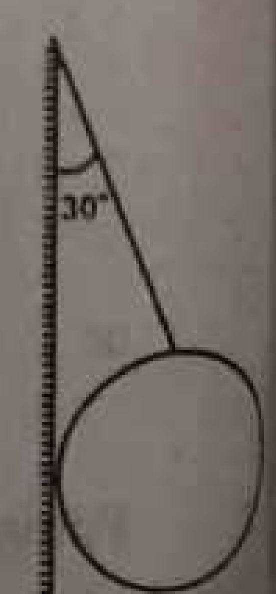
$F \times 25 - 50 \times 13 = 0$

$F \times 25 = 650$

$F = \frac{650}{25}$

$\boxed{F = 26\ N}$

2.18 A uniform sphere of weight 10.0N is held by a string attached to a frictionless wall so that the string makes an angle of 30° with the wall as shown in Fig. 2.24. Find the tension in the string and the force exerted on the sphere by the wall.



Given data:

weight of the sphere = W = 10N

Angle between string and wall = $\theta = 30^\circ$

To find:

Tension in the string = T = ?

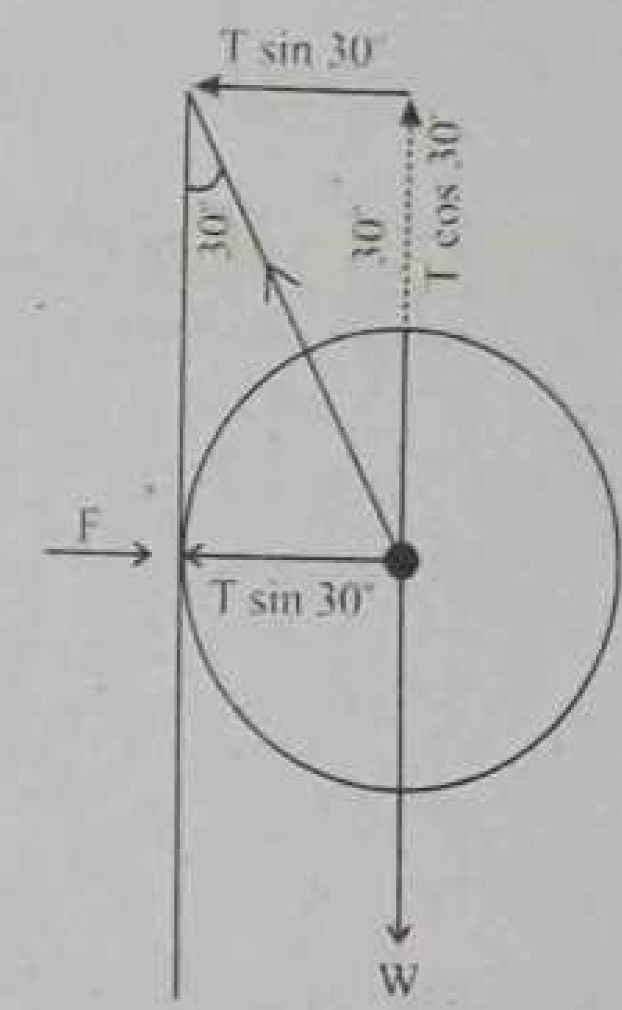
Calculation:

Resolving T into two rectangular components T_x and T_y.

Apply first condition of equilibrium along x- axis i.e., $\sum F_x = 0$

$F = T \sin 30^\circ$

$F = T\left(\frac{1}{2}\right)$ ________ (1)



Apply first condition of equilibrium along y- axis i.e., $\sum F_y = 0$

$T\cos 30^\circ - W = 0$

$T = \frac{W}{\cos 30^\circ}$

$T = \frac{10}{0.866}$

$T = 11.55N$

Or $\boxed{T = 11.6N}$

Putting this value in equ (1) we get

$F = 11.55 \times \frac{1}{2}$

$\boxed{F = 5.77\ N}$

****

Limitation of average velocity

Average velocity does not tell us that the motion is ***steady or variable*** and the path is ***straight or curved***.

If a squash ball comes back to its starting point after bouncing of the wall several times, its total displacement is zero and also its average velocity is zero.

Unit and Direction

SI unit of velocity is **m/sec** (i.e. ms^{-1}). Its dimensions are $[LT^{-1}]$. It is a vector quantity and its direction is along the direction of displacement.

Instantaneous Velocity

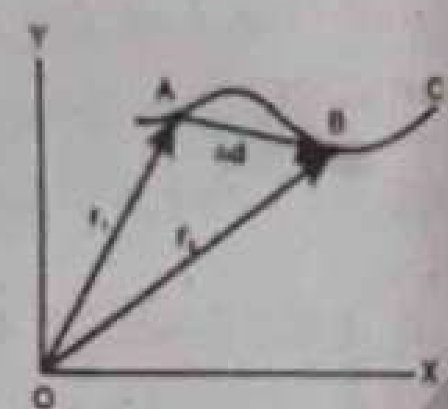
The limiting value of $\frac{\Delta\vec{d}}{\Delta t}$, as time interval Δt following the time t, approaches to zero is called instantaneous velocity.

Mathematically

$$\boxed{\vec{v}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\vec{d}}{\Delta t}}$$

Explanation

Consider a body is moving along the curve as shown in figure. The body moves from point A to point B in time Δt. Let $\vec{r}_1$ is the position vector of point A and $\vec{r}_2$ position vector of point B then displacement of the body during this short time interval is



$$\Delta\vec{d} = \vec{r}_2 - \vec{r}_1$$

If value of Δt is small, the value of $\Delta\vec{d}$ will also become smaller. When Δt approaches to zero the point A approaches point B. In this case $\frac{\Delta\vec{d}}{\Delta t}$ approaches to a limiting value called instantaneous velocity.

Note

The average velocity of the body may be zero even though its instantaneous velocity is not zero.

Uniform velocity

If the body covers equal displacements in equal intervals of time, the body is said to be moving with uniform velocity.

Non-uniform velocity

If the body covers unequal displacements in equal intervals of time, the body is said to be moving with non-uniform velocity.

Condition for uniform velocity

If the average and instantaneous velocity of a body are ***equal*** the body is moving with uniform velocity.

OR

The instantaneous velocity ***does not change***, the body is said to be moving with uniform velocity.

For Your Information

Typical Speeds

Speed, ms^{-1}	Motion
300,000,000	Light, radio waves X-rays, microwaves (in vacuum)
210,000	Earth-Sun travel around the galaxy
29,600	Earth around the sun
1,000	Moon around the earth
990	SR 71 reconnaissance jet
333	Sound (in Air)
267	Commercial jet airline
62	Commercial auto mobile (max)
37	Falcon in a dive
29	Running cheetah
10	100- meters dash (max)
9	porpoise swimming
5	Flying bee
4	Human runing
2	human swimming
0.01	walking ant

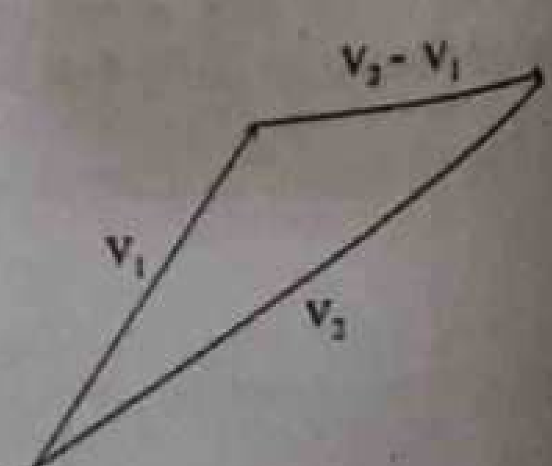
For Your Information

The change in velocity may be due to change in

i) magnitude

ii) direction

iii) In both of above



Acceleration

The time rate of change of velocity of a body is called acceleration.

Unit

SI unit of acceleration is m/sec^2. The dimensions of acceleration are $[LT^{-2}]$

Direction

Acceleration is also a ***vector*** quantity and the direction of acceleration is along the direction of ***change in velocity***.

Average Acceleration

The ratio of the total change in velocity to the total time taken is called average acceleration.

Let $\vec{v}_1$ is the initial velocity of the body which changes to the final velocity $\vec{v}_2$ in time, then the change in velocity is

$$\vec{v}_2 - \vec{v}_1 = \Delta\vec{v} = \text{change in velocity}$$

Then, the average acceleration is given by

$$\boxed{\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}}$$

Instantaneous Acceleration

The limiting value of $\frac{\Delta\vec{v}}{\Delta t}$ as the time interval Δt, following the time t, approaches to zero is called instantaneous acceleration.

Mathematically

$$\boxed{\vec{a}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}}$$

Positive acceleration

If the velocity of a body is ***increasing*** its acceleration is positive.

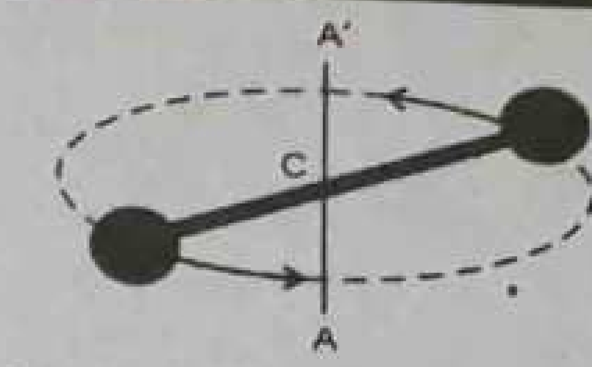
Negative acceleration

If the velocity of a body is ***decreasing***, its acceleration is negative. The negative acceleration is also called retardation or deceleration.

Condition for uniform acceleration

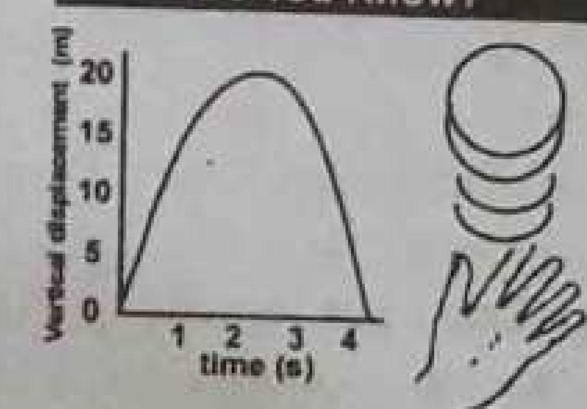
For a body moving with uniform acceleration, its average and instantaneous acceleration are ***equal***.

For Your Information



A measurement of mass independent of gravity. The unknown mass *m* and a calibrated mass m_c are mounted on a light weight rod. If the masses are equal, the rod will rotate without wobble about its centre.

Do You Know?



How the displacement of a vertically thrown ball varies with time?

EXPLANATION:
The variation of displacement with time is parabolic.

Q.2 What are velocity- time graphs? Discuss.

Ans.

Velocity Time Graph

The graph which represents the variation of velocity with time is called velocity-time graph.

Let us consider the case of a body moving along a straight path.

Case I

When the body moves with uniform velocity, velocity- time graph is a ***horizontal straight*** line as shown in figure.

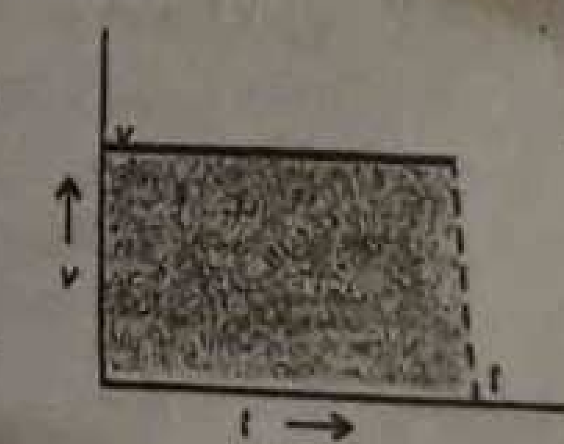


Fig.3.4

Distance covered by the body = Area of rectangle

$= \text{(length) (width)}$

$= vt$

$= S$

Case II

When the body moves with uniformly increasing velocity (i.e. with uniform acceleration.), the velocity- time graph is an ***inclined straight line*** with time axis as shown in figure.

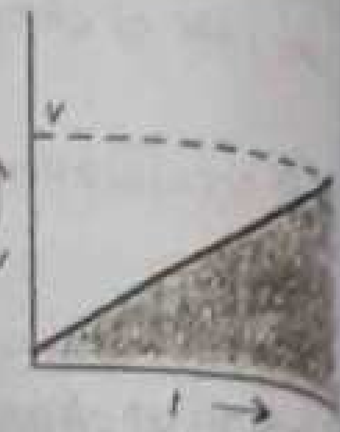


Fig.3.5

Distance covered by the body = Area of triangle

$$= \frac{1}{2}\text{ (base) (height)}$$

$$= \frac{1}{2}vt \quad \left(v_{av} = \frac{0+v}{2} = \frac{1}{2}v\right)$$

$$= v_{av}t$$

$$= S$$

Case III

When the body moves with non-uniformly increasing velocity (i.e. with non-uniform acceleration.), the velocity-time graph is a *curve* as shown in figure.

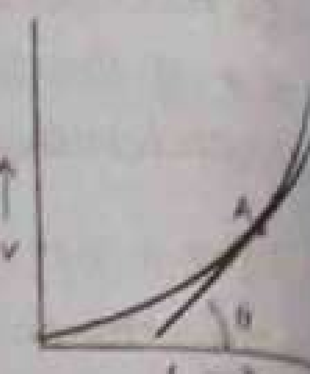


Fig.3.6

Note

The magnitude of instantaneous acceleration is equal to the slope of the ***tangent*** at that point.

Significance of velocity – time graphs.

- Slope of the velocity-time graph is equal to ***average acceleration***.
- Area under the velocity-time graph is equal to ***distance*** covered by the body.

> **DO YOU KNOW?**
> Velocity time graph ... never perpendicular to t... axis as it represent infi... acceleration.

Q.3 Write down the equations of motion for uniformly accelerated bodies?

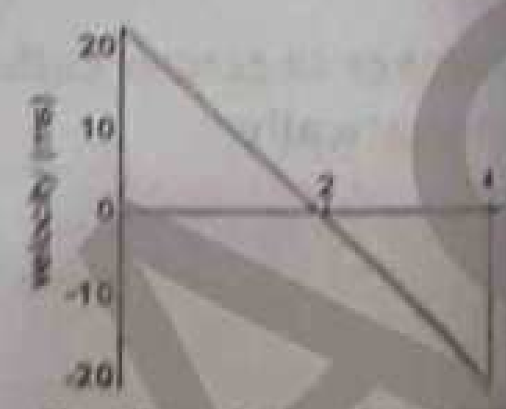
Ans.

Equation of Motion for Uniformly Accelerated Bodies

Consider, a body is moving with uniform acceleration along a straight line. If its initial velocity is v_i and after time interval its final velocity become v_f then

(i) $v_f = v_i + at$ (ii) $S = v_i t + \frac{1}{2}at^2$

(iii) $2aS = v_f^2 - v_i^2$ (iv) $S = \frac{(v_f + v_i)}{2} \times t$



How the velocity of a verti... thrown ball varies with time? Velocity is upwards positive

> **EXPLANATION:**
> For the first half of ... velocity goes on decre... and then becomes ze... maximum height. After ... the ball moves in down... direction with incre... velocity, till it hits ... starting point. Reaching ... starting point, the vel... becomes equal to ... upward initial velocity.

How to apply these equations in problems

- These equations are useful only for ***linear*** motion with ***uniform*** acceleration.
- When the object moves along a ***straight line***, the direction of motion does not change. In such cases, all vectors can be treated like scalars.
- In problems where vectors can be treated like scalars, the direction of initial velocity is taken ***positive***

- A ***negative*** sign is applied to all those quantities whose direction is opposite to that of initial velocity.
- The above equations can also be applied to ***free fall*** motion of bodies by replacing a by g.

About acceleration due to Gravity (g)

In the absence of air all bodies fall freely near the surface of earth under the action of gravity with uniform acceleration called acceleration due to gravity. It is denoted by g.

Its average value near the surface of earth is taken as ***9.8m/sec***^{2}. It is a measure of strength of gravitational field.

The value of g decreases with height as well as depth, and it has the maximum value at the poles. It can also be defined as *gravitational force per unit mass.* [i.e. gravitational field strength $g = \frac{F}{m}$]

> **Do You Know?**
>
> 
>
> At the surface of the Earth, in situations where air friction is negligible, objects fall with the same acceleration regardless of their weights.
>
> **EXPLANATION:**
> According to Newton, the Earth attracts the bodies toward its centre, with force of gravity. The body falls with constant acceleration 'g' due to gravity. The force is called weight of the body, given by,
> W=mg, where g = W/m = 9.8 ms^{-2}, independent of the weight W.
> i.e. (Ratio between weight and mass remains same)

Q.4 Discuss Newton's laws of motion briefly.

Ans.

About Newton's Laws of Motion

Sir Isaac Newton published his empirical laws of motion in his famous book **"principia"** in **1687**. These laws hold good for bodies moving with smaller speed as compare to the speed of light.

For fast moving objects such as atomic particles in accelerators, relativistic mechanics developed by **Ablert Einstein** is applicable.

Newton's First Law of Motion / Law of inertia

Statement

A body at rest will remain at rest and a body moving with uniform velocity will continue to do so, unless unbalanced external force acts on it.

This is also known as law of inertia.

> **An unappreciated anticipation.**
> No body begins to move or comes to rest to itself.
> Abu Ali Sena, (980-1037)
> **EXPLANATION:**
> According to Newton's first law of motion, when no force acts on a body, it will not change its state of rest or of uniform motion. Mathematically, if
> F = 0 then a = 0. The same first law of motion was described by ABU ALI SINA, about 500 years before Newton (1643-1727).

Inertial frame of reference

The frame of reference in which Newton's first law holds is called *inertial frame of reference.* For example earth is approximately an inertial frame of reference.

Newton's Second Law of Motion

Statement

When a force is applied on a body, it produces the acceleration in its own direction. The magnitude of acceleration is directly proportional to applied force and inversely proportional to its mass.

Mathematically, it is expressed as

$$F = m\,a$$

Where F = applied force on body
m = mass of body
a = acceleration produced

~~Mathematical form~~

$$\cancel{\vec{F} = m\vec{a}}$$

Newton's Third Law of Motion

Statement

Action and reaction are equal in magnitude and opposite in direction.

Explanation

When two bodies interact with each other then; action and reaction forces act for the *same* length of time. They never act on the same body but always act on *different* bodies.

Q.5 What is inertia? Explain.

Ans.

Inertia

The property of a body due to which it tends to maintain its state of rest or uniform motion is called inertia.

Explanation

It is a natural resistance to ***acceleration*** that all objects have. The greater the object's mass, the greater this resistance. So,

The mass of the object is a quantitative measure of its inertia.

Examples

- When you make a turn while driving a car, you move the opposite way in which the car turns the corner.
- You get pressed back in your seat when an airplane takes off.
- Your face is smashed against the windshield if your car suddenly stops against a brick wall.

Q.6 Define and explain linear momentum.

Ans.

Momentum

The product of mass and velocity of moving body is called linear momentum.

Mathematically

$$\vec{p} = m\vec{v}$$

Where $\vec{p}$ = the momentum of the body

m = mass of the body

$\vec{v}$ = velocity of the body

Direction

Since $\vec{v}$ is a vector quantity, so momentum is also a vector quantity having same direction is same as that of velocity.

Interesting Information

Throwing a package onto shore from a boat that was previously at rest causes the boat to move out-ward from shore (Newton's third law).

EXPLANATION:
A force F is applied on the package by the man in the boat when it is thrown. As a reaction, a force F will act on the man in opposite direction. As the man is attached with the boat, the force -F is transferred to the boat and the boat moves out-ward from shore.

For Your Information
Action and reaction never balance each other.

Point to Ponder
A car accelerates along a road. Which force actually moves the car?
EXPLANATION:
Reactional force of the road is actually responsible to move the car. The engine of the car simply rotates the wheels but does not move it.

For Your Information
First law of motion gives the definition of force while second law give the measurement of force.

POINT TO PONDER
What is the effect on the speed of A fighter plane chasing another when it opens fire? What happens to the speed of pursued plane when it returns the fire?
EXPLANATION:
When the fighter plane opens fire, its momentum will be in back direction due to reaction force in backward direction and therefore its speed will decrease. When the pursued plane opens fire in back direction, the momentum will act on the plane in forward direction due to reaction in forward direction and therefore. its speed will increase

Unit

SI – unit of momentum is **kg m/sec or N-sec.** Its dimensions are $[MLT^{-1}]$

Physical meaning of momentum

When two bodies of same masses are moving with different velocities, then it is difficult to stop the *faster* one.

Similarly, when two bodies of different masses are moving with same velocities then it is difficult to stop the *massive* one.

This shows that the moving body has a quality due to which it exerts a force on anything that tries to stop it. The quality of moving body which describes the quantity of motion is ***called momentum.***

Q.7 Show that N s is equivalent to kg ms⁻¹.

Ans.

Proof

$$N\,s = kg\frac{m}{s^2}\times s = kg.\frac{m}{s} = kg\,ms^{-1} \quad \left[\because 1N = 1\,kg\frac{m}{s^2}\right]$$

Q.8 How force and momentum are related to each other? State Newton's second law in terms of momentum

Ans.

Momentum and Newton's Second Law of Motion

Consider a body of mass m moving with velocity $\vec{v}_i$. A force $\vec{F}$ is applied on the body for time t and its velocity changes to $\vec{v}_f$

Acceleration produced by the force is

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} \quad \text{\_\_\_\_\_\_ (1)}$$

According to Newton's second law of motion

$$\vec{F} = m\vec{a} \quad \text{\_\_\_\_\_\_ (2)}$$

Using equation (1) in (2), we have

$$\vec{F} = m\frac{\vec{v}_f - \vec{v}_i}{t}$$

OR

$$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{t} \quad \text{\_\_\_\_\_\_ (3)}$$

Where

$m\vec{v}_i$ = initial momentum of body

$m\vec{v}_f$ = final momentum of body

2nd Law in terms of momentum

Time rate of change of momentum of body is equal to the applied force.

Do you wear seat belt

When a moving car stops quickly, the passengers move forward toward the windscreen. Seat belts change the forces of motion and prevent the passengers from moving Thus the chance of injury is greatly reduced.
EXPLANATION:
When you fall towards forward direction, you apply force on the belt. As a reaction the belt applies force on you in back direction and thus you are saved from injury.

Point to Ponder

Which hurt you in the above situations (a) or (b) and think why?
EXPLANATION:
The time of collision Δt in case (a) is smaller and therefore, the impulsive force F will be greater which may hurt you.

This is more general form of Newton's second law of motion. Because it can easily be applicable for the cases when mass is changing.

For example

As rocket accelerates, it loses mass because its fuel is burnt and ejected to provide greater thrust.

Q.9 Define impulse how it is related to momentum?

Ans.

Impulse

When large force acts on a body for a very short interval of time, then the product of force and time for which the force acts, is called impulse of force.

Mathematically

$$\boxed{\vec{I}=\vec{F}\times t} \quad \text{——— (1)}$$

Unit

SI unit of impulse is **kg-m/sec or N-sec**. It is same as that of linear momentum. The dimensions of impulse are $[MLT^{-1}]$. It is a vector quantity.

Relation between impulse and momentum

According to Newton's second law of motion

$$\vec{F}=\frac{m\vec{v}_f-m\vec{v}_i}{t} \quad \text{——— (2)}$$

Using equation (2) in (1), we have

$$\vec{I}=\frac{m\vec{v}_f-m\vec{v}_i}{t}\times t$$

OR
$$\boxed{\vec{I}=m\vec{v}_f-m\vec{v}_i} \quad \text{——— (3)}$$

Thus, impulse = change in momentum of the body

Instantaneous change in momentum of body due to impulsive force is called impulse.

Concept of impulse

Some times the applied force is not constant and it acts for short time. e.g. when a bat hits a cricket ball the ball force varies from instant to instant during collision. In such cases it is more suitable to deal with force and time (i.e. impulse) instead of either quantity alone.

Impulsive Force

Force acting on a body for very short interval of time is called impulsive force.

Q.10 What is an isolated system? State and explain the law of conservation of linear momentum.

Ans.

Isolated system

... a system on which no external agency exerts any force.

Example

... molecules of a gas enclosed in a glass vessel at a constant temperature.

For Your Information

$\vec{F}=\frac{\Delta\vec{p}}{\Delta t}$ is more general form of force than $\vec{F}=m\vec{a}$

Do You Know?

Your hair acts like a crumple (soft) zone on your skull. A force of 5N might be enough to fracture your naked skull (cranium i.e. bony part), but with a covering of skin and hair, a force of 50N would be needed. Why it is so?

EXPLANATION:

The hair on the skull act as a soft zone. For naked skull a force of 5 N is sufficient to fracture because in this case time of collision at is smaller and the impulsive force F will be more effective. In case of the covered skull and hair, at is greater and the force F is not so effective. To produce the fracture, a force 50N instead of 5N will be required.

Does a moving object have impulse?

No, When the body is moving uniform velocity then F = 0 so impulse is zero.

When the body is moving with variable velocity then force is acting continuously. Hence no impulse is produced because no force acts for short interval of time.

Do You Know?

Area under the force time graph represents the impulse.

Law of Conservation of Momentum

Statement

The total linear momentum of an isolated system remains constant.

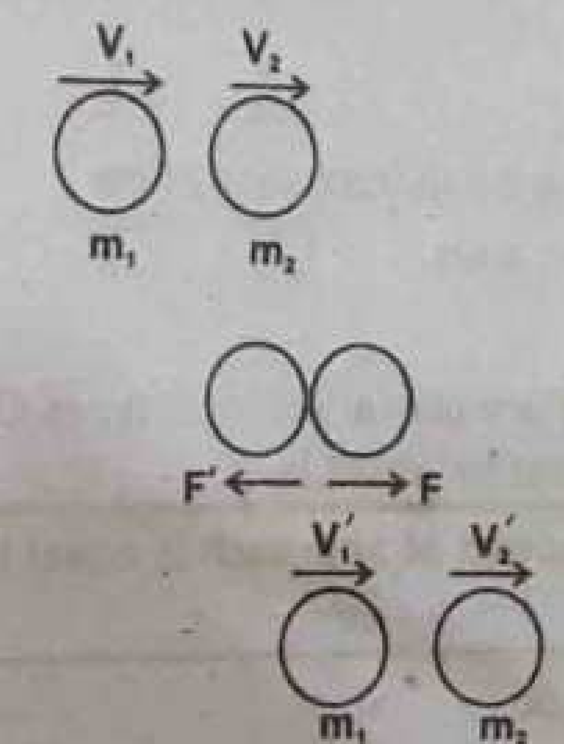
Proof

Consider an isolated system of two smooth non-rotating hard balls of masses m_1 and m_2 moving with velocities $\vec{v}_1$ and $\vec{v}_2$ in the same direction such that v_1 is greater than v_2 (i.e $v_1>v_2$). The two balls collide and their velocities changes to $\vec{v}'_1$ and $\vec{v}'_2$ respectively. Let

$\vec{F}'$ = force exerted on m_1 by m_2

$\vec{F}$ = force exerted on m_2 by m_1

According to Newton's second law of motion



$$\vec{F}'=\frac{m_1\vec{v}'_1-m_1\vec{v}_1}{t}$$

So the change in momentum of mass m_1 is

OR $$m_1\vec{v}'_1-m_1\vec{v}_1=\vec{F}'\times t \quad \text{——— (1)}$$

Similarly

$$\vec{F}=\frac{m_2\vec{v}'_2-m_2\vec{v}_2}{t}$$

So the change in momentum of mass m_2 is

OR $$m_2\vec{v}'_2-m_2\vec{v}_2=\vec{F}\times t \quad \text{——— (2)}$$

Adding equations (1) and (2), we have

$$m_1\vec{v}'_1-m_1\vec{v}_1+m_2\vec{v}'_2-m_2\vec{v}_2=\vec{F}'\times t+\vec{F}\times t$$

OR $$m_1\vec{v}'_1-m_1\vec{v}_1+m_2\vec{v}'_2-m_2\vec{v}_2=\left(\vec{F}'+\vec{F}\right)\times t \quad \text{——— (3)}$$

According to Newton's third law action and reaction are always equal but opposite in direction. So,

$$\vec{F}=-\vec{F}'$$

$$\Rightarrow \quad \vec{F}+\vec{F}'=\vec{0} \quad \text{——— (4)}$$

Using equation (4) in (3), we have

$$m_1\vec{v}'_1-m_1\vec{v}_1+m_2\vec{v}'_2-m_2\vec{v}_2=\vec{0}$$

OR $$\boxed{m_1\vec{v}_1+m_2\vec{v}_2=m_1\vec{v}'_1+m_2\vec{v}'_2}$$

It means that: initial momentum of system = final momentum of system

Thus ***the initial momentum and final momentum for an isolated system before and after the collision remains constant.***

Do You Know?

Which will be more effective in knocking a bear down.

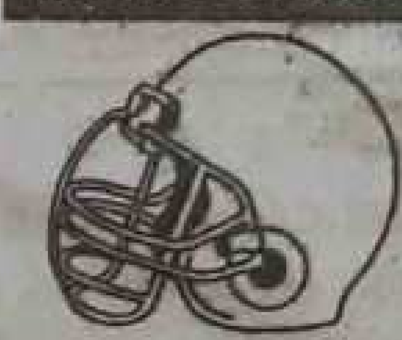
i. a rubber bullet or

ii. a lead bullet of the same momentum

EXPLANATION:

For knocking the bear impulse = F×t acts on the bear. In using the rubber bullet, the time of collision t will be smaller (due to bouncing) and impulsive force F will be larger but in case of lead bullet, the time of collision t will be greater (due to penetration) and the impulsive force F will be smaller. Therefore, the rubber bullet will be more effective, to knock down the bear.

Do You Know?



A motor bicycle's safety helmet is padded so as to extend the time of any collision to prevent serious injury.

EXPLANATION:

The impulse is the product of F and t. The padded helmet will increase the time of collision t and thus the impulsive force F is decreased.

Q.11 Define elastic and inelastic collision?

Ans.

Elastic and Inelastic Collision

Elastic Collision

The collision, in which kinetic energy of the system is conserved, is called elastic collision.

Example

Bouncing back of a hard ball from a marble floor is approximately an elastic collision.

Inelastic Collision

The collision is which the kinetic energy of the system is not conserved is called inelastic collision.

Example

- Bouncing back of a hard ball from sandy floor
- Collision of two tennis balls.

Note

Momentum and *total energy* are conserved in all types of collision.

Perfect elastic collision can not be possible.

> **Do You Know**
> Conditions for one dimensional elastic collision
> * Smooth and hard
> * Non – rotating ba
> * Impact paramete zero. (Head on coll

Q.12 Show that relative speed of approach is equal to relative speed of separation for elastic colli one dimension.

Ans.

Elastic Collision in One Dimension

Consider two smooth, non-rotating hard balls of masses m_1 and m_2 moving in such a way so that their c lie along the same straight line with initial velocities v_1 and v_2 respectively. When they make head on co with each other their velocities becomes v_1' and v_2' respectively as shown in figure.

According to law of conservation of linear momentum

$$m_1v_1+m_2v_2=m_1v_1'+m_2v_2'$$

OR $$m_1v_1-m_1v_1'=m_2v_2'-m_2v_2$$

OR $$m_1(v_1-v_1')=m_2(v_2'-v_2) \quad \text{\_\_\_\_\_\_ (1)}$$

As collision is perfectly elastic, so K.E. is also conserved. i.e.

$$\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2=\frac{1}{2}m_1v_1'^2+\frac{1}{2}m_2v_2'^2$$

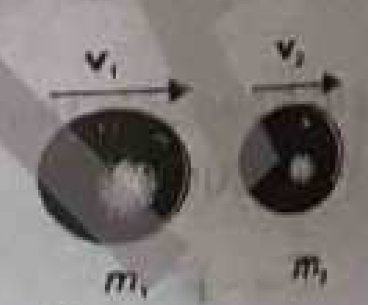
$$\frac{1}{2}m_1v_1^2-\frac{1}{2}m_1v_1'^2=\frac{1}{2}m_2v_2'^2-\frac{1}{2}m_2v_2^2$$

$$\frac{1}{2}m_1(v_1^2-v_1'^2)=\frac{1}{2}m_2(v_2'^2-v_2^2)$$

$$m_1(v_1^2-v_1'^2)=m_2(v_2'^2-v_2^2) \quad \text{\_\_\_\_\_\_ (2)}$$

Dividing equation (2) by (1)

$$\frac{m_1(v_1^2-v_1'^2)}{m_1(v_1-v_1')}=\frac{m_2(v_2'^2-v_2^2)}{m_2(v_2'-v_2)}$$



Before collision

After collision

Fig. 3.9

$$\frac{(v_1-v_1')(v_1+v_1')}{(v_1-v_1')}=\frac{(v_2'-v_2)(v_2'+v_2)}{(v_2'-v_2)}$$

OR $$v_1+v_1'=v_2'+v_2$$

OR $$v_1-v_2=v_2'-v_1' \quad \text{\_\_\_\_\_\_ (3)}$$

OR $$\boxed{v_1-v_2=-(v_1'-v_2')} \quad \text{\_\_\_\_\_\_ (4)}$$

Where

(v_1-v_2) = magnitude of velocity of first ball relative to second ball before the collision.

$(v_2'-v_1')$ = magnitude of velocity of second ball relative to first ball after the collision.

Hence

Relative speed of approach = Relative speed of separation

Q.13 Find the expressions for the velocities of two bodies m_1 and m_2 after elastic collision in one dimension.

Ans.

Determination of velocities after collision

We can calculate the velocities of the masses after collision by solving equations (1), (2) and (3).

Velocity of mass m_1 (i.e. v_1')

From equation (3),

$$v_2'=v_1-v_2+v_1' \quad \text{\_\_\_\_\_\_ (5)}$$

Using equation (5) in (1), we have

$$m_1(v_1-v_1')=m_2[(v_1-v_2+v_1')-v_2]$$

OR $$m_1v_1-m_1v_1'=m_2v_1-m_2v_2+m_2v_1'-m_2v_2$$

OR $$m_1v_1'+m_2v_1'=m_1v_1-m_2v_1+m_2v_2+m_2v_2$$

OR $$(m_1+m_2)v_1'=(m_1-m_2)v_1+2m_2v_2$$

$$\boxed{v_1'=\frac{(m_1-m_2)}{(m_1+m_2)}v_1+\frac{2m_2}{(m_1+m_2)}v_2} \quad \text{\_\_\_\_\_\_ (6)}$$

Velocity of mass m_2 (i.e. v_2')

Using equation (6) in (5), we have

$$v_2'=v_1-v_2+\frac{(m_1-m_2)}{(m_1+m_2)}v_1+\frac{2m_2}{(m_1+m_2)}v_2$$

$$v_2'=\left[1+\frac{(m_1-m_2)}{(m_1+m_2)}\right]v_1+\left[\frac{2m_2}{(m_1+m_2)}-1\right]v_2$$

$$v_2'=\left[\frac{(m_1+m_2)+(m_1-m_2)}{(m_1+m_2)}\right]v_1+\left[\frac{2m_2-(m_1+m_2)}{(m_1+m_2)}\right]v_2$$

$$v_2' = \left[\frac{2m_1}{(m_1+m_2)}\right]v_1 + \left[\frac{m_2-m_1}{(m_1+m_2)}\right]v_2$$

$$\boxed{v_2' = \frac{2m_1}{(m_1+m_2)}v_1 + \frac{(m_2-m_1)}{(m_1+m_2)}v_2} \quad \text{------ (7)}$$

Q.14 Discuss the following cases for the elastic collision in one dimension.

Ans.

Case I

When m_1 and m_2 are equal i.e. $m_1 = m_2 = m$ (say)

In this case, equations (6) and (7) become,

$$v_1' = \frac{(m_1-m_2)}{(m_1+m_2)}v_1 + \frac{2m_2}{(m_1+m_2)}v_2$$

$$v_1' = \frac{(m-m)}{(m+m)}v_1 + \frac{2m}{(m+m)}v_2$$

$$v_1' = \frac{0}{2m}v_1 + \frac{2m}{2m}v_2$$

$$v_1' = 0 + v_2$$

$$v_1' = v_2$$

and

$$v_2' = \frac{2m_1}{(m_1+m_2)}v_1 + \frac{(m_2-m_1)}{(m_1+m_2)}v_2$$

$$v_2' = \frac{2m}{(m+m)}v_1 + \frac{(m-m)}{(m+m)}v_2$$

$$v_2' = \frac{2m}{2m}v_1 + \frac{(0)}{2m}v_2$$

$$v_2' = v_1 + 0$$

$$v_2' = v_1$$

case (i)

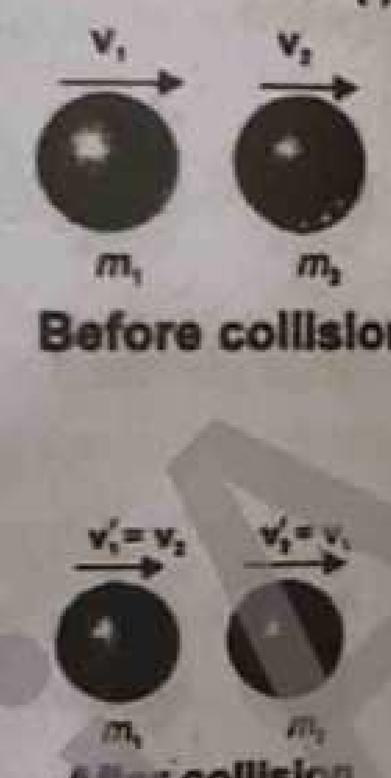


Fig. 3.10

Result

The masses m_1 and m_2 ***exchange*** their velocities after collision.

Case II

When m_1 and m_2 are equal while the target mass is at rest i.e. $m_1 = m_2$ & $v_2 = 0$, In this case, equations (6) and (7) become,

$$v_1' = \frac{(m_1-m_2)}{(m_1+m_2)}v_1 + \frac{2m_2}{(m_1+m_2)}v_2$$

$$v_1' = \frac{(m-m)}{(m+m)}v_1 + \frac{2m}{(m+m)}(0)$$

$$v_1' = 0 + 0$$

$$v_1' = 0$$

and

$$v_2' = \frac{2m_1}{(m_1+m_2)}v_1 + \frac{(m_2-m_1)}{(m_1+m_2)}v_2$$

$$v_2' = \frac{2m}{(m+m)}v_1 + \frac{(m-m)}{(m+m)}(0)$$

$$v_2' = \frac{2m}{2m}v_1 + 0$$

$$v_2' = v_1$$

case (ii)

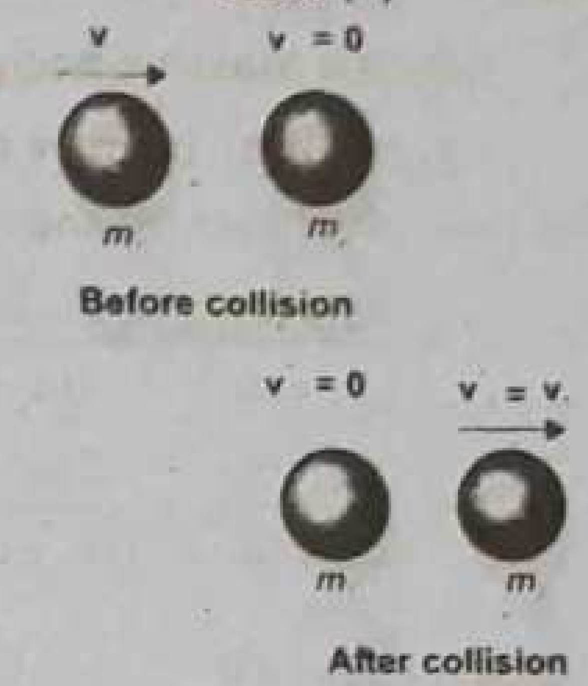


Fig. 3.11

Result

After collision, mass m_1 comes to ***rest*** and m_2 moves with the ***velocity of m_1***.

Case III

When lighter mass m_1 collides with a massive body m_2 at rest i.e. $m_1 << m_2$ or $m_1 = 0$ & $v_2 = 0$.

In this case, equations (6) and (7) become,

$$v_1' = \frac{(m_1-m_2)}{(m_1+m_2)}v_1 + \frac{2m_2}{(m_1+m_2)}v_2$$

$$v_1' = \frac{(0-m_2)}{(0+m_2)}v_1 + \frac{2m_2}{(0+m_2)}(0)$$

$$v_1' = -\frac{m_2}{m_2}v_1 + 0$$

$$v_1' = -v_1$$

and

$$v_2' = \frac{2m_1}{(m_1+m_2)}v_1 + \frac{(m_2-m_1)}{(m_1+m_2)}v_2$$

$$v_2' = \frac{2\times 0}{(0+m_2)}v_1 + \frac{(m_2-0)}{(0+m_2)}(0)$$

$$v_2' = 0 + 0$$

$$v_2' = 0$$

case (iii)

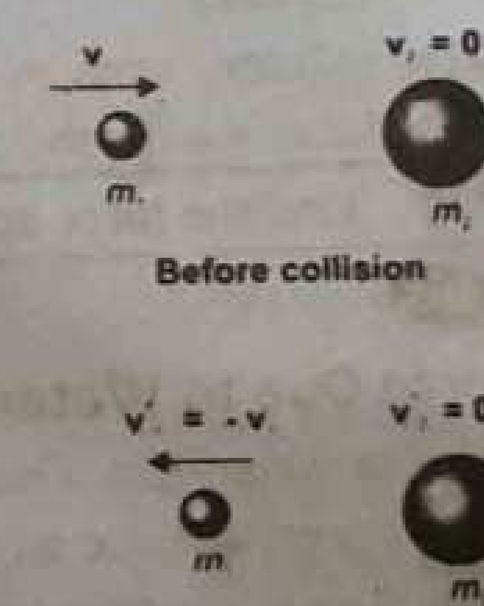


Fig. 3.12

Result

After collision mass m_2 remains at ***rest*** while m_1 ***bounces back*** with the same speed as before collision.

Case IV

When a massive body m_1 collides with a lighter body m_2 at rest

i.e. $m_1 >> m_2$ or $m_2 \approx 0$ & $v_2 = 0$.

In this case, equations (6) and (7) become,

$$v_1' = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_1 + \frac{2m_2}{(m_1 + m_2)} v_2$$

$$v_1' \approx \frac{(m_1 - 0)}{(m_1 + 0)} v_1 + \frac{2m_2}{(m_1 + 0)}(0)$$

$$v_1' \approx \frac{m_1}{m_1} v_1 + 0$$

$$v_1' \approx v_1$$

and

$$v_2' = \frac{2m_1}{(m_1 + m_2)} v_1 + \frac{(m_2 - m_1)}{(m_1 + m_2)} v_2$$

$$v_2' \approx \frac{2m_1}{(m_1 + 0)} v_1 + \frac{(0 - m_1)}{(m_1 + 0)}(0)$$

$$v_2' \approx \frac{2m_1}{m_1} v_1 + 0$$

$$v_2' \approx 2v_1$$

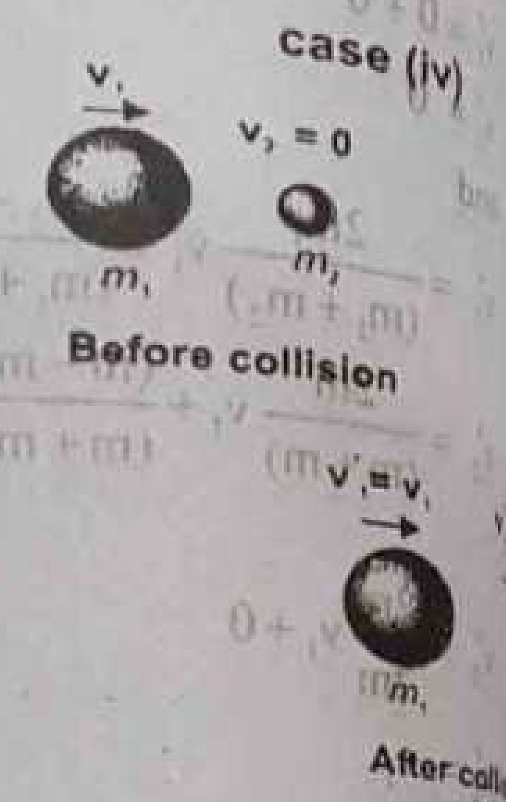


Fig. 3.13

Result

After collision, mass m_1 moves with *same* velocity but the mass m_2 moves with the velocity *double* the speed of m_1.

Q.15 Find the force due to water flow.

Ans.

Force Due to Water Flow

Suppose water from a horizontal pipe strikes a wall normally. Then it exerts a force on the wall. Let initially the velocity of water flow is $\vec{v}$ and on striking the wall, it comes to at rest so final velocity becomes zero. Thus,

initial velocity of water = $\vec{v}_i = \vec{v}$

final velocity of water = $\vec{v}_f = 0$

change in velocity of water = $\Delta \vec{v} = \vec{v}_f - \vec{v}_i = 0 - \vec{v} = -\vec{v}$

mass of the water that hits the wall in time t = m

change in momentum of water = $\Delta \vec{P} = m \Delta \vec{v} = -m\vec{v}$

Then according to Newton's second law of motion, the force $\vec{F}$ exerted on the water is given by

Do You Kno

For Your Infor

When a moving ... quickly, the pa... move forward to... windscreen. Se... 'change the fo... motion and prev... passengers from... Thus the chance o... is greatly reduced.

EXPLANATION

When you fall ... forward directio... apply force on ... As a reaction ... applies force on ... back direction ... you are saved ... injury.

$$\vec{F} = \frac{\text{change in momentum}}{\text{time}}$$

$$\vec{F} = \frac{-m\vec{v}}{t}$$

From Newton's third law of motion, a force of reaction exerted by water on the wall is equal but opposite. So

$$\vec{F} = -\left(\frac{-m\vec{v}}{t}\right)$$

So, $$\boxed{\vec{F} = \frac{m\vec{v}}{t} = \left(\frac{m}{t}\right)\vec{v}}$$

Thus force can be calculated as the product of mass of water striking normally per second and change in velocity.

Example

Suppose water flows from a pipe at 3 kgs^{-1} and its velocity changes from 5 ms^{-1} to zero on striking the wall, then

$$F = \frac{m}{t} v = 3(5 - 0) = 15\ N$$

Q.16 Is momentum conserved, when momentum changes are produced by explosive forces? Explain.

Ans.

DO YOU KNOW?

Explosion is due to the result of internal forces.

Momentum and Explosive Forces

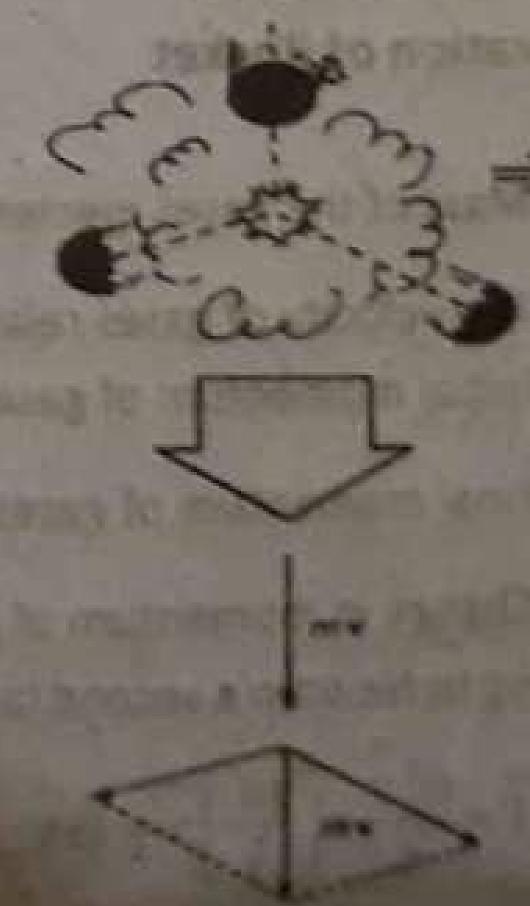
Total momentum remains same when momentum changes occur due to explosive forces within an isolated system.

Examples

1- Explosion of a shell or bomb

Suppose a bomb is falling, in its way it explodes into two pieces. The momentum of the bomb fragments combined by vector addition is equal to the original momentum of falling bomb, as shown in figure.

2- Firing of a rifle



When a bullet of mass m fired is from a rifle of mass M with a velocity $\vec{v}$.

Momentum before fire

Initial momentum is zero as both bullet and rifle are initially at rest.

Momentum after fire

If $\vec{v}'$ is the velocity of recoil of the rifle.

Then, final momentum = $m\vec{v} + M\vec{v}'$

According to the law of conservation of momentum.

momentum before fire = momentum after fire

$$0 = M\vec{v}' + (-m\vec{v})$$

OR $$M\vec{v}' = -m\vec{v}$$

OR $$\boxed{\vec{v}' = \frac{-m\vec{v}}{M}}$$

Which is the velocity of recoil of the rifle. Since mass of rifle is very much greater, so the velocity of recoil of rifle is very small.

Q.17 How is rocket propelled in space? Derive an expression for acceleration of rocket.

Ans.

Rocket Propulsion

Working Principle

Motion of rocket is based on the law of conservation of ***momentum*** and Newton's ***third*** law of motion.

Propulsion

Rocket moves up by ejecting burning gasses from its rear part of engine. When fuel is burned, it turns to high pressure gases. These gases come out from the engine with very high velocity. The rocket gains momentum equal to the momentum of expelled gases but in opposite direction. Rocket continues to gain more and more momentum as long as engine of the rocket continues to expel gases. So the speed of rocket goes on increasing as long as the engines are operating.

Fuel and oxygen [illegible] combustion chamber [illegible] exhaust the chamber [illegible] velocity. The gain in momentum [illegible] the gases equals [illegible] momentum of the rocket [illegible] and rocket push against [illegible] and moves in opposite direction

Fuel of the rocket

Fuel is in the form of ***liquid or solid*** and oxygen. **80%** of the launch mass of the rocket consists of fuel only. For enough upward thrust to overcome the gravity, a typical rocket consumes about **10,000 kg/sec** of fuel. Rocket ejects the burnt gases at speeds of over **4000m/sec.**

Problem of mass

To overcome this problem several rockets are linked together. When one of them performs its function, then it is discarded due to which the rocket moves up with greater speed.

Acceleration of Rocket

Let

Mass of the gases ejected in time $t = m'$

The velocity of gases relative to the rocket $= \vec{v}$

Initial momentum of gases $= 0$

Final momentum of gases $= m'\vec{v}$

Change in momentum of gases $= m'\vec{v} - 0 = m'\vec{v}$

According to Newton's second law of motion, the force exerted on the gases by the rocket is

$$\vec{F} = \frac{m'\vec{v}}{t} = \left(\frac{m'}{t}\right)\vec{v} = m\vec{v} \quad \text{\_\_\_\_\_\_ (i)}$$

Where m = mass of gases ejected per second

The force exerted on the rocket by the gases is

$$\vec{F} = M\vec{a} \quad \text{\_\_\_\_\_\_ (2)}$$

Where M = instantaneous mass of rocket & $\vec{a}$ = acceleration of rocket.

By Newton's third law of motion

$$M\vec{a} = m\vec{v}$$

OR $$\boxed{\vec{a} = \frac{m\vec{v}}{M}} \quad \text{\_\_\_\_\_\_ (3)}$$

As the time passes, the mass M of the rocket decreases due to the burning of fuel and ejection of gases and hence acceleration $\vec{a}$ increases.

> **For your Information**
> If air resistance is taken into consideration then time of rise of a ball is not equal to its time of fall.

> Range of projectile for complimentary angles of projection is same.

Q.18 What is projectile motion? Explain. Derive expression for the instantaneous velocity of a projectile thrown at an angle θ with horizontal

Ans.

Projectile Motion

Two dimensional motion under the constant acceleration due to gravity and inertia is called projectile motion.

Example of projectile motion

(i) A football kicked by a player.
(ii) Bullet fired from gun
(iii) A missile fired from a lunching pad.
(iv) A ball thrown by a cricketer

> **Do You Know?**
> Can there be a motion in two dimensions with acceleration only in one dimension?
> Yes, it becomes possible in projectile motion.

Explanation of projectile motion

Consider the motion of a body thrown horizontally from a certain height with velocity v_x. The body moves forward (due to ***inertia***) as well as downward (due to ***gravity***). As no force acts along the horizontal direction, so the acceleration of body in the horizontal direction is zero ($a_x = 0$). Only force acting on body is force of gravity. So, the body has vertical acceleration equal to g. Hence, the body moves with constant horizontal velocity and constant vertical acceleration. Such a motion is called projectile motion.

Fig.3 [illegible]

Trajectory

The path followed by the projectile is called its trajectory. The trajectory of a projectile is usually a ***parabola***. The path of projectile can be determined by the following equations

$$x = v_{ix}t + \frac{1}{2}a_x t^2 \quad \text{and} \quad y = v_{iy}t + \frac{1}{2}a_y t^2$$

As $a_x = 0$, $v_{ix} = v_x$ and $a_y = -g$, $v_y = v_{iy}$

So $$x = v_{ix}t \quad \text{\_\_\_\_\_\_ (1)}$$

And $$y = v_{iy}t - \frac{1}{2}gt^2 \quad \text{\_\_\_\_\_\_ (2)}$$

> At maximum height, the velocity of the projectile is minimum but not zero.

Special Case: If body is projected horizontally from a certain height. Then

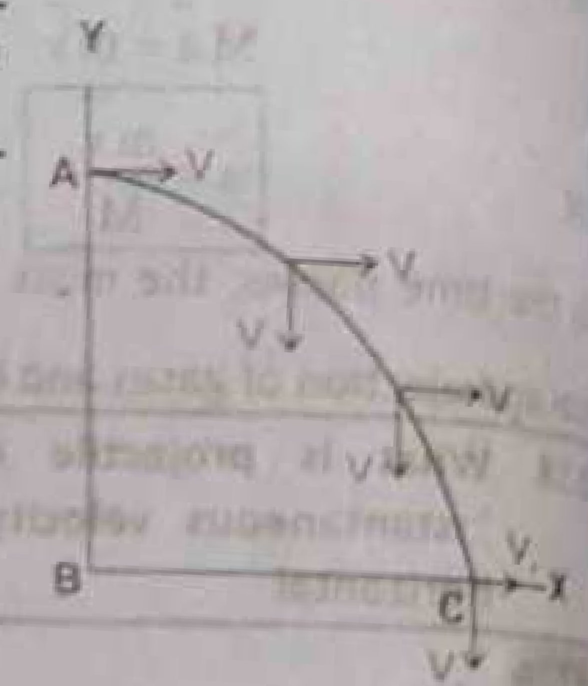
$v_y = v_{iy} = 0 \qquad a_x = 0$

So equ (2) becomes

and $y = \frac{1}{2} g t^2$ ________ (2)

Equation (1) represents the horizontal distance while equation (2) represents the vertical distance of the body

Q.19 Derive expression for the instantaneous velocity of a projectile thrown at an angle θ with horizontal.



Ans.

Instantaneous Velocity (v_t)

Suppose a projectile is fired with initial velocity v_i at an angle θ with horizontal.

Let

Horizontal component of initial velocity $= v_{ix} = v_i \cos\theta$

Vertical component of initial velocity $= v_{iy} = v_i \sin\theta$

Horizontal component of velocity

Since there is no horizontal force along horizontal axis so acceleration $a_x = 0$.

So, Horizontal component of velocity at any instant t is

$$v_{fx} = v_{ix} + a_x t$$

$$v_{fx} = v_{ix} \quad (\because a_x = 0)$$

OR $\boxed{v_{fx} = v_{ix} = v_i \cos\theta}$ ________ (1)

Vertical component of velocity

Vertical component of velocity at any instant t is

$$v_{fy} = v_{iy} + a_y t$$

OR $\boxed{v_{fy} = v_i \sin\theta - gt}$ ________ (2) $[\because a_y = -g \text{ and } v_{iy} = v_i \sin\theta]$

Magnitude of velocity

Thus the magnitude of velocity at any instant t is

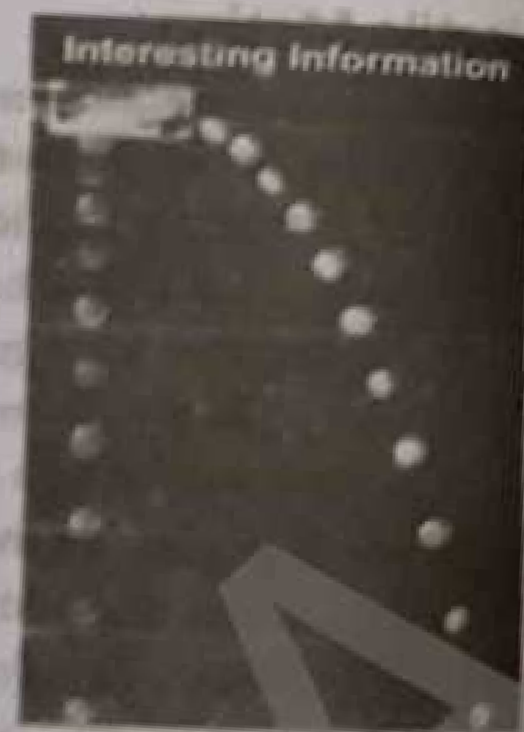
$\boxed{v = \sqrt{v_{fx}^2 + v_{fy}^2}}$ ________ (3)

Direction of velocity

Let resultant velocity makes an angle φ with the horizontal. Then

$$\tan\phi = \frac{v_{fy}}{v_{fx}}$$

$$\phi = \tan^{-1}\frac{v_{fy}}{v_{fx}}$$



A photograph of two balls released simultaneously from a mechanism that allows one ball to drop freely while the Other is projected horizontally. At any time the two balls are at the same level, i.e., their vertical displacements are equal.

EXPLANATION:

Since vertical acceleration $a_y = g$ in both cases so at any time, their vertical velocities are equal. Hence the two balls are at the same level, i.e., their vertical displacements are equal.

Q.20 A projectile is thrown with initial velocity v_i making an angle θ with the horizontal. Find its

(a) Maximum height **(b) Time of flight**

(c) Range **(d) Maximum range**

Ans.

Max-Height of the projectile

The maximum vertical distance covered by the projectile is called the maximum height of the projectile.

For the determination of height we take

Initial vertical velocity of projectile $= v_{iy} = v_i \sin\theta$

Vertical acceleration $= a_y = -g$

Vertical velocity at highest point $= v_{fy} = 0$

Maximum height $= y = H = ?$

Now according to equation of motion,

$$2aS = v_f^2 - v_i^2$$

OR $2a_y y = v_{fy}^2 - v_{iy}^2$

$$2(-g)H = 0 - (v_i \sin\theta)^2$$

$$-2gH = -v_i^2 \sin\theta^2$$

$\boxed{H = \frac{v_i^2 \sin^2\theta}{2g}}$ ________ (1)

Time of flight

The time taken by body to cover the distance from the place of projection to the place where it hits the ground is called the time of flight.

As the body goes up and comes back to same level so it covers no vertical distance. i.e.

$$y = 0$$

Initial vertical velocity of projectile $= v_{iy} = v_i \sin\theta$

acceleration due to gravity $= a_y = -g$

time of flight $= t = ?$

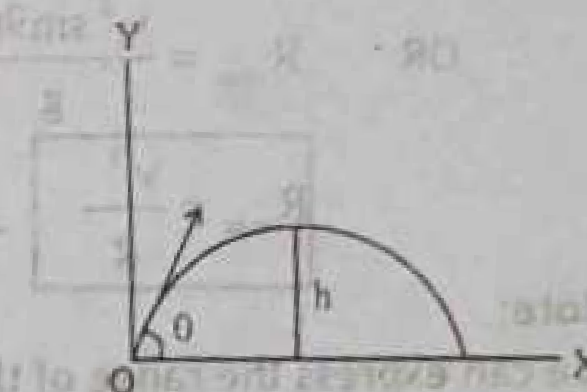
As $S = v_i t + \frac{1}{2} a t^2$

OR $y = v_{iy} t + \frac{1}{2} a_y t^2$

OR $0 = (v_i \sin\theta) t - \frac{1}{2} g t^2$

OR $\frac{1}{2} g t^2 = (v_i \sin\theta) t$

$\boxed{t = \frac{2 v_i \sin\theta}{g}}$ ________ (2)



For Your Information

$$H = \frac{1}{4} R_{max}$$

For Your Information

The factor which remains constant during the projectile motion are
* horizontal velocity
* both x and y components of acceleration

Range of projectile

Maximum distance which a projectile covers in the horizontal direction is called the range of projectile.

Then $x = v_{ix} t + \frac{1}{2} a_x t^2$ becomes

$$R = v_i \cos\theta \times \frac{2 v_i \sin\theta}{g} + 0$$

OR $R = \frac{v_i^2 (2 \sin\theta \cos\theta)}{g}$

$\boxed{R = \frac{v_i^2 \sin 2\theta}{g}}$ ________ (3) $[\because 2 \sin\theta \cos\theta = \sin 2\theta]$

Maximum Range

The range of the projectile will be maximum when $\sin 2\theta$ has maximum value.
i.e.

$$\sin 2\theta = 1$$

OR $2\theta = \sin^{-1}(1)$

OR $2\theta = 90^\circ$

OR $\theta = 45^\circ$

So, equation $R = \dfrac{v_i^2 \sin 2\theta}{g}$ becomes

$$R_{max} = \frac{v_i^2 \sin 2(45^\circ)}{g}$$

OR $R_{max} = \dfrac{v_i^2 \sin 90^\circ}{g}$

$$\boxed{R_{max} = \frac{v_i^2}{g}} \quad \text{\_\_\_\_\_ (4)} \quad [\because \sin 90^\circ = 1]$$

Note:

We can express the range of the projectile in terms of maximum range as $\boxed{R = R_{max}\sin 2\theta}$

Q.21 Define ballistic flight, ballistic missile and ballistic trajectory.

Ans.

Ballistic Flight

When a projectile is given an initial push and is then allowed to move freely due to inertia and under the action of gravity, then such a flight is called a ballistic flight.

Ballistic Missile

The un-powered and unguided missile is called ballistic missile.

Ballistic Trajectory

The path followed by the ballistic missile is called ballistic trajectory.

Explanation

Ballistic missile moves due to the super position of two independent motions:

- a straight line inertial flight along the direction of launch
- vertical gravity fall.
- The gravity and inertia are responsible for the parabolic path of projectile.

Effect of inertia

Due to inertia an object should move straight off in the direction in which it is thrown, at constant speed equal to its initial speed particularly in empty space.

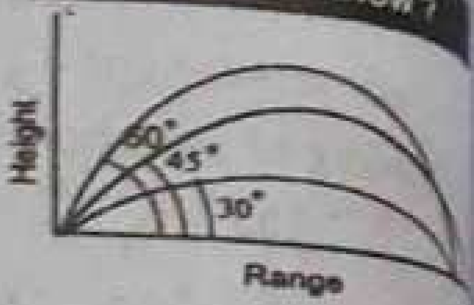
Effect of gravity

Due to gravity straight path changes into a curved trajectory.

For flat earth (For short ranges)

The trajectory of projectile is *parabolic*.

Do You Know?



Water is projected from two ... pipes at the same speed-from ... an angle of 30° and from the ... at 60°. Why are the ranges equal?

EXPLANATION:

The range of a projectile is given ...

$$R = \frac{v_i^2}{g}\sin 2\theta$$

As $\sin 2\theta = \sin 2 \times 30^\circ$ ... $\sin 60^\circ = 0.866$

And $\sin 2\theta = \sin 2 \times 60^\circ$ = ... $120^\circ = 0.866$

Therefore, R is same for both ... angles.

For an angle less than 45°, the ... reached by the projectile and the ... both will be less. When the angle ... projection is larger than 45°, the ... attained will be more but the ... again less.

EXPLANATION:

The range is given by, $R = \dfrac{v_i^2}{g}\sin 2\theta$

The range depends on $\sin 2\theta$.
When $\theta = 45^\circ$, the range becomes maximum that is

$$R_{max} = \frac{v_i^2}{g}\sin 90^\circ = \frac{v_i^2}{g}$$

The values $\sin 2\theta$ for all other angles ... or greater than 45° are less than 1, therefore, the range, regarding all other angles is smaller than the range at 45°.
The height h is given as,

$$h = \frac{v_i^2 \sin^2\theta}{g}$$

The height depends on angle θ ... greater values of θ, the height will ... greater and smaller for smaller values of θ.

For spherical earth

Ballistic trajectory will be *elliptical*.

The major element of warfare is to shoot the missile on a selected distant spot.

Uses of Ballistic Missiles

The ballistic missiles are useful only for *short ranges*.

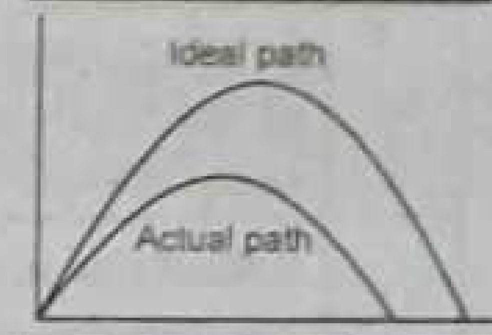
For large ranges and greater precision powered and remote control guided missiles are used.

At high speed and for long trajectories the ***air friction*** is not negligible and some times the force of air friction is more than gravity. It affects both its horizontal and vertical motion. Therefore, we cannot neglect the ***aerodynamic forces***.

Effect of aerodynamic forces

The air friction creates difficulties for this purpose. So the angle of projection needs a high degree of precision.

For Your Information



In the presence of air friction, the trajectory of a high speed projectile fall short of a parabolic path.

EXPLANATION:

Due to air friction, the vertical and horizontal velocities decrease and therefore, (the height and range both decrease which make fall short of a parabolic trajectory, as shown in the figure.

FORMULAE

1	Displacement of a particle	$\vec{d} = \Delta\vec{r} = \vec{r}_2 - \vec{r}_1$	
2	Average velocity	$\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$	
3	Instantaneous velocity	$\vec{v} = \lim\limits_{\Delta t \to 0}\dfrac{\Delta\vec{d}}{\Delta t}$	
4	Average acceleration	$\vec{a}_{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t}$	$\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$
5	Instantaneous acceleration	$\vec{a} = \lim\limits_{\Delta t \to 0}\dfrac{\Delta\vec{v}}{\Delta t}$	
6	2nd law of motion	$\vec{F} = m\vec{a}$	
7	Linear momentum	$\vec{P} = m\vec{v}$	
8	2nd law of motion in terms of momentum	$\vec{F} = \dfrac{m\vec{v}_f - m\vec{v}_i}{t}$	
9	Impulse	$\vec{I} = \vec{F} \times t$	$\vec{I} = \vec{F} \times t = m\vec{v}_f - m\vec{v}_i$
10	Law of conservation of linear momentum	$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$	

11	Relation between relative velocity of approach and relative velocity of separation	$\vec{v}_1 - \vec{v}_2 = -(\vec{v}_1' - \vec{v}_2')$
12	Velocity of mass m_1 after collision in one dimensional elastic collision	$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$
13	Velocity of mass m_2 after collision in one dimensional elastic collision	$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$
14	Force due to water flow	$F = \frac{m}{t} v$
15	Recoil velocity of a rifle	$v' = -\frac{mv}{M}$
16	Acceleration of rocket	$a = \frac{mv}{M}$
17	Horizontal distance of an object thrown horizontally from height h	$x = v_{ix} t$
18	Vertical distance of an object thrown horizontally from height h	$y = \frac{1}{2} g t^2$
19	x-component of instantaneous velocity of a projectile	$v_{fx} = v_{ix} = v_i \cos\theta$
20	y-component of instantaneous velocity of a projectile	$v_{fy} = v_i \sin\theta - gt$
21	Instantaneous velocity of a projectile	$v = \sqrt{v_{fx}^2 + v_{fy}^2}$
22	Height of projectile	$H = \frac{v_i^2 \sin^2\theta}{2g}$
23	Time flight of projectile	$T = \frac{2v_i \sin\theta}{g}$
24	Range of projectile	$R = \frac{v_i^2 \sin 2\theta}{g}$
25	Maximum range of projectile	$R_{max} = \frac{v_i^2}{g}$

Multiple Choice Questions

♦ *Four possible answers to each statement are given below. Tick (✓) the correct answer:*

1. Which of the following can be zero if a body is in motion for some time?
   (a) Speed (b) Displacement
   (c) Distance covered (d) None
2. If the displacement covered by body is zero, then what can you say about its distance?
   (a) It is negative (b) It may and may not be zero
   (c) It must be zero (d) It cannot be zero
3. The slope of the velocity time graph for retarded motion is:
   (a) Zero (b) Positive
   (c) Negative (d) Neutral
4. Two bodies are moving in opposite direction with velocity v. What is relative velocity between them
   (a) $\sqrt{2}v$ (b) 2v
   (c) v (d) Zero
5. Area under velocity time graph represents.
   (a) Force (b) Displacement
   (c) Distance (d) Acceleration
6. Instantaneous and average velocities becomes equal if body has ________:
   (a) Zero acceleration (b) Uniform acceleration
   (c) Variable acceleration (d) Moves in a circle
7. Inertia of an object is measure by its ________
   (a) Volume (b) Density
   (c) Mass (d) Temperature
8. 2nd law of motion defines ________
   (a) Inertia (b) Acceleration
   (c) Velocity (d) Both a and b
9. When a body moves in a straight line then its displacement coincides with ________
   (a) Distance (b) Force
   (c) Velocity (d) None
10. The rate of change in momentum of a body falling freely is equal to its ________
    (a) K.E. (b) Momentum
    (c) Power (d) Weight
11. A small sports car collides head-on with a massive truck. Which vehicle experiences the greater impact force (in magnitude)?
    (a) The car (b) The truck
    (c) They experience the same force (d) None of these
12. In above question, which vehicle experiences the greater acceleration?
    (a) The car (b) The truck
    (c) They experience the same acceleration (d) None of these

13. **A base ball of mass m is thrown upward with some initial speed. If air resistance is neglected, the force acting on the ball when it reaches its peak is ________**

(a) mg and upward (b) mg and downward
(c) Zero (d) None of these

14. **A body is moving in a straight line such that the distance covered by it in time t is proportional to the square of the time t. The acceleration of the body is:**

(a) Constant (b) Zero
(c) Increasing (d) Decreasing

15. **SI unit of impulse is equivalent to**

(a) Force (b) Momentum
(c) Acceleration (d) None

16. **Taking off rocket can be explained by ________**

(a) 1st law of motion (b) 2nd law of motion
(c) 3rd law of motion (d) None

17. **Which component of acceleration is zero in projectile motion?**

(a) Horizontal (b) Vertical
(c) Both (d) None

18. **A fighter plane drops a bomb when it is at the top of enemies target. Bomb misses the target due to**

(a) Due to bad weather (b) Due to action of gravity
(c) Due to horizontal component of velocity (d) All of above

19. **A ball is thrown horizontally from the top of tower. What happens to the horizontal component of its velocity?**

(a) First increased then decrease (b) Increases
(c) Remains same (d) None of these

20. **What is angle of projection, so that the horizontal range is equal to maximum height?**

(a) $Tan^{-1}(4)$ (b) $Tan^{-1}(3)$
(c) $Tan^{-1}(2)$ (d) $Tan^{-1}(1)$

ANSWERS

1. b	2. b	3. c	4. b	5. c	6. a	7. c	8. b	9. a	10. d
11. c	12. a	13. b	14. a	15. b	16. c	17. a	18. c	19. c	20. a

Short Questions of Exercises

Q.3.1 What is the difference between uniform and variable velocity? From the explanation of variable velocity, define acceleration. Give SI units velocity and acceleration?

(Bwp 2004, D.G.Khan 2005-2006, Lhr 2010-

Ans. Difference between uniform and variable velocity:

- In case of uniform velocity the body covers ***equal displacements*** in equal intervals of time but in case of variable velocity it covers ***unequal displacements*** in equal intervals of time.
- In case of variable velocity, the magnitude or direction of velocity or both may ***change*** whereas in uniform velocity both the magnitude and direction ***remains the same***.

Definition of acceleration:

The time rate of a change of velocity of a body is called acceleration.

Mathematically,

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

where $\vec{v}_i$ = initial velocity of body

$\vec{v}_f$ = final velocity of the body

$\Delta \vec{v}$ = change in velocity during time Δt

S.I units of Acceleration & velocity:

SI unit of velocity is m/sec

SI units of acceleration is m/sec^2

Q.3.2 An object is thrown vertically upward. Discuss the sign of acceleration due to gravity, relative to velocity, while the object is in air?

(Bwp 2003, Mir Pur 2004, Fsd 2008)

Ans. Sign of acceleration due to gravity relative to velocity is ***negative***.

Explanation:

All those quantities are assigned to be negative sign whose direction is opposite to the direction of initial velocity.

So when the object is thrown upward, the direction of gravitational pull is opposite to the direction of initial velocity so the sign of acceleration due to gravity relative to velocity is *negative*.

Q.3.3 Can the velocity of an object reverse direction when acceleration is constant? If so, give an example.

(Federal 2003-2005, Lhr 2005, Sgd 2005, Grw 2005-2010, Fsd 2008, Mir Pur 2009, Lhr 2010-2011)

Ans. ***Yes***, it can be possible.

Example:

When a body is thrown vertically upward its velocity goes on decreasing due to gravity and becomes zero at the maximum height. After that it will reverse its direction of velocity, but the acceleration remains constant during whole flight (*i.e* $9.8\ m/s^2$).

Q.3.4 Specify the correct statement:

a. An object can have a constant velocity even its speed is changing.
b. An object can have a constant speed even its velocity is changing.
c. An object can have a zero velocity even its acceleration is not zero.
d. An object subjected to a constant acceleration can reverse its velocity.

Ans. Statements (b), (c) and (d) are correct.

Explanation:

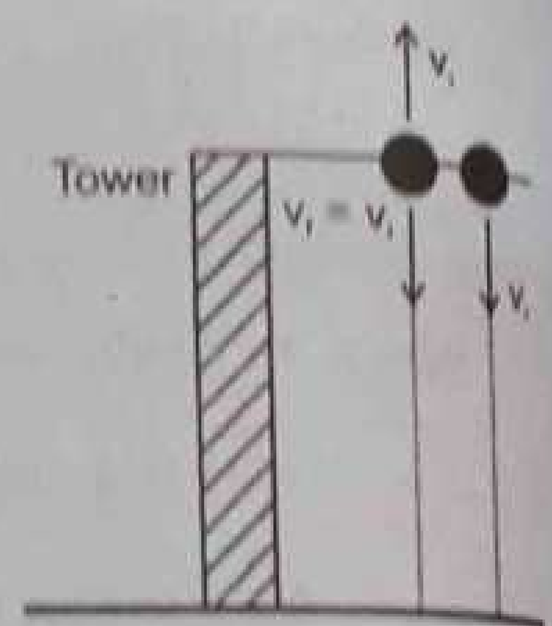
(a) As when speed changes, velocity also changes so it is ***not true*** statement.

(b) when a body moves in a circle its speed is constant but its velocity changes due to change in direction. So it is a ***true*** statement.

(c) when a moving object is stopped by applying the sudden hard brakes, velocity becomes zero at that instant but acceleration is not. So it is ***true*** statement.

(d) when an object is thrown vertically up, after reaching it maximum height, its velocity reverses but its acceleration remains constant (i.e 9.8 m/s²). So it is also ***true*** statement.

5 A man standing on the top of a tower throws a ball straight up with initial velocity v_i and at the same time throws a second ball straight downward with the same speed. Which ball will have large speed when it strikes the ground? Ignore air friction.



s. Both the balls hit the ground with ***same*** speed.

Explanation:

The ball which is thrown vertically up with velocity v_i will have same velocity v_i when it reaches back to the top of tower. So the two balls have ***same*** downward velocity at top of tower. Hence they hit the ground with same final velocity.

Q.3.6 Explain the circumstances in which the velocity $\vec{v}$ and acceleration $\vec{a}$ of a car are;

(i) Parallel

(ii) Anti-parallel

(iii) Perpendicular to one another

(iv) $\vec{v}$ is zero but $\vec{a}$ is not zero

(v) $\vec{a}$ is zero but $\vec{v}$ is not zero

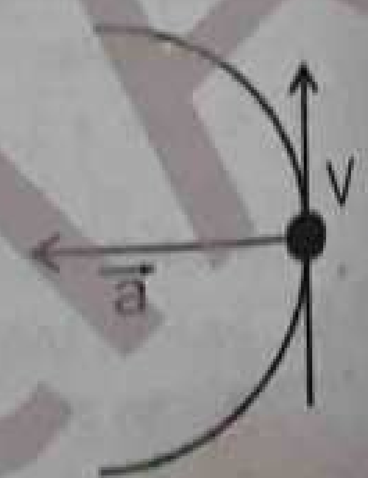
(Fsd 2005, Federal 2005, Mir Pur 2009, Mtn 2009, Grw 2005-2009-2010)

Ans. (i) Parallel:

If the velocity of the car is ***increasing*** along a straight path then velocity $\vec{v}$ and acceleration $\vec{a}$ are parallel.

(ii) Anti-parallel

If the velocity of the car is ***decreasing*** along a straight path then velocity $\vec{v}$ and acceleration $\vec{a}$ are anti parallel. For example when the brakes are applied to a moving car.



(iii) perpendicular to each other

If the car is moving in a ***circular path*** then the velocity $\vec{v}$ and $\vec{a}$ are mutually perpendicular.

(iv) $\vec{v}$ is zero but $\vec{a}$ is not zero

If moving car is ***stopped*** by applying sudden hard brakes then at that instant the velocity of the car $\vec{v}$ becomes zero but the acceleration is not zero.

(v) $\vec{a}$ is zero but $\vec{v}$ is not zero

When car moves with ***uniform*** velocity then $\vec{v}$ is not zero but $\vec{a}$ is zero.

Q.3.7 Motion with constant velocity is a special case of motion with constant acceleration. Is this statement true? Discuss.

(Rwp 2005, Fsd 2008, Lhr 2009)

Ans. ***Yes***, it is true statement.

Reason:

When the body moves with uniform velocity then the ***change in velocity is zero.*** So acceleration is zero. It remains zero as far as the body moves with constant velocity. Hence acceleration remains constant (i.e., 0m/sec²) throughout the motion of the body.

Q.3.8 Find the change in momentum for an object subjected to a given force for a given time and state law of motion in terms of momentum?

(Bwp 2004, Federal 2005, Grw 2008)

Ans. Let m = mass of the body $\quad \vec{v}_i$ = initial velocity $\quad$ F = applied force

$\vec{v}_f$ = final velocity $\quad$ t = time interval

Then
$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

According to Newton's second law of motion,
$$\vec{F} = m\vec{a}$$
$$\vec{a} = \frac{\vec{F}}{m}$$

Comparing equations (1) and (2) we get
$$\frac{\vec{F}}{m} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$
$$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$$

where $m\vec{v}_i$ represents the initial momentum while $m\vec{v}_f$ represents final momentum.

Thus we can say

Second law in terms of momentum

The time ***rate of change of momentum*** of body is equal to the applied force.

Q.3.9 Define impulse and show that how it is related to linear momentum?

(Federal 2003, Sgd 2003, Lhr 2004, Bwp 2006, Grw 2008, Lhr 2009-2010-2011)

Ans. Impulse

When a large force acts on a moving body for a short interval of time then the product of force and time is called impulse.

Relation
$$\vec{I} = \vec{F} \times t$$

As force is the time rate of change of momentum So
$$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{t}$$

or $\quad \vec{F} \times t = m\vec{v}_f - m\vec{v}_i$

$$I = m\vec{v}_f - m\vec{v}_i \qquad (I = \vec{F} \times t)$$

So the impulse is equal to the instantaneous *change in momentum* of the body.

Q.3.10 State the law of conservation of linear momentum, pointing out the importance of isolated system. Explain, why under certain conditions, the law is useful even though the system is not completely isolated?

(Mtn 2003, D.G.Khan 2005, Grw2011)

Ans: Law of conservation of momentum

Total linear momentum of an isolated system always remains constant.

Importance of Isolated system

Law of conservation of momentum holds only for isolated systems otherwise it is not valid.

Application for not completely isolated systems

When the effect of external force (like frictional and gravitational forces) is *negligibly small* as compared to the forces between the interacting objects, then this law become applicable.

Q.3.11 Explain the difference between elastic and inelastic collisions. Explain how would a bouncing ball behave in each case? Give plausible reasons for the fact that K.E is not conserved in most cases?

(Bwp 2005-2008, Grw 2005, Lhr 2009)

Ans. Difference between elastic and inelastic collision:

In case of elastic collision the K.E of the system is conserved while in case of inelastic collision the K.E is not conserved.

But the total linear momentum and the total energy of the system remains constant in both types of collision.

Behaviour of bouncing ball:

When a hard ball is dropped onto a marble floor, it rebounds to very nearly the initial height. It loses negligible amount of energy in the collision with the floor. Then such collision is approximately an elastic collision.

But if the ball is not able to reach the initial height then there is a loss of kinetic energy and such collision is called inelastic collision.

Solid reasons for loss of kinetic energy:

In case of inelastic collision, the loss of kinetic energy is due to

- friction of ball with floor
- friction of ball and air
- sound

Q.3.12 Explain what is meant by projectile motion? Derive the expression for

a. the time of flight

b. the range of projectile.

Show that the range of projectile is maximum when projectile is thrown at an angle of 45° with horizontal.

(Federal 2003, D.G.Khan 2005, Mir Pur 2006-2009)

Ans. The two dimensional motion under the constant acceleration due to gravity and inertia is called projectile motion.

Examples A foot ball kicked by a player.
A ball thrown by a cricketer.
A missile fired from a launching pad.

Time of flight

The time taken by body to cover the distance from the place of projection to the place where it just to hit the ground is called the time of flight.

Since the projectile covers no vertical distance after its total time of flight. So S = h = 0. If $v_i \sin\theta$ is the vertical component of initial velocity Then,

$$S = v_i t + \frac{1}{2}at^2$$

$$0 = (v_i \sin\theta)\, t - \frac{1}{2}gt^2$$

$$\frac{1}{2}gt^2 = (v_i \sin\theta)t$$

$$t = \frac{2v_i \sin\theta}{g}$$

Range of the projectile

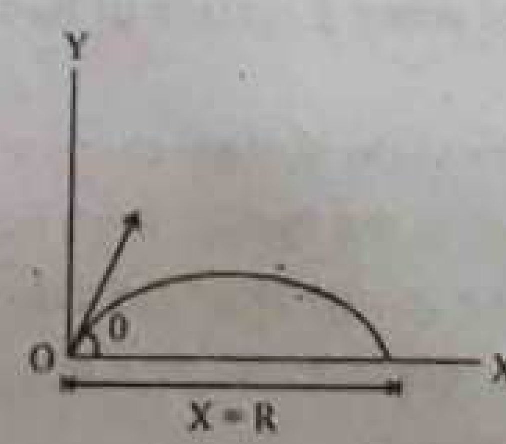
Maximum distance which a projectile covers in the horizontal distance is called the range of projectile. If $v_i\cos\theta$ is horizontal component of initial velocity then range of projectile R for the total time of flight t can be expressed as,

$$R = v_{ix} \times t$$

putting values, we get

$$R = v_i\cos\theta \times \frac{2v_i\sin\theta}{g}$$

$$R = \frac{v_i^2(2\sin\theta\cos\theta)}{g}$$



$$R = \frac{v_i^2\sin 2\theta}{g} \qquad (2\sin\theta\cos\theta = \sin 2\theta)$$

Maximum Range

The range of the projectile is maximum when the value of $\sin 2\theta$ has maximum value. The maximum value of Sine function is one. Thus

$$\sin 2\theta = 1$$

$$2\theta = \sin^{-1}(1)$$

$$2\theta = 90^\circ$$

$$\theta = \frac{90^\circ}{2}$$

$$\theta = 45^\circ$$

So

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$R_{max} = \frac{v_i^2 \sin 2(45^\circ)}{g}$$

$$R_{max} = \frac{v_i^2 \sin 90^\circ}{g}$$

$$R_{max} = \frac{v_i^2}{g}$$

Q.3.13 At what point or points in its path does a projectile have its minimum speed, its maximum speed? (Grw 2011)

Ans: **Minimum speed:**

The speed of the ball is minimum at its ***maximum height*** because at this point the vertical component of velocity become zero

Maximum speed:

The speed of ball is maximum at

i) its point of ***projection***

ii) the point just to ***hit the ground*** (point of landing).

Q.3.14 (a) What is meant by ballistic trajectory?

a. The paths followed by an un-powered and unguided projectile.

b. The path followed by the powered and unguided projectile

c. The path followed by un-powered but guided projectile.

d. The path followed by powered and guided projectile.

Ans: (a) is the correct answer

Q.3.14 (b) What happens when a system of two bodies undergoes an elastic collision? Identify the correct answer.

a. The momentum of the system changes.

b. The momentum of the system does not change

c. The bodies come to rest after collision.

d. The energy conservation law is violated.

Ans. (b) is the correct answer

(Mir Pur 2004-2006-2009, D.G.Khan 2005-2006, Mtn 2004-2005-2006, Grw 2003-2008, Bwp 2007-2008, Fsd 2008)

Solved Examples

Example 3.1:

The velocity-time graph of a car moving on a straight road is shown in Fig. 3.7. Describe the motion of the car and find the distance covered.

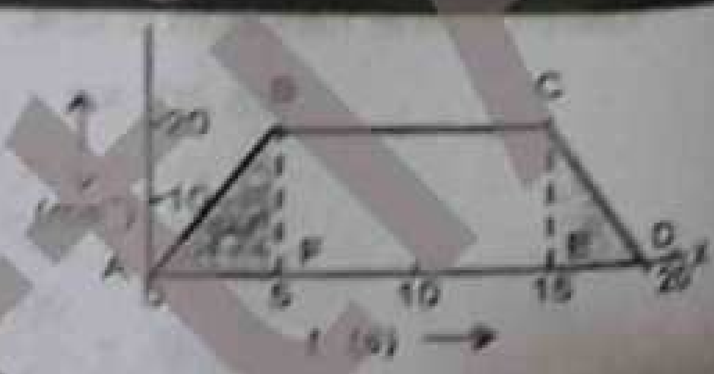


Fig.3.7

Solution:

Motion of car from A to B:

Graph shows that car start from rest and its velocity increases uniformly to 20 ms^{-1} in 5 seconds. Its average acceleration is given by

$$a = \frac{\Delta v}{\Delta t}$$

or $$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{20 - 0}{5 - 0} = \frac{20}{5}$$

$$\boxed{a = 4\ ms^{-2}}$$

Motion of car from B to C:

The graph further tells us that the velocity of the car remains constant from 5^{th} to 15^{th} second, it means that the value of acceleration is zero, that is $a = 0$.

Motion of car from C to D:

The velocity decreases uniformly to zero from 15^{th} to 19^{th} second.

The acceleration of the car during last 4 second is

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{0 - 20}{19 - 15} = \frac{-20}{4}$$

$$\boxed{a = -5\ ms^{-2}}$$

The negative sign indicates that the velocity of the car decreases during these four seconds.

Total Distance Covered by Car:

Distance covered = Area of ΔABF + Area of rectangle BCEF + Area of ΔCDE

$$S = \left(\frac{1}{2}\right)(20)(5) + (20)(10) + \left(\frac{1}{2}\right)(20)(4)$$

$$S = 50 + 200 + 40$$

$$\boxed{S = 290\ m}$$

Example 3.2

A 1500 kg car has its velocity reduced 20 ms^{-1} to 15 ms^{-1} in 3.0 s. How large was the average retarding force.

Given Data:

Mass of the car = m = 1500 kg

Initial velocity = $\vec{v_i}$ = 20 ms^{-1}

Final velocity = $\vec{v_f}$ = 15 ms^{-1}

Time = t = 3.0 s

To Find:

Average retarding force = $\vec{F}$ = ?

Calculation:

According to Newton's sound law in term of momentum.

$$\vec{F} = \frac{mv_f - mv_i}{t}$$

putting values, we get

$$\vec{F} = \frac{1500 \times 15 - 1500 \times 20}{3}$$

$$\vec{F} = \frac{22500 - 30000}{t}$$

$$\vec{F} = \frac{-750}{3}$$

$$\vec{F} = \boxed{-2500N}$$

The negative sign shows that the force is retarding force.

Example 3.3

Two spherical balls of 2.0 kg and 3.0 kg masses are moving towards each other with velocities 6.0 ms^{-1} and 4 ms^{-1} respectively. What must be the velocity of the smaller ball after collision, if the velocity of the bigger ball is 3.0 ms^{-1}?

Given Data:

Mass of smaller ball $= m_1 = 2.0$ kg
Mass of bigger ball $= m_2 = 3.0$ kg
Initial velocity of smaller ball $= v_1 = 6.0\ ms^{-1}$
Initial velocity of bigger ball $= v_2 = 4ms^{-1}$
Final velocity of bigger ball $= v_2' = 3.0\ ms^{-1}$

To Find:

Final velocity of smaller ball $= v_1' = ?$

Calculation:

According to law of conservation of momentum.
Momentum before collision = Momentum after collision

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

As both the balls are moving toward one another, so their velocities are of opposite sign.
Let the direction of smaller ball is positive and bigger ball is negative, thus

$$m_1 v_1 + m_2(-v_2) = m_1 v_1' + m_2(-v_2')$$

putting values, we get

$$2 \times 6 + 3 \times (-4) = 2 \times v_1' + 3 \times (-3)$$
$$12 - 12 = 2v_1' - 9$$
or $$2v_1' - 9 = 0$$
or $$2v_1' = 9$$
$$v_1' = 4.5\ ms^{-1}$$

Example 3.4:

A 70g ball collides with another ball of mass 140g. The initial velocity of the first ball is 9 ms^{-1} to the right while the second ball is at rest. If the collision were perfectly elastic, what would be the velocity of the two balls after the collision?

Given Data:

Mass of the first ball $= m_1 = 70g = 0.07$ kg
Mass of the second ball $= m_2 = 140g = 0.14$ kg
Velocity of the first ball before collision $= v_1 = 9\ ms^{-1}$
Velocity of the second ball before collision $= v_2 = 0$

To Find:

Velocity of first ball after collision $= v_1' = ?$
Velocity of second ball after collision $= v_2' = ?$

Calculation:

As the collision is perfectly elastic, so

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \left(\frac{2m_2}{m_1 + m_2}\right) v_2$$

Putting values, we get

$$v_1' = \left(\frac{0.07 - 0.14}{0.07 + 0.14}\right) \times 9 + 0$$
$$v_1' = -\frac{0.07}{0.21} \times 9$$
$$v_1' = -3ms^{-1}$$

Now

$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_2$$

Putting values, we get

$$v_2' = \left(\frac{2 \times 0.07}{0.07 + 0.14}\right) \times 9 + 0 \quad \text{as } v_2 = 0$$
$$v_2' = \frac{0.14}{0.21} \times 9$$
$$v_2' \boxed{= 6ms^{-1}}$$

Example 3.5

A 100g golf ball is moving to the right with a velocity of 20 ms^{-1}. It makes a head on collision with an 8kg steel ball, initially at rest. Compute velocities of the balls after collision.

Given Data:

Mass of the golf ball $= m_1 = 100g = 0.1$ kg
Velocity of golf ball before collision $= v_1 = 20\ ms^{-1}$
Mass of the steel mass $= m_2 = 8$ kg
Velocity of the steel ball before collision $= v_2 = 0$

To Find:

Velocity of golf ball after collision $= v_1' = ?$
Velocity of steel ball after collision $= v_2' = ?$

Calculation:

As $$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \left(\frac{2m_2}{m_1 + m_2}\right) v_2$$

putting values, we get

$$v_1' = \left(\frac{0.1 - 8}{0.1 + 8}\right) \times 20 + 0 \quad (\text{As } v_2 = 0)$$
$$v_1' = \frac{-7.9}{8.1} \times 20$$
$$\boxed{v' = -19.5\ ms^{-1}}$$

Now

$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_2$$

putting values, we get

$$v_2' = \left(\frac{2 \times 0.1}{0.1 + 8}\right) \times 20 + 0 \quad (\text{As } v_2 = 0)$$
$$v_2' = \frac{0.2}{8.1} \times 20$$
$$v_2' \boxed{= 0.5\ ms^{-1}}$$

Example 3.6

A hose pipe ejects water at a speed of 0.3 ms^{-1} through a hole of area 50 cm^2. If the water strikes a wall normally, calculates the force on the wall, assuming the velocity of the water normal to the wall is zero after striking.

Given Data:

Speed of water = $v_1 = 0.3\ ms^{-1}$
Area of the hole of water = $A = 50\ cm^2 = 0.005\ m^2$
Speed of water normal to the wall = $v_2 = 0$
Density of water = $\rho = 1000\ kg\ m^{-3}$

To Find:

Force exerted by water on the wall = F = ?

Calculation:

Volume of water striking the wall per second = rate of flow

$$= (\text{Area})(\text{Velocity})$$
$$= 0.005 \times 0.3$$
$$= 0.0015\ m^3/s$$

Now

Mass of the water striking the wall per second = $\dfrac{\text{Mass}}{\text{time}}$ = density × $\dfrac{\text{volume}}{\text{time}}$

$$\frac{m}{t} = 1000 \times 0.0015$$
$$\frac{m}{t} = 1.5\ kg/s$$

Force exerted by water on the wall is

$$F = \left(\frac{m}{t}\right)v$$
$$F = (1.5) \times (0.3)$$
$$\boxed{F = 0.45\ N}$$

Example 3.7

A ball is thrown with a speed of 30 ms^{-1} in a direction 30° above the horizon. Determine the height to which it rises, the time of flight and the horizontal range.

Given Data:

Speed of the ball = $v_i = 30\ ms^{-1}$
Angle = $\theta = 30°$ with horizontal
Acceleration due to gravity = $g = 9.8\ ms^{-2}$

To Find:

(i) Height attained by the ball = h = ?
(ii) Time of flight = t = ?
(iii) Horizontal range = R = ?

Calculation:

(i) **For height:**

As $$h = \frac{v_i^2 \sin^2\theta}{2g}$$

Putting values, we get

$$h = \frac{(30)^2(\sin 30°)^2}{2\times 9.8}$$
$$h = \frac{900 \times (0.5)^2}{19.6}$$
$$\boxed{h = 11.5\ m\ (\text{approx})}$$

(ii) **Time of flight:**

As $$t = \frac{2v_i \sin\theta}{g}$$

Putting values, we get

$$t = \frac{2\times 30\times \sin 30°}{9.8}$$

or $$t = \frac{60 \times 0.5}{9.8}$$
$$\boxed{t = 3.1\ s}$$

(iii) **Horizontal range:**

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Putting values, we get

$$R = \frac{(30)^2 \sin(2\times 30°)}{9.8}$$
$$R = \frac{900\times \sin 60°}{9.8}$$
$$R = \frac{900 \times 0.866}{9.8}$$
$$\boxed{R = 79.5\ m}$$

Example 3.8

In example 3.7 calculate the maximum range and the height reached by the ball if the angles of projection are (i) 45° (ii) 60°.

Given Data:

Speed of the ball = $v_i = 30\ ms^{-1}$
Angle = $\theta = 45°$ and 60°
Acceleration due to gravity = $g = 9.8\ ms^{-2}$

To Find:

(i) $\left.\begin{array}{l}\text{Maxmimum range} = R_{max} = ?\\ \text{Height attained} = h = ?\end{array}\right\}$ For $\theta = 45°$

(ii) $\left.\begin{array}{l}\text{Maxmimum range} = R_{max} = ?\\ \text{Height attained} = h = ?\end{array}\right\}$ For $\theta = 60°$

Calculation:

(i) Maximum Range (For $\theta = 45°$)

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Putting values, we get

$$R = \frac{(30)^2\sin(2\times45^\circ)}{9.8}$$

$$R = \frac{900\times \sin 90^\circ}{9.8}$$

$$R = \frac{900\times 1}{9.8}$$

$$\boxed{R = 91.8\text{ m}}$$

Now

Height (For $\theta = 45^\circ$)

$$h = \frac{v_i^2\sin^2\theta}{2g}$$

Putting values, we get

$$h = \frac{(30)^2\sin^2 45^\circ}{2\times9.8}$$

$$h = \frac{900\times(0.707)^2}{19.6}$$

$$\boxed{h = 22.9\text{ m}}$$

(ii) Maximum Range (For $\theta = 60^\circ$)

$$R = \frac{v_i^2\sin2\theta}{g}$$

Putting values, we get

$$R = \frac{(30)^2\sin(2\times60^\circ)}{9.8}$$

$$R = \frac{900\times\sin120^\circ}{9.8}$$

$$R = \frac{900\times0.866}{9.8}$$

$$\boxed{R = 79.5\text{ m}}$$

Now

Height (For $\theta = 60^\circ$)

$$h = \frac{v_i^2\sin^2\theta}{2g}$$

Putting values, we get

$$h = \frac{(30)^2(\sin60^\circ)^2}{2\times9.8}$$

$$h = \frac{900\times0.866}{19.6}$$

$$\boxed{h = 34.4\text{ m}}$$

Exercise Problems

3.1 **A helicopter is ascending vertically at the rate of 19.6 ms^{-1}. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground.**

Given data:

Initial velocity of the helicopter = v_i = 19.6 ms^{-1} (upward)

Net vertical distance covered by the stone = S = -156.8m (downward)

(There is negative sign, being displacement opposite to initial velocity)

Acceleration due to gravity = g = -9.8 ms^{-2}

To find:

Time = t = ?

Calculation:

Using the following equation of motion

$$S = v_i t + \frac{1}{2}gt^2$$

putting the values, we get

$$-156.8 = 19.6t + \frac{1}{2}(-9.8)t^2$$

$$-156.8 = 19.6t - 4.9t^2$$

$$4.9t^2 - 19.6t^2 - 156.8 = 0$$

$$4.9(t^2 - 4t - 32) = 0$$

$$t^2 - 4t - 32 = 0$$

making factors, we get

$$t^2 - 8t + 4t - 32 = 0$$

$$t(t-8) + 4(t-8) = 0$$

$$(t-8)(t+4) = 0$$

OR $t - 8 = 0,\ t = 8$ sec

OR $t + 4 = 0,\ t = -4$ sec

But time cannot be negative, thus

$\boxed{t = 8\text{ sec}}$ (i.e. time taken by stone)

3.2. **Using the following data, draw a velocity- time graph on a straight road of a motorbike.**

Velocity (ms^{-1})	0	10	20	20	20	20	0
Time (s)	0	30	60	90	120	150	180

Use the graph to calculate

(a) the initial acceleration

(b) the final acceleration and

(c) the total distance traveled by the motorcyclist.

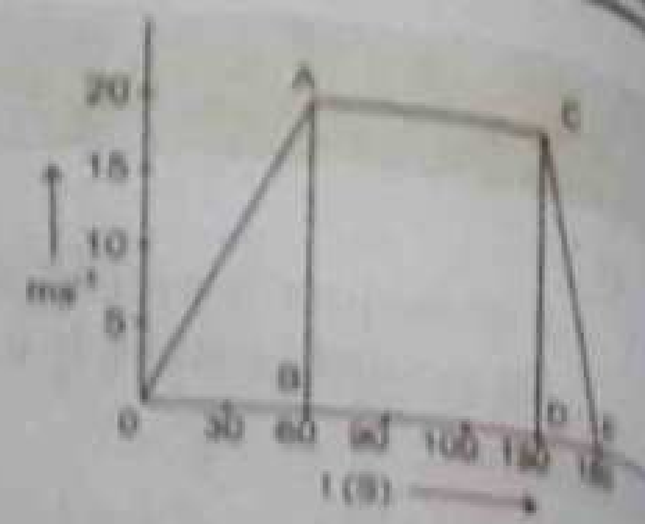
Calculation:

From the given data, we have to draw graph and then find,

(a) The initial acceleration $= a_i = ?$

(b) The final acceleration $= a_f = ?$

(c) The total distance covered $= S = ?$



From graph

(a) - Initial acceleration = a_i

a_i = slope of the graph OA

$$a_i = \frac{AB}{OB}$$

$$a_i = \frac{v_2 - v_1}{t_2 - t_1}$$

$$a_i = \frac{20-0}{60-0}$$

$$a_i = \frac{20}{60}$$

$$\boxed{a_i = 0.33\,ms^{-2}}$$

(b) Final acceleration = a_f

a_f = slope of the graph 'CE'

$$a_f = \frac{CD}{DE}$$

$$a_f = \frac{0-20}{180-150}$$

$$a_f = \frac{-20}{30}$$

$$\boxed{a_f = -0.67\,ms^{-2}}$$

Negative sign shows that the velocity decreases during last 30 seconds.

(c)- The Total Distance-Covered

The total distance covered is numerically equal to the area under velocity- time graph. Thus,

S = (Area of ΔOAB) + (Area of rectangle ABDC) + (Area of Δ CDE)

$$S = \frac{1}{2}(OB)(AB) + (BD)(AB) + \frac{1}{2}(DE)(CD)$$

$$S = \frac{1}{2} \times 60 \times 20 + 90 \times 20 + \frac{1}{2} \times 30 \times 20$$

$$S = 600 + 1800 + 300$$

$$S = 2700\,m$$

Or $\boxed{S = 2.7\,km}$

3.3 **A, proton moving with speed of $1.0\times10^7\,ms^{-1}$ passes through a 0.020 cm thick sheet of paper and emerges with a speed of $2.0\times10^6\,ms^{-1}$. Assuming uniform deceleration, find retardation and time taken to pass through the paper.**

Given Data:

Initial speed of proton $= v_i = 1.0\times10^7\,ms^{-1}$

Distance covered $= S = 0.02\,cm = 0.02\times10^{-2}\,m$

Final speed of electron $= v_f = 2.0\times10^6\,ms^{-1}$

To find:

Retardation $= a = ?$

Time taken by proton $= t = ?$

Calculations:

Using the third equation of motion

$$2aS = v_f^2 - v_i^2$$

putting values, we get

$$2\times a\times 0.02\times10^{-2} = (2\times10^6)^2 - (1\times10^7)^2$$

$$a = \frac{4\times10^{12} - 1\times10^{14}}{2\times0.02\times10^{-2}}$$

$$a = \frac{-96\times10^{12}}{0.04\times10^{-2}}$$

$$a = -2400\times10^{14}\,ms^{-2}$$

Or $\boxed{a = -2.4\times10^{17}\,ms^{-2}}$

Now using the equation ,for calculating time 't'

$$v_f = v_i + at$$

$$t = \frac{v_f - v_i}{a}$$

Putting value, we get

$$t = \frac{2\times10^6 - 1\times10^7}{-2.4\times10^{17}}$$

$$t = \frac{-8\times10^6}{-2.4\times10^{17}}$$

$$\boxed{t = 3.33\times10^{-11}\,sec.}$$

3.4 **Two masses m_1 and m_2 are initially at rest with a spring compressed between them . what is the ratio of the magnitude of their velocities after the spring has been released?**

Given Data:

First mass $= m_1$

Second mass $= m_2$

Initial velocity of $m_1 = v_1 = 0$

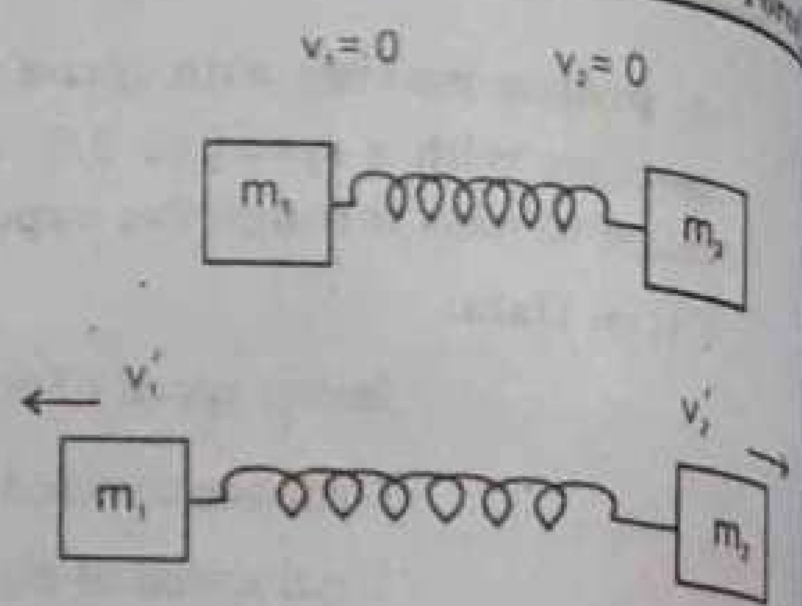
Initial velocity of $m_2 = v_2 = 0$

To find:

Final velocity of $m_1 = v_1' = ?$

Final velocity of $m_2 = v_2' = ?$

Ratio of their velocities after release $= \frac{v_1'}{v_2'} = ?$



Calculations:

According to law of conservation of momentum

Initial momentum = final momentum

$$m_1v_1 + m_2v_2 = m_1v_1' + (-m_2v_2')$$

Where negative sign shows that v_1 and v_2 are opposite in direction

$$m_1(0) + m_2(0) = m_1v_1' + (-m_2v_2')$$

$$0 = m_1v_1' + (-m_2v_2')$$

$$m_1v_1' = m_2v_2'$$

$$\boxed{\frac{v_1'}{v_2'} = \frac{m_2}{m_1}}$$

which is the ratio of their velocities after the spring has been released.

3.5 An amoeba of mass 1.0×10^{-12} kg propels itself through water by blowing a jet of water through a tiny orifice. The amoeba ejects water with a speed of $1.0\times10^{-4}\,ms^{-1}$ and at a rate of $1.0\times10^{-13}\,kgs^{-1}$. Assume that the water is being continuously replenished so that the mass of the amoeba remains the same.

(a) if there were no force on amoeba other than the reaction force caused by the emerging jet, what would be the acceleration of the amoeba?

(b) If amoeba moves with constant velocity through water, what is force of surrounding water (exclusively of jet) on the amoeba?

Given Data:

Mass of amoeba $= M = 1.0\times10^{-12}\,kg$

Speed of ejected water $= v = 1.0\times10^{-4}\,ms^{-1}$

Mass of water ejected per second $= \frac{m}{t} = 1.0\times10^{-13}\,kgs^{-1}$

To find:

(a) Acceleration of amoeba = a=?

(b) Force of water on amoeba = F=?

Calculation:

F = mass per second x speed of ejected water

$$F = \frac{m}{t} \times v$$

Putting values, we get

$$F = 1.0\times10^{-13}\times1.0\times10^{-4}$$

$$\boxed{F = 1.0\times10^{-17}\,N}$$

(a) **Acceleration of amoeba**

Using Newton's second law of motion

$$F = Ma$$

$$\text{Or} \quad a = \frac{F}{M}$$

putting values, we get

$$a = \frac{1.0\times10^{-17}}{1.0\times10^{-12}}$$

$$a = 1.0\times10^{-17+12}$$

$$\boxed{a = 1.0\times10^{-5}\,ms^{-2}}$$

3.6 A boy places a fire cracker of negligible mass in an empty can of 40 g mass. He plugs the end with a wooden block of mass 200 g. After igniting the firecracker, he throws the can straight up. It explodes at the top of its path. If the block shoots out with a speed of 3.0 ms^{-1}, how fast will the can be going?

Given Data:

Mass of can $= m_1 = 40\text{ g} = 40\times10^{-3}\,kg$

Mass of wooden block $= m_2 = 200g = 200\times10^{-3}\,kg$

Final speed of block $= v_2' = 3\text{ ms}^{-1}$

To find:

Final speed of can $= v_1' = ?$

Calculations:

As both can and block are initially at rest, so their initial velocities are $v_1 = 0$ and $v_2 = 0$

According to law of conservation of momentum,

Total initial momentum = Total final momentum

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$m_1\times0 + m_2\times0 = m_1v_1' + m_2v_2'$$

$$0 = m_1v_1' + m_2v_2'$$

$$v_1' = \frac{-m_2v_2'}{m_1}$$

$$v_1' = \frac{-200\times10^{-3}\times3}{40\times10^{-3}}$$

$$\boxed{v_1' = -15\,m/\text{sec}}$$

The negative sign shows that the can will shoot in the opposite direction to that of block.

3.7. An electron ($m = 9.1\times10^{-31}\,kg$) travelling at $2.0\times10^{7}\,ms^{-1}$ undergoes a head on collision with a hydrogen atom ($m = 1.67\times10^{-27}\,kg$) which is initially at rest. Assuming the collision to be perfectly elastic and a motion to be along a straight line, find the velocity of hydrogen atom?

Given data:

Mass of electron $= m_1 = 9.1\times10^{-31}\,kg$

Velocity of electron $= v_1 = 2.0\times10^{7}\,ms^{-1}$

Mass of hydrogen atom $= m_2 = 1.67\times10^{-27}\,kg$

Velocity of Hydrogen atom before collision $= v_2 = 0$

To find:

Velocity of Hydrogen atom after collision $= v_2' = ?$

Calculation:

As the collision is perfectly elastic, so

$$v_2' = \frac{2m_1}{[m_1+m_2]}v_1 + \frac{[m_2-m_1]}{[m_1+m_2]}v_2$$

putting values, we get

$$v_2' = \frac{2\times 9.1\times 10^{-31}\times 2\times 10^{7}}{9.1\times 10^{-31}+1.67\times 10^{-27}} + 0$$

$$v_2' = \frac{36.4\times 10^{-23}}{1.67\times 10^{-27}}$$

Or $\quad v_2' = 2.178\times 10^{4}$

Or $\quad \boxed{v_2' = 2.18\times 10^{4}\,ms^{-1}}$

3.8. A truck weighing 2500 kg and moving with a velocity of 21 ms^{-1} collides with stationary car weighing 1000 kg. The truck and the car move together after the impact. Calculate their common velocity.

Given data:

Mass of the truck $= m_1 = 2500$ kg

Initial velocity of the truck $= v_1 = 21\ ms^{-1}$

Mass of the car $= m_2 = 1000$ kg

Initial velocity of the car $= v_2 = 0$

To find:

common (i-e combined)velocity after collisions=?

i-e $\quad v_1' = v_2' = v = ?$

Calculation:

According to law of conservation of momentum

Initial momentum = final momentum

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$m_1v_1 + m_2v_2 = m_1v + m_2v \qquad \text{As} \quad v_1' = v_2' = v$$

or $\quad m_1v_1 + m_2v_2 = (m_1+m_2)v$

or $\quad v = \dfrac{m_1v_1 + m_2v_2}{m_1+m_2}$

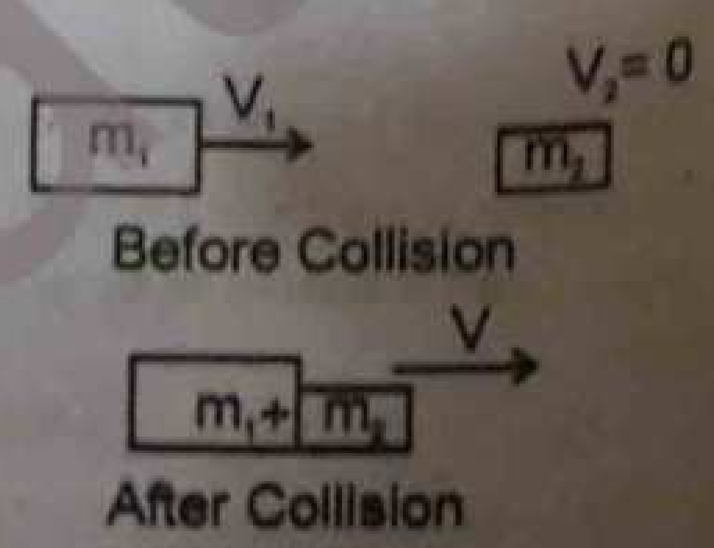
putting values, we get

$$v = \frac{2500\times 21 + 1000\times 0}{2500+1000},$$

$$v = \frac{2500\times 21}{2500+1000}$$

$$v = \frac{52500}{3500}$$

$$\boxed{v = 15\,ms^{-1}}$$



3.9. Two blocks of masses 2.0 kg and 0.50 kg are attached at the two ends of a compressed spring. The elastic potential energy stored in the spring is 10J. Find the velocities of the blocks if the spring delivers its energy to the blocks when released.

Given data:

Mass of first block $= m_1 = 2.0\,kg$

Mass of second block $= m_2 = 0.5\,kg$

Elastic potential energy of spring = P.E = 10 J

Initial velocity of mass $m_1 = v_1 = 0$

Initial velocity of mass $m_2 = v_2 = 0$

To find:

Final velocity of $m_1 = v_1' = ?$

Final velocity of $m_2 = v_2' = ?$

Calculation:

According to law of conservation of momentum

Initial momentum = final momentum

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

Putting values, we get

$$2.0\times 0 + 0.5\times 0 = 2.0\times v_1' + 0.5\times v_2'$$

$$0 = 2v_1' + 0.5\,v_2'$$

Or $\quad v_2' = \dfrac{-2}{0.5}v_1'$

$$v_2' = -4\,v_1' \quad \ldots\ldots\ldots\ldots\ldots\ (1)$$

According to law of conservation of energy

Loss of P.E. = gain in K.E.

$$10 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

$$10 = \frac{1}{2}\times 2\times v_1'^2 + \frac{1}{2}\times 0.5\times v_2'^2$$

putting values , we get

$$10 = v_1'^2 + \frac{0.5}{2}v_2'^2$$

Or $\quad 20 = 2v_1'^2 + 0.5v_2'^2$

$$20 = 2v_1'^2 + (0.5)(-4v_1')^2 \qquad \text{As} \quad v_2' = -4v_1'$$

$$20 = 2v_1'^2 + (0.5)(16v_1'^2)$$

Or $\quad 20 = 2v_1'^2 + 8v_1'^2$

$$10v_1'^2 = 20$$

Or $\quad v_1'^2 = 2$

Or $\quad \boxed{v_1' = 1.41\,ms^{-1}}$

Putting value of v_1' in equ. (1), we get

$$v_2' = -4\times 1.41$$

Or $\quad \boxed{v_2' = -5.64\,ms^{-1}}$

3.10 A Foot ball is thrown upward with an angle of 30° with respect to the horizontal. To throw a 40 m pass what must be the initial speed of the ball?

Given Data:

Angle of Projection with horizontal = $\theta = 30^{\circ}$

Range of football = R = 40 m

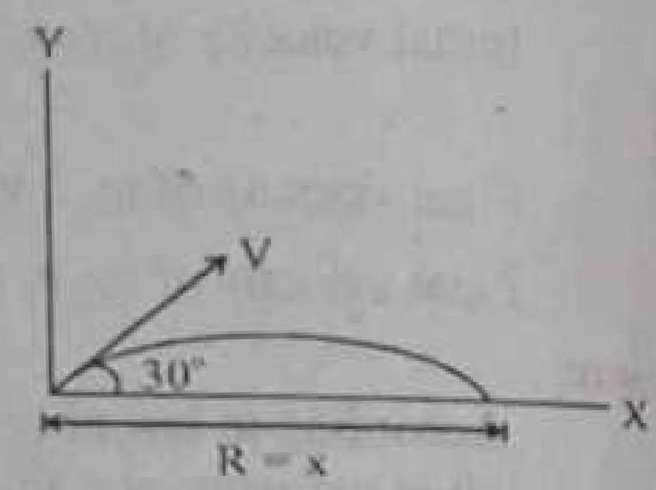
Acceleration due to gravity $= g = 9.8 ms^{-1}$

To find:

Initial speed of football $= v_i' = ?$

Calculation:

Using the equation



$$R = \frac{v_i^2 \sin 2\theta}{g}$$

or $$v_i^2 = \frac{R \times g}{\sin 2\theta}$$

Putting values, we get

$$v_i^2 = \frac{40 \times 9.8}{\sin 2(30^{\circ})}$$

Or $$v_i^2 = \frac{40 \times 9.8}{\sin 60^{\circ}}$$

Or $$v_i^2 = \frac{392}{0.866}$$

Or $$v_i^2 = 452.65$$

Or $$v_i = 21.27 ms^{-1}$$

$$\boxed{v_i = 21.3 ms^{-1}}$$

3.11 A ball is thrown horizontally from a height of 10 m with velocity of 21 ms^{-1}. How for off it hit the ground and with what velocity?

Given Data:

Initial horizontal velocity $= v_{ix} = 21 ms^{-1}$

Initial vertical velocity $= v_{iy} = 0$

Height from the ground $= y = 10 m$

Acceleration due to gravity $= g = 9.8 ms^{-2}$

To find:

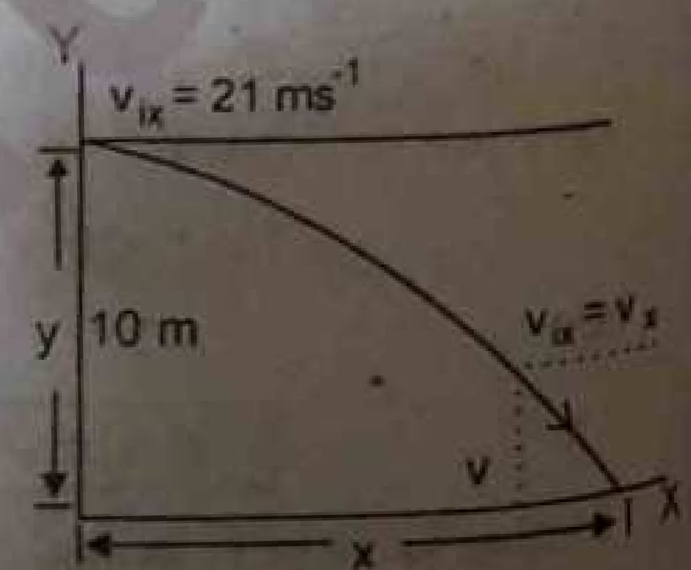
Horizontal distance $= x = ?$

Velocity to hit the ground $= v = ?$

Calculations:

The first thing is to find the time to reach the ground from vertical fall,

Using the following equation



$$y = v_i t + \frac{1}{2} g t^2$$

Putting the values, we get

$$10 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$$

$$10 = 0 + 4.9 t^2$$

$$t^2 = \frac{10}{4.9}$$

$$t^2 = 2.04$$

Or $\boxed{t = 1.42 sec.}$

Now horizontal distance is given by

$$x = v_{ix} \times t$$

Putting values, we get

$$x = 21 \times 1.42$$

$$x = 29.82 m$$

$$x = 30 m$$

For calculating v (magnitude of resultant velocity).

We need v_{fx} and v_{fy}

As $v_{fx} = v_x = v_{ix} = 21 ms^{-1}$ (As horizontal velocity remains same)

For vertical velocity

$$v_{fy} = v_{iy} + gt,$$

$$v_{fy} = 0 + 9.8 \times 1.42$$

$$v_{fy} = 14 ms^{-1}$$

Now resultant velocity $= v = \sqrt{(v_{fx})^2 + (v_{fy})^2}$

putting values, we get

$$v = \sqrt{(21)^2 + (14)^2}$$

$$v = \sqrt{441 + 196}$$

$$v = \sqrt{637}$$

$$\boxed{v = 25 ms^{-1}}$$

3.12 A bomber dropped a bomb at a height of 490 m when its velocity along the horizontal was 300 kmh^{-1}.

(a) How long was it in air?

(b) At what distance from the point vertically below the bomber at the instant the bomb was dropped, did it strike the ground?

Given Data:

Height of bomber $= y = h = 490 m$

Horizontal velocity of bomber $= v_{ix} = 300 mh^{-1} = \frac{300 \times 1000}{60 \times 60} = 83.3 ms^{-1}$

Initial vertical velocity of the bomb $= v_{iy} = 0$

Acceleration due to gravity $= g = 9.8 ms^{-2}$

To find:

(a) Time taken by the bomber in air $= t = ?$

(b) Horizontal distance covered by bomb $= x = ?$

Calculations:

(a) For calculating time, using the equation,

$$y = v_{iy}t + \frac{1}{2}gt^2$$

Putting values, we get

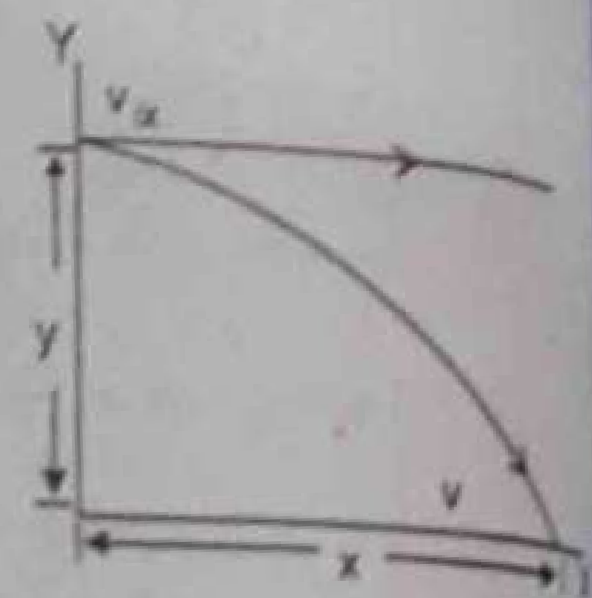
$$490 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$$

$$490 = 4.9t^2$$

or $$t^2 = \frac{490}{4.9}$$

or $$t^2 = 100$$

$$\boxed{t = 10 sec}$$



(b) Horizontal distance covered $= x = v_{ix} \times t$

$$x = 83.3 \times 10$$

$$\boxed{x = 833m}$$

Yes, the bomb will hit the ground

3.13 Find the angle of a projectile for which its maximum height and horizontal range are equal.

Given Data:

Maximum height = horizontal range

To find:

Angle of projection = θ = ?

Calculations:

As maximum height $= H = \frac{v_i^2 \sin^2\theta}{2g}$

Range of projectile $= R = \frac{v_i^2 \sin 2\theta}{g}$

According to given condition,

$$H = R$$

$$\frac{v_i^2 \sin^2\theta}{2g} = \frac{v_i^2 \sin 2\theta}{g}$$

Or $$\frac{\sin^2\theta}{2} = 2\sin\theta\cos\theta$$

Or $$\sin\theta = 4\cos\theta$$

Or $$\frac{\sin\theta}{\cos\theta} = 4$$

Or $$\tan\theta = 4$$

Or $$\theta = \tan^{-1}(4)$$

Or $$\boxed{\theta = 76^\circ}$$

3.14 Prove that for angles of projection, which exceed or fall short of 45° by equal amounts, the range are equal.

Given Data:

For maximum range, θ = 45°

To find:

To prove that range is same for angle of projection $\theta_1 = 45^\circ + \phi$

And is same for angle of projection $\theta_2 = 45^\circ - \phi$

Calculations:

We have to prove that range is same for the angles $\theta = 45^\circ + \phi$ and $\theta = 45^\circ - \phi$. As, formula for range is

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Case (i):-

$$R_1 = \frac{v_i^2 \sin 2(45^\circ + \varphi)}{g}$$

$$R_1 = \frac{v_i^2 \sin(90^\circ + 2\varphi)}{g}$$

Since $\sin(90^\circ + \theta) = \cos\theta$

$$R_1 = \frac{v_i^2 \cos(2\varphi)}{g}$$

Case (ii):-

$$R_2 = \frac{v_i^2 \sin 2(45^\circ - \varphi)}{g}$$

$$R_2 = \frac{v_i^2 \sin(90^\circ - 2\varphi)}{g}$$

Since $\sin(90^\circ - \theta) = \cos\theta$

$$R_2 = \frac{v_i^2 \cos(2\varphi)}{g}$$

Since $R_1 = R_2$, so the range remains unaffected when the angle of projection exceed or fall from 45° by equal amount.

3.15 A SLBM(submarine launched ballistic missile) is fired from a distance of 3000 km. If the Earth is considered flat and the angle of launch is 45° with horizontal, find the velocity with which the missile is fired and the time taken by SLBM to hit the target.

Given Data:

Range of ballistic missile = R = 3000 km = 3×10^6 m

Angle of launch = θ = 45°

To find:

Initial velocity of missile $= v_i = ?$

Time taken by SLBM to hit the target = t = ?

Calculation:

As $$R = \frac{v_i^2 \sin 2\theta}{g}$$

Or $$v_i^2 = \frac{Rg}{\sin 2\theta}$$

putting values, we get

$$v_i^2 = \frac{3 \times 10^6 \times 9.8}{\sin(2 \times 45^\circ)} = \frac{29.4 \times 10^6}{\sin 90^\circ}$$

$$v_i^2 = 29.4 \times 10^6$$

Or $$v_i = 5.42 \times 10^3 ms^{-1}$$

$$\boxed{v_i = 5.42 kms^{-1}}$$

Time taken by SLBM i.e. (total time of flight)

$$t = \frac{2v_i \sin\theta}{g}$$

Putting values, we get

$$t = \frac{2 \times 5.42 \times 10^3 \times \sin 45^\circ}{9.8}$$

$$t = \frac{2 \times 5.42 \times 10^3 \times 0.707}{9.8}$$

$$t = \frac{7.66 \times 10^3}{9.8}$$

$$t = 0.78 \times 10^3$$

$$t = 780\text{sec.}$$

or $$t = \frac{780}{60}\text{min}$$

$$\boxed{t = 13\text{min}}$$

****

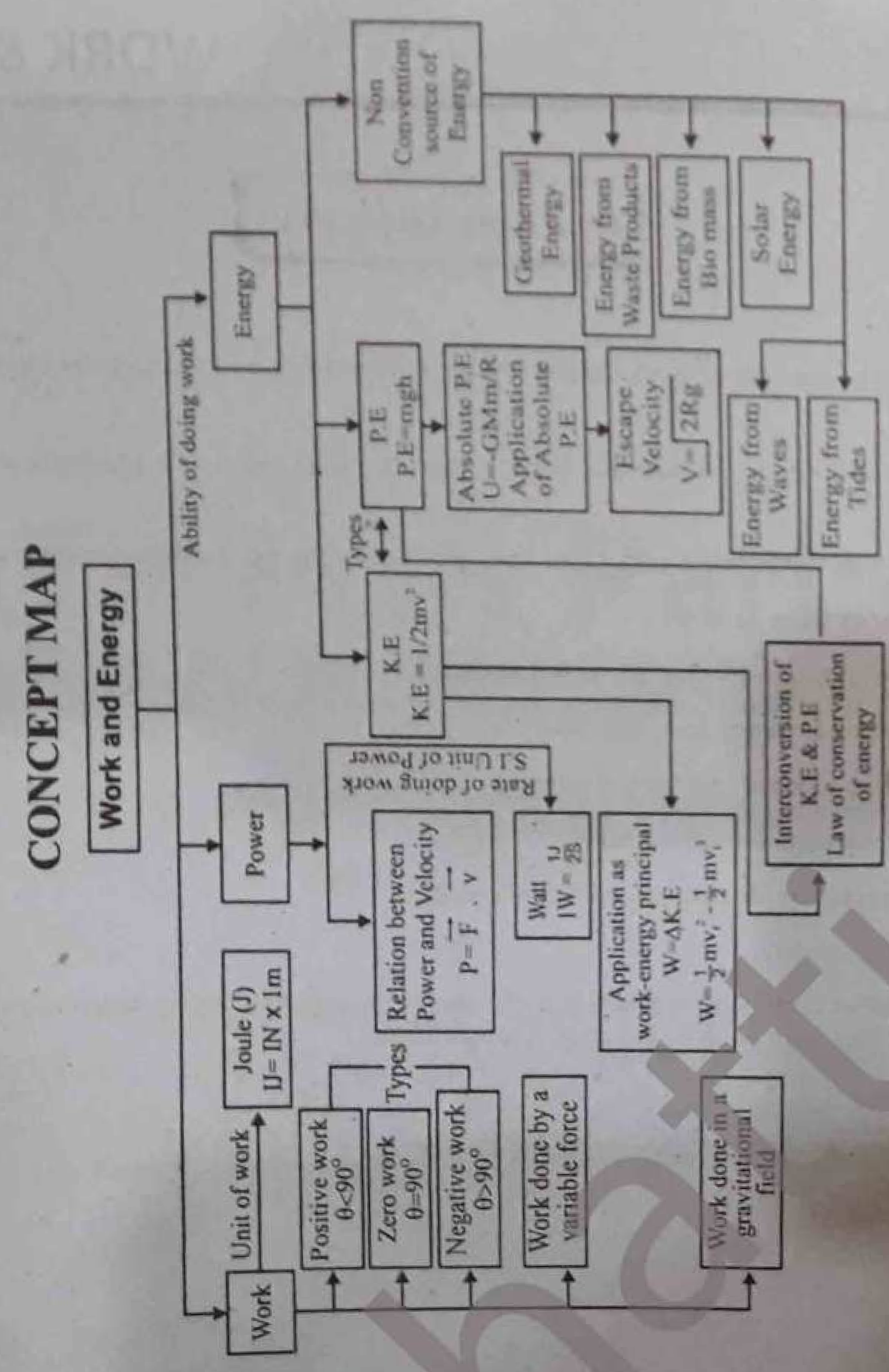
Chapter 4

WORK & ENERGY

Learning Objectives

1. Understand the concept of work in terms of product of force and displacement in the direction of the force.
2. Understand and derive the formula Work = mgh for work done in a gravitational field near Earth's surface.
3. Understand that work can be calculated from area under the force-displacement graph.
4. Relate power to work done.
5. Define power as the product of force and velocity.
6. Quote examples of power from everyday life.
7. Explain the two types of mechanical energy.
8. Understand the work-energy principle.
9. Derive an expression for absolute potential energy.
10. Define escape velocity.
11. Understand that in resistive medium loss of potential energy of a body is equal to gain in kinetic energy of the body plus work done by the body against friction.
12. Give examples of conservation of energies from everyday life.
13. Describe some non-conventional sources of energy.

Chapter No. 4



Introduction

The energy of an object changes if an exchange of energy occurs between the object and its environment. Such a transfer can occur due to a *force* or due to an exchange of *heat*.

The transfer of energy via force is a process called doing work. Doing work is the act of transferring the energy. Work, then, is transferred energy. Energy transferred to the object is positive work. Energy transferred from the object is negative work.

Work is often thought in terms of physical or mental effort. In Physics, however, the term involves two things (i) *force* (ii) *displacement*.

Q.1 What do you understand by the term work? Explain.

> **Do You Know?**
> Work has the same dimension as that of torque.

Ans.

Work Done by Constant Force

Work done on a body by a constant force is defined as *the product of magnitude of displacement and the component of force in the direction of displacement.*

Mathematical form

Let

$\vec{F}$ = constant force applied on a body.

Fig. 4.1

$\vec{d}$ = displacement of the body

θ = angle between force and displacement

Then the work done on the body is,

$$W = (F\cos\theta)\,d$$

OR $$W = Fd\cos\theta$$

OR $$\boxed{W = \vec{F}\cdot\vec{d}}$$

Another definition

Work can also be defined as *the dot product of force and displacement.*

Unit of Work

Work is a *scalar* quantity. SI unit of work is Nm called *joule*.

> **For Your Information**
> 1 joule = 10^7 ergs

Definition of joule

When one newton force acts on the body and the body covers a distance of one meter in the direction of force, the work done is said to be one joule.

Dimension of work

The dimension of work is $[ML^2T^{-2}]$

Special cases

1) If $\theta < 90°$, work done is positive.
2) If $90° < \theta \leq 180°$, work done is negative.
3) If $\theta = 90°$, no work is done.
4) If $\theta = 0°$, work done is maximum.

Graphical representation of Work

Graphically, the area under the force-displacement curve represents the work done by force.

If we plot graph between force and displacement then,

Area under the graph = (OP)(OR)

= Fd

So = W

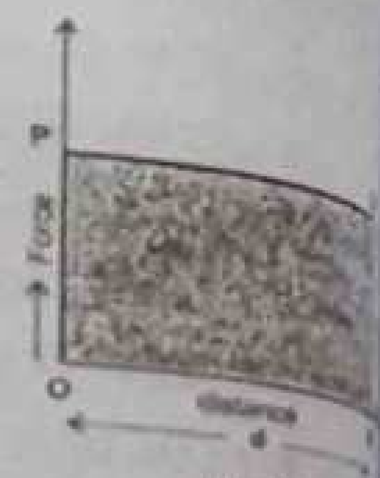


Fig 4.1

If force $\vec{F}$ makes an angle θ with horizontal. Then the graph is plotted between F $\cos\theta$ and d.

Q.2 How much work is being done by upward force when a person holding the pail moving forward?

Ans.

In this case ***no work*** is being done

Reason

because the angle between $\vec{F}$ and $\vec{d}$ is 90°.

So $W = Fd\cos 90°$

$W = 0$

Fig 4.2(a)

Q.3 How much work is being done on the wall (shown in figure)?

Ans.

No work is being done

Reason

because the displacement of wall is zero.

So $W = F(0)\cos\theta$

$W = 0$

Fig 4.2(b)

Q.4 How can we calculate the work done by variable force?

Ans.

Work Done by a Variable Force

In many cases, the force is not constant, but it varies in magnitude or direction or in both.e.g.

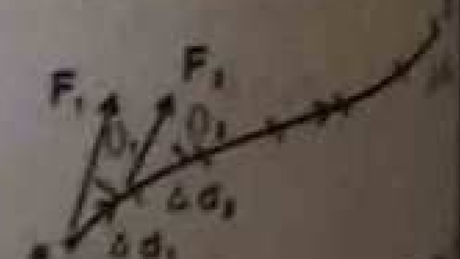
- Force of gravity acting on a rocket moving away from earth.
- Force exerted by spring increases by the amount of stretch.

Let us consider the path of particle in xy-plane from point a to b as shown in figure.

Divide the path into n short intervals of displacements $\Delta\vec{d}_1, \Delta\vec{d}_2, \ldots\ldots, \Delta\vec{d}_n$.

The forces acting during these intervals are $\vec{F}_1, \vec{F}_2, \ldots\ldots, \vec{F}_n$ respectively.

The force is considered to be approximately constant for each interval of displacement.



A particle acted upon by variable force, moves ... the path shown from ... to point b.

So work done for the first interval is,

$$\Delta W_1 = \vec{F}_1 . \Delta\vec{d}_1 = F_1\cos\theta_1 \Delta d_1$$

Similarly,

$$\Delta W_2 = \vec{F}_2 . \Delta\vec{d}_2 = F_2\cos\theta_2 \Delta d_2$$

and up to nth interval

$$\Delta W_n = \vec{F}_n . \Delta\vec{d}_n = F_n\cos\theta_n \Delta d_n$$

Now the total work done in moving the body from point a to b is.

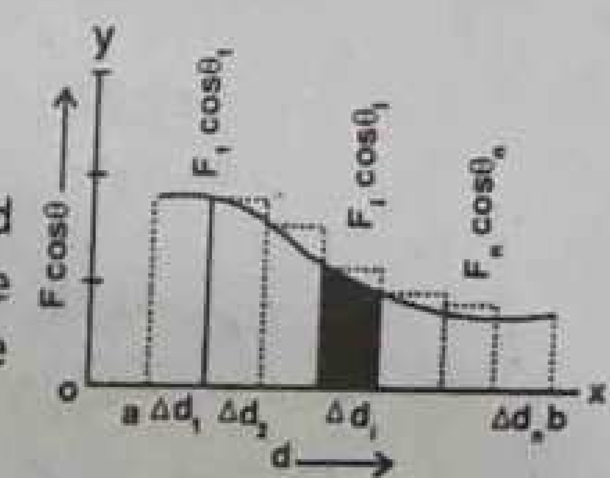
$$W = (\Delta W_1 + \Delta W_2 + \ldots\ldots + \Delta W_n)$$

OR $$W = (F_1\cos\theta_1 \Delta d_1 + F_2\cos\theta_2 \Delta d_2 + \ldots\ldots + F_n\cos\theta_n \Delta d_n)$$

OR $$W = \sum_{i=1}^{n} F_i\cos\theta_i \Delta d_i$$

How to calculate work graphically

To calculate the work done plot F $\cos\theta$ versus d. Area under the graph is divided into n rectangles from a to b. Area of each rectangle represents the work done during that interval. The total work done is equal to sum of areas of all the rectangles.



For more accurate calculation of work

The work done can be calculated more accurately, if we subdivide the distance into a large number of intervals so that each Δd becomes very small. i.e. $\Delta d \to 0$

So $$W = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i\cos\theta_i \Delta d_i$$

In this case the total area of rectangles is equal to area under F$\cos\theta$ verses d graph.

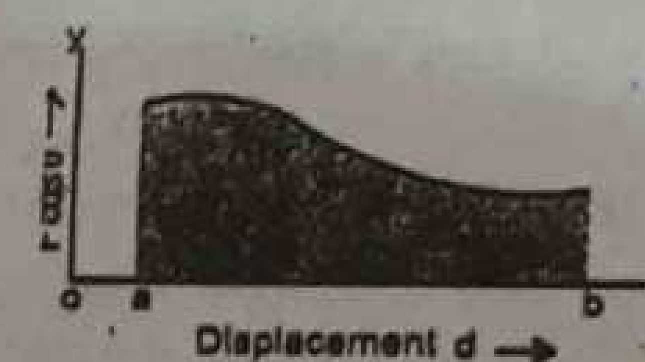


Fig. 4.6

Q.5 What is gravitational field? Show that gravitational field is a conservative field. OR Show that in gravitational field

(i) Work done in gravitational field is independent of path followed.

(ii) Work done along a closed path in a gravitational is zero.

Gravitational field

The space around the earth in which its gravitational force acts on a body is called the gravitational field.

Sign convention for wok done in Gravitational field

- If displacement is in the direction of gravitational force, the work is positive
- If displacement is against the direction of gravitational force, the work is negative.
- If displacement is perpendicular to the direction of gravitational force, the work done is zero.

Work Done in Gravitational Field

Let us consider a body of mass m .The body is displaced from A to B along different paths with constant velocity in gravitational field as shown in figure.

Gravitational force acting on the body F=W= mg

Path - I (Work done along path ADB)

Work done by gravitational force along path ADB is

$$W_{ADB} = W_{A\to D} + W_{D\to B}$$

Now $W_{A\to D} = mg(AD)\cos 90^\circ$

$$= mg\,(AD)\,(0)$$
$$= 0$$

And $W_{D\to B} = mg(DB)\cos 180^\circ$

$$= mg\,(DB)\,(-1) \qquad [\because \cos 180^\circ = -1]$$
$$= -mgh \qquad [DB = h]$$

So

$$W_{ADB} = W_{A\to D} + W_{B\to D}$$
$$= 0 + (-mgh)$$
$$W_{ADB} = -mgh \text{ ______ } (1)$$

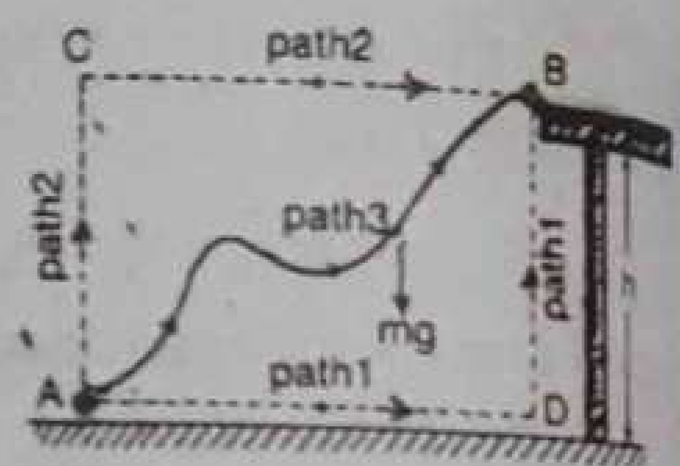


Fig. 4.8

Path -II (Work done along path ACB)

Work done by gravitational force along path ACB is

$$W_{ACB} = W_{A\to C} + W_{C\to B}$$

Now $W_{A\to C} = mg(AC)\cos 180^\circ$

$$= mg\,(AC)\,(-1)$$
$$= -mgh$$
$$W_{C\to B} = mg(CB)\cos 90^\circ$$
$$= mg\,(CB)\,(0)$$
$$= 0$$

Thus $W_{ACB} = W_{A\to C} + W_{C\to B}$

$$= -mgh + 0$$
$$W_{ACB} = -mgh \text{ ______ } (2)$$

Path -III (Work done along curved path)

In order to calculate work done along curve path, divide the path into horizontal and vertical steps as shown figure.

Work done along horizontal steps

No work is done for these steps because mg is perpendicular to the displacement for horizontal steps. So

$$W_H = 0$$

Work done along vertical steps

Work done along vertical displacements is

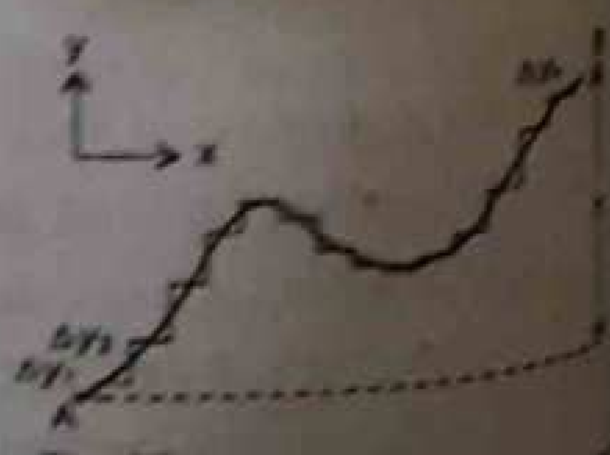


Fig. 4.9

A smooth path may be replaced by a series of infinitesimal x and y displacements. Work is done only during the y displacements

$$W_V = mg\Delta y_1\cos 180^\circ + mg\Delta y_2\cos 180^\circ + \dots\dots\dots + mg\Delta y_n\cos 180^\circ$$

OR $= -mg\Delta y_1 - mg\Delta y_2 - mg\Delta y_3 - \dots\dots - mg\Delta y_n \quad [\because \cos 180^\circ = -1]$

OR $= -mg(\Delta y_1 + \Delta y_2 \dots\dots\dots + \Delta y_n)$

OR $W_V = -mgh \qquad [\because (\Delta y_1 + \Delta y_2 + \dots\dots + \Delta y_n) = h]$

Thus, the work done along curved path is

$$W_{AB} = W_H + W_V$$
$$= 0 + (-mgh)$$
$$W_{AB} = -mgh \text{ ______ } (3)$$

From equations (1), (2) and (3) it is clear that ***work done is independent of the path followed.*** i.e.

$$W_{ADB} = W_{ACB} = W_{AB} = -mgh$$

Work done along a closed

Consider a closed path ADBA. The body is moved from A to D, D to B and then from B to A. the total work done is equal to sum of work done along these paths.

So,

$$W_{A\to D} = mg(AD)\cos 90^\circ$$
$$= mg\,(AD)\,(0)$$
$$= 0$$

Now $W_{D\to B} = mg(DB)\cos 180^\circ$

$$= mg\,(DB)\,(-1)$$
$$= -mgh$$

And for the curved path

$$W_{B\to A} = mg(\Delta y_1)\cos 0^\circ + mg(\Delta y_2)\cos 0^\circ + \dots\dots + mg(\Delta y_n)\cos 0^\circ$$
$$= mg(\Delta y_1 + \Delta y_2 + \dots\dots\dots\dots\dots + \Delta y_n) \quad [\because \cos 0^\circ = 1]$$
$$= mgh \qquad [\because (\Delta y_1 + \Delta y_2 + \dots\dots + \Delta y_n) = h]$$

So,

$$W_{ADBA} = W_{A\to D} + W_{D\to B} + W_{B\to A}$$
$$= 0 + (-mgh) + (mgh)$$
$$W_{ADBA} = 0$$

Hence the ***work done along a closed path is zero.***

For Your Information

1 hp = 746 watt = 550 ft - lb

Conservative Field

The field, in which work done is independent of the path followed is called conservative field

OR

The field, in which work done along a closed path is zero, is called conservative field.

Examples

1) Gravitational field
2) Electric field

Conservative force

1. Work done by conservative force is independent of path followed.
2. Work done by conservative force along a closed path is zero.
3. Examples of conservative force are gravitational force, elastic spring force and electric force etc.

For Your Information

<u>Conservative Forces</u>

Gravitational force
Elastic spring force
Electric force

<u>Non Conservative Forces</u>

Frictional force
Air resistance
Tension in a string
Normal force
Propulsion force of a rocket
Propulsion force of a motor

Q.6 Define power and instantaneous power. Give its unit.

Ans.

Power

Power is defined as *work done per unit time.*

OR

Power is defined as *the rate of doing work.*

Average Power

It is defined as *total work done divided the total time taken.*

Mathematically

$$\boxed{P_{av} = \frac{\Delta W}{\Delta t}}$$

Where

ΔW = the work done and Δt = time taken

Instantaneous Power

Instantaneous power is defined as *the limiting value of* $\frac{\Delta W}{\Delta t}$ *as time* Δt, *following the time t approaches zero.*

So, $$\boxed{P_{ins} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}}$$

Unit of Power

Power is a scalar quantity. SI unit of power is *joule/second* called watt.

Dimensions of power are $[ML^2T^{-3}]$

Definition of watt

The power is said to be one watt if *one joule of work is done in one second.*

Q.7 Define commercial unit of electrical energy.

Ans.

Commercial Unit of electrical energy

The commercial unit of electrical energy is **kilowatt-hour.**

Kilowatt-hour

Kilowatt-hour is *the work done in one hour by an agency whose power is one kilowatt.*

So, 1kWh = 1000 W × 3600 sec = 1000 J/sec × 3600 sec

= 3600000 J

= 3.6×10^6 J

1kWh = 3.6 MJ

For your Informa...

Approximate Pow...

Device	Power
Jumbo Jet Aircraft	[illegible]
Car at 90 kmh^{-1}	[illegible]
Electric heater	[illegible]
Colour T.V	[illegible]
Flash Light (two cells)	[illegible]
Pocket calculator	[illegible]

Q.8 Show that instantaneous power, $P = \vec{F} \cdot \vec{v}$

Ans. **Proof**

Let $\vec{F}$ is the constant force acting on a moving body and $\vec{v}$ is constant velocity of the body

Then the power delivered to the body at any instant is given by

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

OR $$P = \lim_{\Delta t \to 0} \frac{\vec{F} \cdot \Delta \vec{d}}{\Delta t} \qquad [\because \Delta W = \vec{F} . \Delta \vec{d}\,]$$

OR $$P = \vec{F} \cdot \left(\lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t} \right)$$

OR $$\boxed{P = \vec{F} \cdot \vec{v}} \qquad [\because \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t} = \vec{v}\,]$$

Another Definition of power

Power can also be defined as the *scalar product of force and velocity.*

Q.9 Define energy. Give the two types of mechanical energy.

Ans.

Energy

The ability of a body to do work is called energy.

Types of Mechanical energy

1. Kinetic Energy 2. Potential energy

Kinetic Energy

Energy possessed by a body due to its motion is called kinetic energy.

Mathematically

$$\boxed{K.E = \frac{1}{2} mv^2}$$

Where m is the mass of body moving with velocity v.

Kinetic energy may be translational, rotational etc.

Potential Energy

The energy possessed by a body because of its position in a force field or because of its physical condition is called potential energy.

Gravitational Potential Energy

The potential energy due to gravitational field near the surface of the earth at a height h is

$$\boxed{P.E. = mgh}$$

Do You Know

It takes about 9×10^{9} J to make a car and the car then uses about 1×10^{12} J of energy from petrol in its life time.

For Your Information

Approximate Energy Values

Source	Energy (J)
Burning 1 ton coal	30 × 10[illegible]
Burning 1 litre petrol	5 × 10[illegible]
K.E. of car at 90 kmh^{-1}	1 × 10[illegible]
Running Person at 10 kmh^{-1}	3 × 10[illegible]
Fission of one atom of uranium	1.8 × 10[illegible]
K.E. of a molecule of air	6 × 10[illegible]

For Your Information

An body may have energy without having momentum.

Elastic Potential Energy

The energy stored in a compressed / stretched spring is called elastic potential energy.

Elastic potential energy = $\frac{1}{2}kx^2$

Where k spring constant and x is the extension.

The units of energy are the same as those of work.

Q.10 State and prove work-energy principle.

Ans.

Work-Energy Principle

Statement

Work done on a body is equal to the change in its kinetic energy.

Proof

Let

m = mass of body

v_i = initial velocity of the body

F = force applied on the body

d = distance covered by the body

v_f = final velocity of the body

The work done on the body is

Work done = F d ______ (1)

Now, according to equation of motion

$2ad = v_f^2 - v_i^2$ (as S = d)

OR $d = \frac{1}{2a}(v_f^2 - v_i^2)$ ______ (2)

And according to Newton's second law of motion

F = ma ______ (3)

Using equations (2) and (3) in (1), we get

$$\text{Work done} = ma\frac{1}{2a}(v_f^2 - v_i^2) = \frac{1}{2}m(v_f^2 - v_i^2)$$

OR $\text{Work done} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

OR Work done = final K.E. – initial K.E. = change in K.E.

Note

- If a body is raised from the surface of earth, the work done changes its gravitational P.E.
- If a spring is a compressed, the work done on it is equal to the increase in elastic potential energy.

For Your Information

If p is the momentum and E is the energy of mass m then

$$E = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

Tid-bits

All the food you eat in one day has about the same energy as 1/3 litre of petrol.

Q.11 Define absolute potential energy. Derive relation for absolute P.E. of body of mass m at distance r from the center of earth.

Ans.

Absolute Potential Energy

The absolute gravitational potential energy of an object at a certain position is *the work done by gravitational force in displacing the object from that position to infinity where the force of gravity becomes zero.*

Calculation of Absolute P.E

Let a body of mass m is displaced in space from point 1 to N in the gravitational field. The gravitational force does not remain constant during this displacement.

The relation P.E = mgh is true only near the surface of the earth where gravitational force is constant. As gravitational force varies inversely to the square of the distance from the surface of earth $\left(\text{i.e. } F \propto \frac{1}{r^2}, \text{inverse square law}\right)$, so it can not be applied in this case.

In order to calculate the work done by gravitational force, the distance between 1 to N is divided into equal small steps of lengths Δr, so that the value of force remains constant for each step.

Work done during 1st step (1 to 2)

Suppose

m = mass of the body

M = mass of earth

r_1 = distance of point 1 from the center of the earth

r_2 = distance of point 1 from the center of the earth

Calculation of r

The distance between the center of this step and center of the earth is

$r = \frac{r_1 + r_2}{2}$ ______ (1)

Also displacement of body from point 1 to 2 is

$\Delta r = r_2 - r_1$ ______ (2)

OR $r_2 = \Delta r + r_1$ ______ (3)

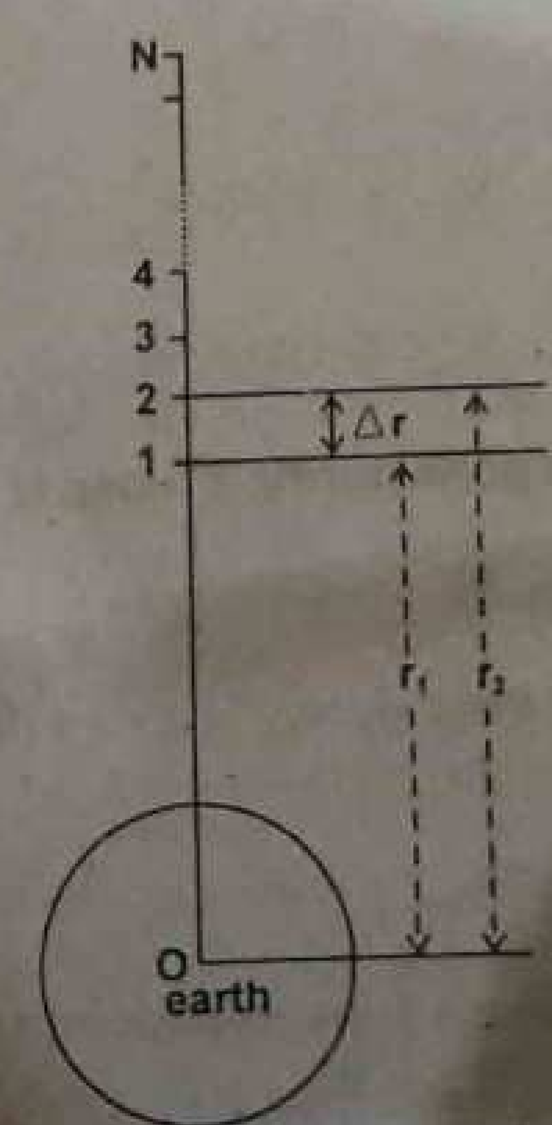
Using equation (3) in (1)

$$r = \frac{r_1 + \Delta r + r_1}{2}$$

$$r = \frac{2r_1 + \Delta r}{2}$$

$$r = \frac{2r_1}{2} + \frac{\Delta r}{2}$$

$r = r_1 + \frac{\Delta r}{2}$ ______ (4)



Squaring both sides, we have

$$r^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2$$

$$= r_1^2 + \frac{\Delta r^2}{4} + 2r_1\frac{\Delta r}{2}$$

$$= r_1^2 + \frac{\Delta r^2}{4} + r_1\Delta r$$

Since $(\Delta r)^2 << r_1^2$, so this term can be neglected as compared to r_1^2

$$r^2 = r_1^2 + r_1\Delta r$$

$$= r_1^2 + r_1(r_2 - r_1)$$

$$= r_1^2 + r_1r_2 - r_1^2$$

$$\boxed{r^2 = r_1 r_2} \quad \text{\_\_\_\_\_\_\_\_ (5)}$$

Now, the gravitational force F at the center of this step is

$$F = \frac{GMm}{r^2} \quad \text{\_\_\_\_\_\_\_\_ (6)}$$

Using equation (5) in (6), we have

$$F = \frac{GMm}{r_1r_2} \quad \text{\_\_\_\_\_\_\_\_ (7)}$$

Thus the work done during 1st step is

$$W_{1\to2} = \vec{F}\cdot\Delta\vec{r}$$

$$= F\,\Delta r\cos 180^\circ$$

$$(\because \vec{F} \text{ is opposite to } \Delta\vec{r})$$

$$= -F\Delta r$$

$$= -G\frac{Mm}{r_1r_2}(r_2 - r_1) \quad \text{[using equations (2) \& (7)]}$$

$$W_{1\to2} = -GMm\left[\frac{r_2}{r_1r_2} - \frac{r_1}{r_1r_2}\right]$$

$$W_{1\to2} = -GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Work done during 2nd step (2 to 3)

$$W_{2\to3} = -GMm\left[\frac{1}{r_2} - \frac{1}{r_3}\right]$$

Work done during last step (N-1 to N)

For last step work done is,

$$W_{N-1\to N} = -GMm\left[\frac{1}{r_{N-1}} - \frac{1}{r_N}\right]$$

Total work done from point 1 to N

$$W_{total} = W_{1\to2} + W_{2\to3} + \ldots\ldots + W_{N-1\to N}$$

Do You Know?

There more energy reaching Earth in 10 days of sunlight than in all the fossil fuels on the Earth.

$$= -GMm\left[\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \ldots\ldots + \left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)\right]$$

$$W_{total} = -GMm\left[\frac{1}{r_1} - \frac{1}{r_N}\right]$$

Tit-bits

More coal has been used since 1945 than was used in the whole of history before that.

Absolute P.E

If N lies at infinity, then

$$r_N = \infty \text{ and } \frac{1}{r_N} = \frac{1}{\infty} = 0$$

So $$W_{total} = -GMm\left[\frac{1}{r_1} - 0\right] = -GMm\left[\frac{1}{r_1} - \frac{1}{\infty}\right]$$

OR $$W_{total} = \frac{-GMm}{r_1} \quad \text{\_\_\_\_\_\_\_\_ (8)}$$

In general, the absolute gravitational potential of body at distance r from the center of the earth is

$$\boxed{U = \frac{-GMm}{r}} \quad \text{\_\_\_\_\_\_\_\_ (9)}$$

Absolute P.E on the surface of Earth

When the body lies at the surface of the earth then, r = R. So, equation (9) becomes

$$\boxed{U_g = \frac{-GMm}{R}} \quad \text{\_\_\_\_\_\_\_\_ (10)}$$

Where R is the radius of the earth and the negative sign represents that gravitational field of earth for mass m is attractive.

Note :

- When the body moves away from earth's surface, r increases, U increases. (i.e. it becomes less negative.)
- When the body falls towards the earth's surface, r decreases, U decreases. (i.e. it becomes more negative.)

Note: To calculate P.E. choice of zero reference point

- The choice of zero point is *arbitrary*.
- We can take the ***surface*** of the Earth or the point at ***infinity*** as zero P.E reference.
- The difference of P.E from one point to another is significant
- The change in P.E as we move a body above the Earth's surface will always ***positive***.

Q.12 Define escape velocity and derive the mathematical expression for escape velocity?

Ans

Escape Velocity

The initial velocity of a body with which it goes out of the earth's gravitational field is called escape velocity

Explanation

When a body is thrown upward, it returns back after reaching a certain height. That is due to gravitational force acting downward. If we increase the initial velocity of the body then it gains more height. If we go on increasing the initial velocity, then at certain velocity, the body it will not return back to the ground. This particular velocity is called *escape velocity*.

Expression for escape velocity

We know that the absolute P.E. of a body of mass m on the surface of earth is

$$U_g = \frac{-GMm}{R} \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

As the body goes out of gravitational field, it P.E. becomes zero.

So, $\text{Increase in P.E} = 0 - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$

Thus the initial K.E. needed by the body to reach infinity (i.e. out of gravitational field) is

$$\text{Initial K.E} = \frac{GMm}{R}$$

OR $$\frac{1}{2} m v_{esc}^2 = \frac{GMm}{R}$$

Where m = the mass of the body; M = mass of the earth and, R = radius of earth

OR $$v_{esc}^2 = \frac{2GM}{R}$$

OR $$\boxed{v_{esc} = \sqrt{\frac{2GM}{R}}} \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

This is expression for the escape velocity of a body.

Another formula for escape velocity

As the gravitational force for a mass m placed on the surface of the earth of mass is

$$F = \frac{GMm}{R^2}$$

But $$F = mg$$

So $$mg = \frac{GMm}{R^2}$$

OR $$g = \frac{GM}{R^2}$$

OR $$GM = gR^2$$

Thus equation (2) becomes,

$$v_{esc} = \sqrt{\frac{2gR^2}{R}}$$

OR $$\boxed{v_{esc} = \sqrt{2gR}} \quad \text{\_\_\_\_\_\_\_\_ (3)}$$

Value of escape velocity on Earth

As $g = 9.8 m/sec^2$ and $R = 6.4 \times 10^6 m$

So $$v_{esc} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$= 11.2 \times 10^3 \text{ m/sec}$$

OR $$= 11.2 \times 10^3 \text{ m/sec}$$

OR $$\boxed{v_{esc} = 11.2 \text{ km/sec}}$$

For Your Information

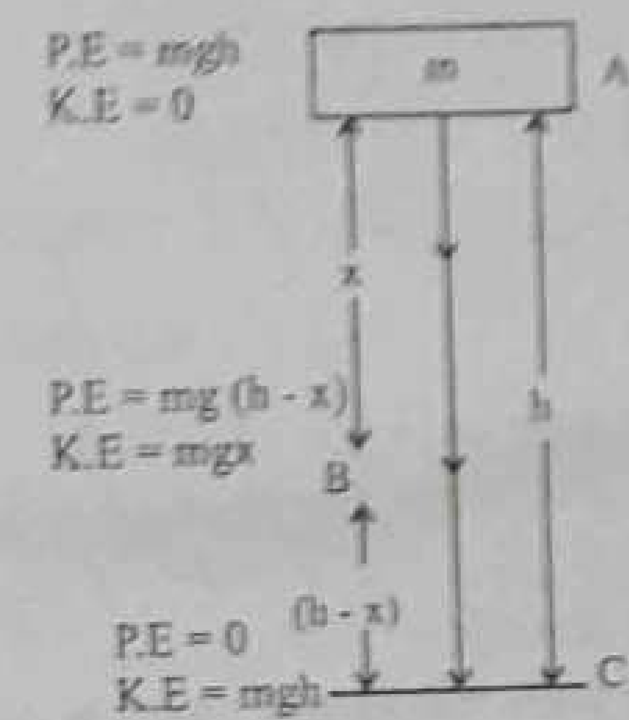
Some Escape speeds (km^{-1})	
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Nepyune	22.4
Uranus	25.4
Saturn	37.0
Jupiter	61

Q.13 Discuss interconversion of potential energy and kinetic energy.

Ans

Inter-conversion of Potential Energy and Kinetic Energy

Consider a body is falling freely under the action of gravity through a non-resistive medium. Suppose a body of mass *m* is at rest at height *h* above the surface of earth.



At position A

Potential energy of the body = mgh

Kinetic energy of body = 0

The total energy at A is

$$E_A = mgh + 0$$

$$E_A = mgh \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

Change in P.E. & K.E. when body falls

At position B

Let

downward distance covered by the body = x

height of body from the surface of earth = *(h-x)*

Potential energy of the body = mg (h-x) = mgh − mgx

Kinetic energy of body $= \frac{1}{2} m v_B^2$ ________ (2)

Where v_B is the velocity of body at point B.

Calculation of v_B

By equation of motion

$$2ad = v_f^2 - v_i^2$$

OR $$v_f^2 = 2ad + v_i^2$$

OR $$v_B^2 = 2gx + 0 \qquad [\because v_i = 0, v_f = v_B, d = x \text{ and } a = g]$$

OR $$v_B^2 = 2gx$$

Putting this value of v_B^2 in equation (2), we have

$$K.E = \frac{1}{2} m (2gx) = mgx$$

So, total energy at position B

$$E_B = mg(h - x) + mgx$$

OR $$E_B = mgh - mgx + mgx$$

OR $$E_B = mgh \quad \text{\_\_\_\_\_\_\_\_ (3)}$$

At position C [Just before hitting the ground]

Downward distance covered by the body = h

Height of the body from the surface of earth = 0

Potential energy of the body = 0

Kinetic energy of body $= \frac{1}{2} m v_C^2$ ________ (4)

Where v_C is the velocity of body at point C.

Calculation of v_C

By equation of motion

$$2ad = v_f^2 - v_i^2$$

OR $$v_f^2 = 2ad + v_i^2$$

OR $$v_C^2 = 2gh + 0 \qquad [\because v_i = 0, v_f = v_C, d = h \text{ and } a = g]$$

OR $$v_C^2 = 2gh$$

Putting this value of v_C^2 in equation (4), we have

$$K.E = \frac{1}{2} m (2gh) = mgh$$

So, total energy at position B

$$E_C = 0 + mgh$$

OR $$E_C = mgh \qquad \text{\_\_\_\_\_\_\_\_ (5)}$$

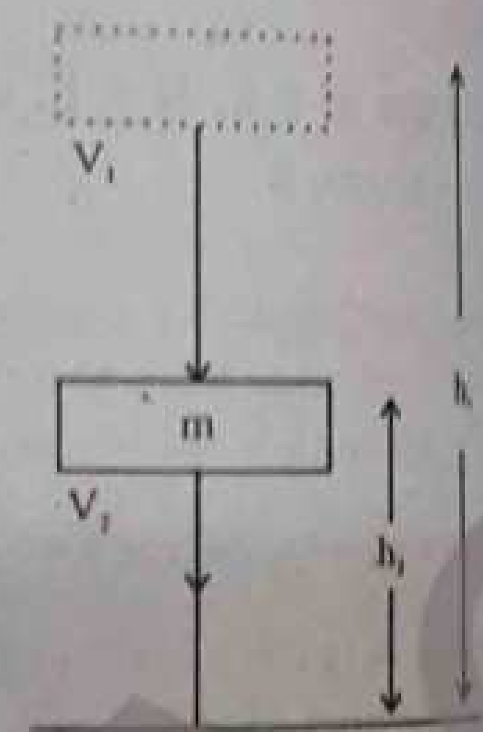
Conclusion

If a body falls from a height h_1, then at height h_2 above the surface of earth

Loss in P.E. = Gain in K.E.

$$\Rightarrow \quad mgh_1 - mgh_2 = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$$

OR $$\boxed{mg(h_1 - h_2) = \frac{1}{2} m (v_2^2 - v_1^2)}$$



Where v_1 and v_2 are the velocities at height h_1 and h_2 respectively.

- P.E. and K.E. are *interconvertible* but the total energy remains unchanged.
- In the absence of friction, *loss in P.E. = gain in K.E.*
- In the presence of friction, *loss in P.E. = gain in K.E. + work done against friction*

i.e. $$mgh = \frac{1}{2} mv^2 + fh$$

Q.14. State law of conservation of energy. Why new sources of energy has to be developed if energy is conserved?

Ans.

Conservation of Energy

Statement

Energy cannot be destroyed. It can be transformed from one form into another, but the total amount of energy remains constant.

Conservation of mechanical energy

The K.E and P.E are the different forms of mechanical energy.

The total mechanical energy of the body is equal to the sum of K.E and. P.E.

P.E may change into K.E. Similarly, the K.E may also be Chang into P.E, but total energy remains constant.

For Your Information

Source of energy	Original Source
Source	Sun
Bio mass	Sun
Fossil fuels	Sun
Wind	Sun
Waves	Sun
Hydro electric	Sun
Tides	Moon
Geothermal	Earth

Mathematically ,total energy = P.E. + K. E.= constant

This is the special case of conservation of energy.

Need for new sources

In daily life we observe many energy changes from one form to another. At last all energy transfers heat the environment and energy is lost in the form of *heat* which is useless. So, useful energy *is decreasing* though total energy is conserved. That is why we need to develop new sources of energies.

Energy Sources

Renewable	Nonrenewable
Hydroelectric	Coal
Wind	Natural Gas
Tides	Oil
Geothermal*	Uranium
Biomass	Oil shale
Sunlight	Tar sands
Ethanol/Methanol**	

* Individual fields may run off

** Renewable when made from bio mass

Q.15 Discuss the different non-conventional energy sources.

Ans.

Non-Conventional Energy Sources

These sources are not very common these days. Some of them are given below;

1.Energy from Tides

Gravitational force of the *moon* produces tides in the sea. The tides raise the water in the sea roughly *twice* a day.

Explanation

Water at high tide can be trapped in a basin by constructing a dam. Dam is filled at high tide. Then, water is released in a controlled way at low tide to derive the turbines. The dam is filled again for next high tide and the fall of water also derive the turbines. This process is used to generate the current.

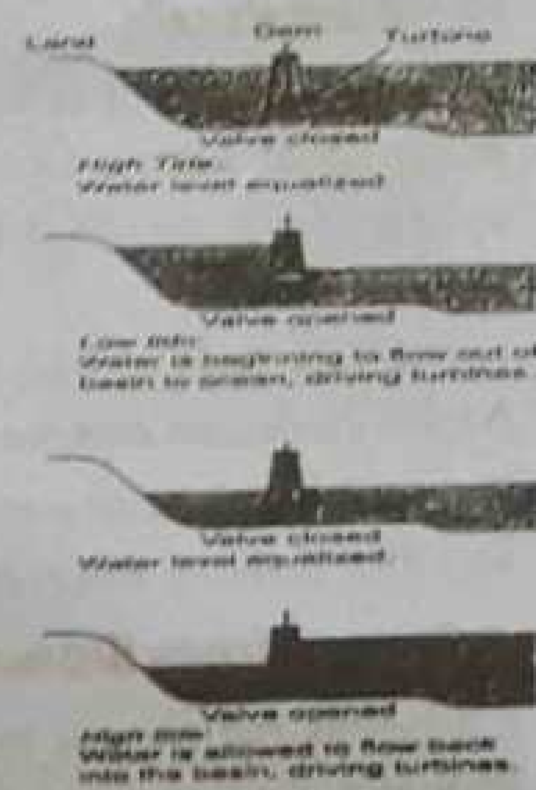


Fig. 4.13

2. Energy from Waves

The tides and winds blow across the surface of ocean. These waves produce strong water waves. The energy of these waves can generate electricity.

Salter's duck

The device which is used for this purpose was invented by Professor Salter called Salter's duck.

Fig. 4.14

For Your Information

The rapid growth of human population has put a strain on our natural resources. A sustainable society maximizes the benefit from each resource. Minimizing the use of energy is an other method of conservation. We can save energy by,

(1) turning off lights and electrical appliances when not use.

(2) using fluorescent bulbs instead of incandescent bulbs

(3) using sunlight in offices, commercial centers and houses during daylight hours

(4) Taking short hot showers

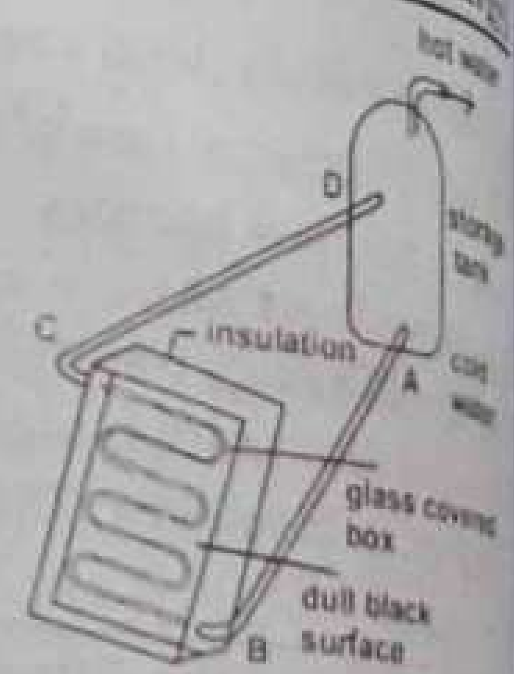
It has two parts (1) Duck float (2) Balance float.

The wave energy produces the movement in duck float relative to balance float.

The relative motion of duck float is used to run electricity generators.

3. Solar Energy

Sun is the major source of energy on earth. ***"Solar energy at normal incidence outside the earth's atmosphere per second per unit area is about 1.4kW/m^2, which is referred as solar constant."*** While passing through atmosphere, the total energy is reduced due to reflection, scattering and absorption by dust particles, water vapours and other gases. On a clear day at noon, the solar energy at the surface of the earth is **1kW/m^2**.



Solar energy is used to heat the water by solar reflection and thermal absorbers. It can also use to generate electrical energy. For this purpose, the surface of the collector is blackened to absorb the heat energy. It can heat up the water up to 70° C.To get higher temperature; we use the reflectors or lens of large size.

Photo voltaic cells

Sunlight can be converted into electrical energy by ***photo voltaic cells***. They are made up of silicon. Electrons in the silicon gain energy from sunlight to create voltage. Solar panels are expensive but of long life time and have low running cost.

For cloudy days or nights, electric energy can be stored during the Sun light in ***Nickel cadmium batteries*** by connecting them to solar panels.

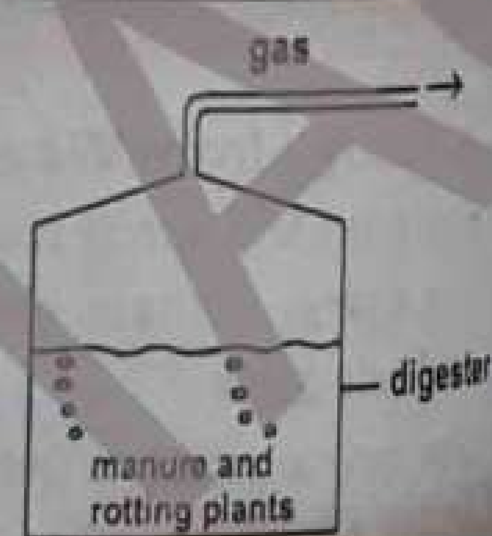
They are used to power satellites. They are also used in remote ground based weather stations and rain forests communication and also in solar calculators.

Do You Know?

Population can be reduced if
(1) People use mass transport
(2) Use geothermal, Solar, hydroelectrical and wind energy as alternative forms of energy.

4. Energy from Biomass

It includes the organic materials such as crop residue, natural vegetation, trees, animal dung and sewage. Biomass energy refers to the use of materials as fuel. Two common methods of conversion of biomass into fuels are direct combustion and fermentation.



Direct Combustion Method

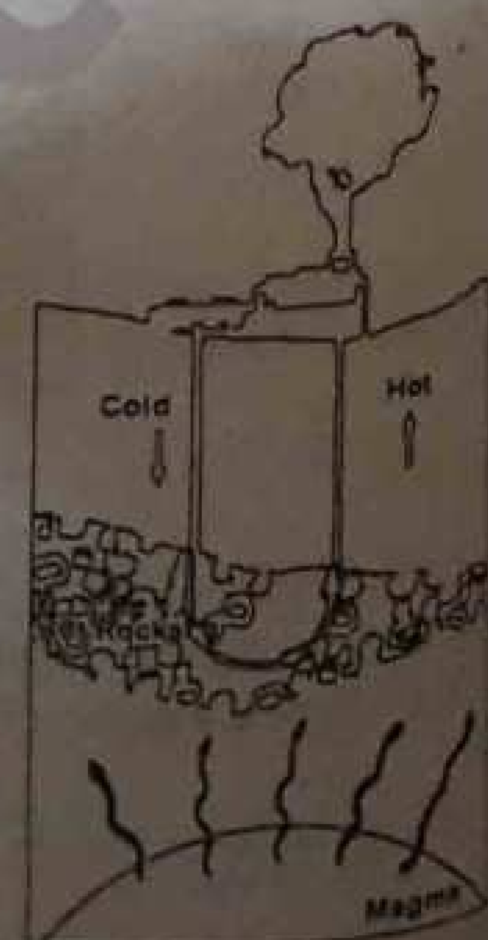
Direct combustion method is applied to get energy from waste product commonly known as solid waste and confined it into chamber and ignite it. The heat produces is then use in a boiler to run the turbine of generator.

Fermentation Method

Bio fuel such as ethanol is a replacement of gasoline, which is obtained by fermentation of biomass using enzymes and by decomposition through bacterial action in the absence of air. The rotting of biomass in a closed tank called a ***digester*** produces biogas which can be piped out to use for cooking and heating.

5. Energy from Waste Products

It is probably the most commonly used conversion process, in which waste material like wood waste, crop residue and municipal solid waste is burnt in a confined container. Heat produced in this way can be used in a boiler to produce steam that can operate turbine generator.



6. Geothermal Energy

The heat extracted from inside the earth is in the form of hot water or steam. It can be generated by following methods.

Radioactive Decay

Due to decay of radioactive elements, the energy heating the rocks is constantly being released.

Residual Heat of the Earth

Hot igneous rocks within 10Km of the earth are present in molten form. They conduct heat energy from interior part of the earth. The temperature of these rocks is **200°C or more.**

Compression of Material

In deep inside the earth, the compressed materials cause for the generation of heat energy.

In some places, water beneath the ground is in contact with hot rocks and is raised to high temperature and pressure in form of hot springs, geysers or steam vents. The steam produced in this way can run the turbines or for heating purpose.

Geo thermal energy may cause to produce hot geyser releasing with an explosive column into the air. They usually occur in volcanic region and erupt with irregular intervals. This extraction of geo thermal seriously disturbs geyser system by reducing heat flow and aquifer pressure.

"Aquifer is a layer of rock holding water that allows water to percolate through it with pressure."

FORMULAE

1	Work done by constant force	$W = \vec{F} \cdot \vec{d}$	
2	Work done by variable force	$W = \sum_{i=1}^{n} F_i \Delta d_i \cos\theta_i$	
3	Power	$P = \vec{F} \cdot \vec{v}$	$P = \frac{W}{t}$
4	Average Power	$P_{av} = \frac{\Delta W}{\Delta t}$	
5	Instantaneous Power	$P_{ins} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$	
6	Kinetic Energy	$K.E = \frac{1}{2} mv^2$	$K.E = \frac{p^2}{2m}$
7	Gravitational Potential Energy	$P.E = mgh$	
8	Work Energy Principle	$W = \frac{1}{2} mv_f^2 - \frac{1}{2} v_i^2$	
9	Absolute P.E	$U = \frac{-GMm}{r}$	$U = \frac{-GMm}{r}$ (at surface of earth)
10	Escape Velocity	$v = \sqrt{2gR}$	$v = \sqrt{\frac{GM}{R}}$
11	Conservation of Energy	$mg(h_1 - h_2) = \frac{1}{2} m(v_2^2 - v_1^2)$	$mg = \frac{1}{2} mv^2 + fh$

Multiple Choice Questions

◆ *Four possible answers to each statement are given below. Tick (✓) the correct answer:*

1. **Which of the following types of forces cannot do the work on the particle ?**
   (a) Frictional force (b) Gravitational force
   (c) Elastic force (d) Centripetal force
2. **Relation between horse power and watt is:**
   (a) 1hp = 766 watt (b) 1 hp = 746 watt
   (c) 1hp = 736 watt (d) 1hp = 796 watt
3. **Proton, Neutron, electron and α particle have same momentum, which particle have greater K.E**
   (a) α (alpha) particle (b) Proton
   (c) Neutron (d) Electron
4. **Slope of work-time graph is**
   (a) Work (b) Displacement
   (c) Power (d) None
5. **When two protons are brought together, then**
   (a) Their K.E. increases (b) P.E. decreases
   (c) P.E. increases (d) P.E. remains same
6. **A person holds a bucket of weigh 60N. He walks 7m along the horizontal and then climb up ve 5m. The work done by man is:**
   (a) 720J (b) 420J
   (c) 300J (d) None
7. **The SI unit of power (watt) is equivalent to**
   (a) Kgm^2s^{-3} (b) $kgms^{-2}$
   (c) kgm^2s^{-2} (d) None
8. **Two masses of 1gm and 4gm are moving with same K.E. The ratio of their linear moment are:**
   (a) 1 : 16 (b) 1 : 2
   (c) $\sqrt{2} : 1$ (d) 4 : 1
9. **Pick out the conventional source of energy:**
   (a) Wind energy (b) Coal energy
   (c) Tidal energy (d) Bio mass energy
10. **Escape velocity of a body depends upon the**
    (a) Mass of the planet (b) Mass of the body
    (c) Velocity of the plant (d) None
11. **KWh is the unit of:**
    (a) Mass (b) Work
    (c) Power (d) Time
12. **Which one is non-renewable energy source?**
    (a) Wind (b) Tides
    (c) Uranium (d) All of these

13. **Absolute P.E of a body at infinity is**
    (a) Zero (b) Negative
    (c) infinity (d) Both a & b
14. **Which one of the following does not effect the orbital velocity of a satellite ?**
    (a) mass of planet (b) mass of satellite
    (c) pull of earth (d) None
15. **When speed of a body is double then.**
    (a) K.E is doubled (b) Its acceleration is doubled
    (c) P.E is doubled (d) Its momentum is doubled
16. **Consumption of energy by a 60-watt bulb in 2 second is:**
    (a) 120J (b) 60J
    (c) 30J (d) 0.02J
17. **Dot product of force and velocity is:**
    (a) Pressure (b) Work
    (c) Momentum (d) Power
18. **A force of 10N displaces a body by 6m in 3 seconds the power of agency applying the force is:**
    (a) 180 W (b) 5 W
    (c) 1.8 W (d) None
19. **If we go above the surface of earth, the gravitational constant**
    (a) Remains constant (b) Becomes zero
    (c) Varies inversely to the square of the distance (d) Varies directly to the square of the distance
20. **The typical source of wave energy is**
    (a) sun (b) moon
    (c) earth (d) mars

ANSWERS

1. d	2. b	3. d	4. c	5. c	6. c	7. a	8. b	9. b	10. a
11. b	12. c	13. a	14. b	15. d	16. a	17. d	18. d	19. a	20. a

Short Questions of Exercise

Q.4.1 A person holds a bag of groceries while standing still, talking to a friend. A car is stationary with its engine running. From the stand point of work, how are these two situations similar?

Ans. In both cases work done is <u>zero</u>.

Reason:

As the person and the car both are at rest. So the displacement is <u>zero</u>. (i.e., d = 0)

Work done = $Fd\cos\theta$

$W = F(0)\cos\theta$

$W = 0$

So the work done becomes zero.

Q.4.2 Calculate the work done in kilo joules in lifting a mass of 10 kg (at a steady velocity) throug vertical height of 10 m?

(Mtn 2006-2009, Lhr 2006, Fsd 2006, D.G.Khan 2006, Bwp 2007-2008, Lhr 2009,Grw 20

Ans. **Data:** $m = 10kg$

$h = 10m$

$W (kJ) = ?$

Calculation: As the work done is equal to increase in its P.E. So,

$W = mgh$

$W = 10 \times 9.8 \times 10$

$W = 980\ J$

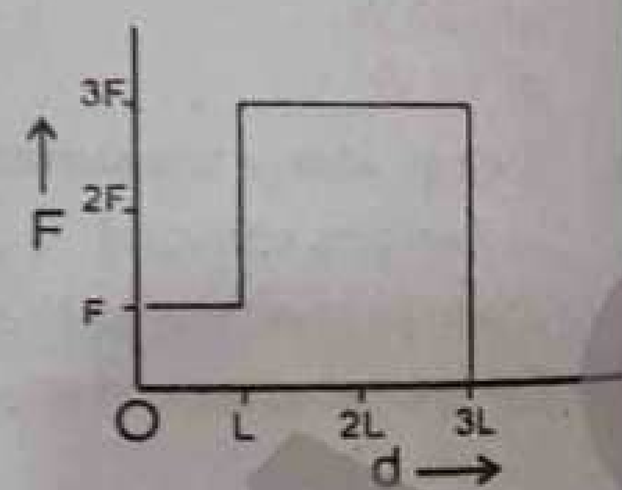
$W = \frac{980}{1000} kJ$

Result: $W = 0.980\ kJ$

Q.4.3 A force F acts through a distance L. the force is then increased to 3F, and then acts through a furthe distance of 2 L. draw the work diagram to scale.

Ans. **Force displacement graph:**

As area under the force displacement graph is equal to the work done by the body. So



$W = (F)(L) + (3F)(2L)$

$W = FL + 6FL$

$W = 7FL$

Q.4.4 In which case is more work done? When a 50 kg bag of books is lifted through 50 cm, or when a 5 kg crate is pushed through 2m across the floor with a force of 50 N? (Lhr 2010-20

Ans.

Case I:	Case II:
Data:	***Data:***
$m_1 = 50\ kg$	$m_2 = 50\ kg$
$h = 50cm = 0.5\ m$	$d = 2m$
$W_1 = ?$	$F = 50\ N$
	$W_2 = ?$
Calculation:	***Calculation:***
$W_1 = m_1gh$	$W_2 = Fd$
$= 50 \times 9.8 \times 0.5$	$= (50)(2)$
$= 245\ J$	$= 100\ J$

Result: More work is done in <u>*case I*</u>.

Q.4.5 An object has 1 J of potential energy. Explain what does it mean?

(Lhr 2004, Sgd 2005, Mtn 2005, Federal 2005, Mir Pur 2006, Grw 2005-2009, Bwp 2008, Grw 2011)

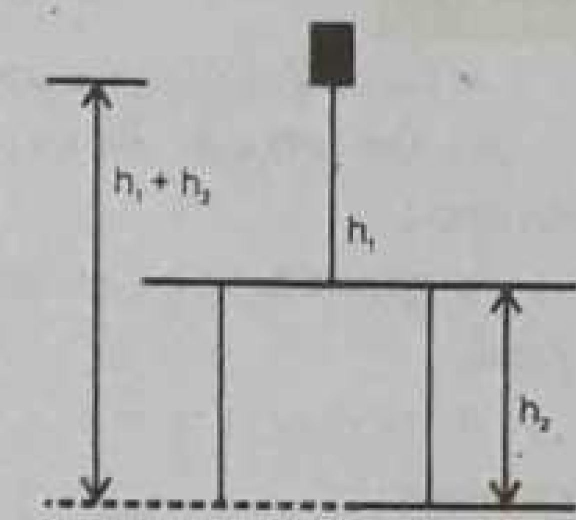
Ans. We know that energy is defined as the ability of a body to do work. So an object has one joule potential energy means that the body has capacity to do a work of one joule.

Q.4.6 A ball of mass m is held at a height h_1 above a table. The table top is at a height h_2 above the floor. One student says that the ball has potential energy mgh_1, but another says that it is $mg(h_1 + h_2)$ Who is correct?

Ans. <u>Both</u> of them are correct.

Reason

Since P.E is always measured with respect to some reference point. Therefore, we can say that first student has measured P.E *with respect to <u>table top</u>* (mgh_1) while the second student has measured P.E *with respect to floor* = $mg(h_1 + h_2)$



Q.4.7 When a rocket re-enters the atmosphere, its nose cone becomes very hot. Where does this heat energy come from?

(Grw 2003-2009, Mtn 2004-2009, Lhr 2005-2006, D.G.Khan 2005, Bwp 2004, Lhr 2010-2011)

Ans. **Source of heat energy**

When the rocket re-enters the atmosphere, then some of its kinetic energy is used in doing ***<u>work against friction</u>*** with dust particles and air, which is converted into heat. Due to this heat energy, its nose cone becomes very hot.

Q.4.8 What sort of energy is in the following:

(a) compressed spring.

(b) water in a high dam.

(c) A moving car. (Sgd 2003, Mtn 2004, Rwp 2005, Bwp 2007, Lhr 2009)

Ans. a) A compressed spring has ***<u>elastic potential energy</u>***.

b) Water in the high dam has ***<u>gravitational potential energy</u>***.

c) A moving car has ***<u>kinetic energy</u>***.

Q.4.9 A girl drops a cup from a certain height, which breaks into pieces. What energy changes are involved? (Grw 2005, Bwp 2006)

Ans. **Energy changes**

A cup thrown from certain height losses its gravitational potential energy and gain its K.E. When it strikes the ground then a part of this kinetic energy is used to break the cup and rest of the energy converts into;

(i) Sound energy

(ii) K.E of moving pieces

(iii) Heat energy.

Q.4.10 A boy uses a catapult to thrown a stone which accidentally smashes a green house window. List the possible energy changes? (Grw 2011)

Ans. **Possible Energy changes:**

The following energy changes occur,

(i) When the boy throws the stone the elastic potential energy changes into ***<u>kinetic energy</u>***.

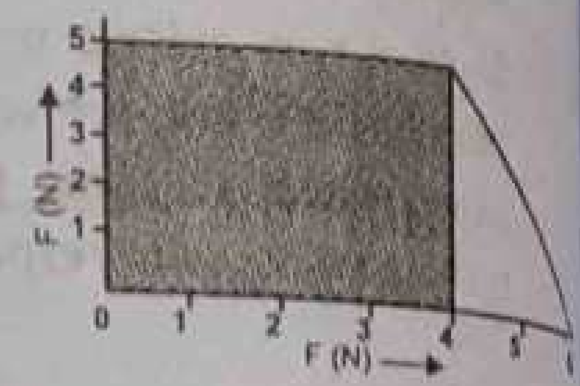
(ii) When stone hits the window, a part of kinetic energy used to ***<u>break the window</u>*** into pieces. (Lhr 2006)

(iii) Rest of the energy converted into ***<u>heat and sound</u>***.

Solved Examples

Example 4.1

A force F acting on an object varies with distance x as shown in Fig. 4.7. Calculate the work done by the force as the object moves from x = 0 to x = 6 m.



Given Data:

A force acting on an object varies with distance x as shown in Fig.

To Find:

Work done = W = ?

Calculation:

Total work done = total area under the curve

= area of rectangle + area of triangle

$= (4m \times 5N) + \frac{1}{2}(2m \times 5N)$

$= 20\text{ Nm} + 5\text{ Nm}$

$= 25\text{ Nm}$

$= 25J$

Example 4.2

A 70 kg man runs up a long flight of stairs in 4 s. The vertical height of the stairs is 4.5 m. calculate its power output in watts.

Given Data:

Mass of the man = m = 70 kg

Time taken = t = 4 sec.

Height of the stairs = h = 4.5 m

To Find:

Power output = P = ?

Calculation:

As power $= P = \frac{W}{t}$

or $P = \frac{mgh}{t}$

Putting values, we get

$P = \frac{70 \times 9.8 \times 4.5}{4}$

$P = \frac{3087}{4}$

P = 771.5 watt.

Or $P = 7.7 \times 10^2$ watt

Example 4.3

A brick of mass 2.0 kg is dropped from a rest position 5.0 m above the ground. What is the velocity at a height of 3.0 m above the ground?

Given Data:

Mass of brick = m = 2 kg

Initial velocity of brick = $v_1 = 0$

Initial height = $h_1 = 5m$

Final height = $h_2 = 3m$

To Find:

Velocity at height 3m above the ground = v_2 = ?

Calculation:

Loss of P.E = Gain in K.E.

$mg(h_1 - h_2) = \frac{1}{2}m(v_2^2 - v_1^2)$

Putting values, we get

$2 \times 9.8(5-3) = \frac{1}{2} \times 2(v_2^2 - 0^2)$

$2 \times 9.8 \times 2 = v_2^2$

Or $v_2^2 = 39.2$ (As $v_2 = v$)

So $v = 6.3\text{ ms}^{-1}$

Exercise Problems

4.1 **A man pushes a lawn mower with a 40 N force directed at an angle of 20° downward from the horizontal. Find the work done by the man as he cuts a strip of grass 20 m long.**

Given Data:

Applied force = F = 40 N

Angle = θ = 20°

Length of strip of grass = d = 20m

To find:

Work done= W= ?

Calculation:

As work $= W = \vec{F}.\vec{d} = Fd\cos\theta$

Or $W = Fd\cos\theta$

Putting the values, we get

$W = 40 \times 20 \times \cos 20^\circ$

$W = 40 \times 20 \times 0.93$

$W = 751.68J$

$\boxed{W = 7.5 \times 10^2 J}$

4.2 **A rain drop $(m = 3.35 \times 10^{-5} kg)$ falls vertically at a constant speed under the influence of the forces of gravity and friction. In falling through 100m, much work is done by (a) gravity and (b) friction.**

Given Data:

Mass of rain drop $= m = 3.35 \times 10^{-5} kg$

Height = h = 100m

To find:

Work done by gravity = W_g = ?

(a) work done by friction = W_f = ?

Calculation:

(a):- Work done by gravity = $W_g = \vec{F}.\vec{h}$

$W_g = F h \cos\theta$

As gravitational force is along the displacement, therefore angle between mg and h is 0^o

So $W_g = mgh \cos 0^o$

$W_g = mgh$

$W_g = 3.35\times10^{-5}\times9.8\times100$

$W_g = 3283\times10^{-5}$

$\boxed{W_g = 0.0328J}$

(b):- Work done by friction $= W_f = \vec{F}.\vec{h}$

$W_f = f h \cos\theta$

As friction is opposite to displacement, therefore angle between frictional force and h is 180^o.

So $W_f = f h \cos 180^o$ $(\because \cos 180^o = -1)$

$W_f = -fh$

As the drop is falling at constant speed so $f = W = mg$

$W_f = -mgh$

Putting values, we get

$W_f = -3.35\times10^{-5}\times9.8\times100$

$\boxed{W_f = -0.0328J}$

4.3 **Ten bricks, each 6.0 cm thick and mass 1.5 kg, lie flat on a table. How much work is required to stack them one on the top of another?**

Given Data:

Mass of each brick $= m = 1.5kg$

Height of each brick $= h = 6cm = 0.06m$

Number of brick $= n = 10$

To find:

Work required to stack them one on the top of another = W = ?

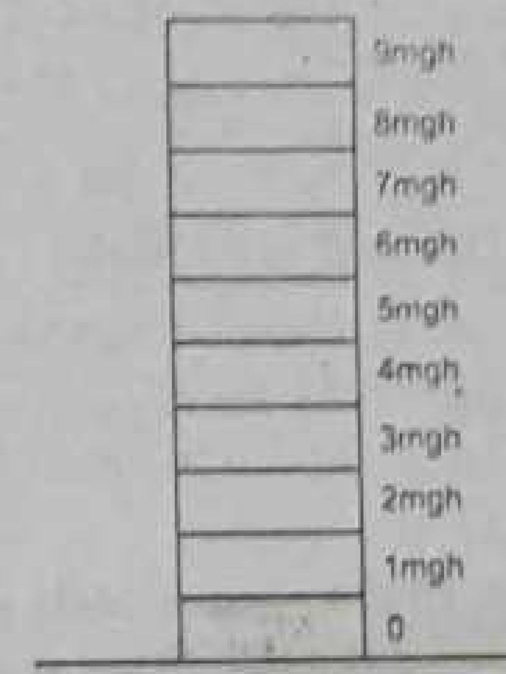
Calculation:

...ork will be done on the first brick lying flat on the table. (i.e $W_1 = 0$). When we place other bricks on it ...y one, work is done in the form of P.E, according to their height. Hence we must add all these potential ...es to get the total work done in terms of P.E. Thus work done for 2^{nd} brick when place on to the Ist brick

$W_2 = mg.h \cos 0^o = mgh$

work done for 3^{rd} brick when place on to the two bricks

$W_3 = mg.(2h) \cos 0^o = 2mgh$



Similarly

$W_4 = 3mgh$

$W_5 = 4mgh$

$W_6 = 5mgh$

$W_7 = 6mgh$

$W_8 = 7mgh$

$W_9 = 8mgh$

$W_{10} = 9mgh$

Total work done is

$W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 + W_9 + W_{10}$

$W = 0 + mgh + 2mgh + 3mgh + 4mgh + 5mgh + 6mgh + 7mgh + 8mgh + 9mgh$

$W = 45mgh$

Putting values, we get

$W = 45\times1.5\times9.8\times0.06$

$W = 39.69$

$\boxed{W = 40J}$ (approximately)

4.4 **A car of mass 800 kg traveling at 54 kmh^{-1} is brought to rest in 60 metres. Find the average retarding force on the car. What has happened to original kinetic energy?**

Given Data:

Mass of car = m = 800 kg

Initial velocity = $v_i = 54\,km/h = \frac{54\times1000}{60\times60} = 15\,m/s$

Final velocity = $v_f = 0$

Distance covered = d = 60 m

To find:

(i) Average retarding force = F = ?

(ii) What happened to original K.E. = ?

Calculation:

(i) As according to work-energy principle,

$$Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Or $$Fd = \frac{1}{2}m(v_f^2 - v_i^2)$$

Putting values, we get

Or $F(60) = \frac{1}{2} \times 800(0-(15)^2)$

$60F = 400 \times (-225)$

$F = -\frac{400 \times 225}{60}$

Or $\boxed{F = -1500N}$

Negative sign shows that it is retarding force.

(ii) As the velocity of the car is decreasing, so its kinetic energy also decreases and becomes zero due to frictional force.

4.5 A 1000 kg automobile at the top of an incline 10metre high and 100 m long is released and rolls down the hill. What is its speed at the bottom of the incline if the average retarding force due to friction is 480 N?

Given Data:

Mass of automobile = m = 1000 kg
Height of incline = h=10m
Length of incline = s = 100m
Average retarding force = f= 480 N

To find:

Final speed of automobile at bottom of incline = v = ?

Calculations:

As

Using the relation loss of P.E = gain in K.E + work done against friction

$$mgh = \frac{1}{2}mv^2 + fd$$

$$mgh - fd = \frac{1}{2}mv^2$$

Putting values, we get

$$1000 \times 9.8 \times 10 - 480 \times 100 = \frac{1}{2} \times 1000v^2$$

$$98000 - 48000 = 500\,v^2$$

$$50000 = 500\,v^2$$

or $v^2 = \frac{50000}{500}$

$v^2 = 100$

or $\boxed{v = 10\ ms^{-1}}$

4.6 100 m^3 of water is pumped from a reservoir into a tank, 10 m higher than the reservoir, in 20 minutes. If density of water is 1000 kg m^{-3}, find

(a) the increase in P.E.

(b) the power delivered by the pump.

Given Data:

Volume of water = $V = 100m^3$

Height of the tank = $h = 10m$
Time taken = $t = 20\min = 1200sec$
Density of water $\rho = 1000kg/m^3$

To find:

(a) Increase in P.E = mgh = ?
(b) Power delivered by pump = P = ?

Calculations:

As density = $\frac{mass}{volume}$

Or Mass = volume × density

Or $m = V \times \rho$

Or $m = 100 \times 1000$

$m = 100000$ kg

(a) Increase in P.E .= mgh

Increase in P.E .= 100000 × 9.8 × 10

$\boxed{\text{Increase in P.E .} = 9.8 \times 10^6 J}$

(c) Power delivered by pump =

$$P = \frac{work}{time}$$

$$P = \frac{P.E.}{t}$$

$$P = \frac{9.8 \times 10^6}{1200}$$

$$P = 8166.6$$

$$P = 8.16 \times 10^3 watt$$

Or $\boxed{P = 8.2kW}$

4.7. A force (thrust)of 400 N is required to overcome road friction and air resistance in propelling an automobile at 80 kmh^{-1}. What power (KW) must the engine develop?

Given data:

Force (thrust) = F = 400 N

Velocity of engine = v = 80 km/h = $\frac{80 \times 1000}{60 \times 60} = 22.22\,m/s$

To find:

Power = P=?

Calculation:

As $P = \vec{F}.\vec{v}$

Or $P = Fv\cos\theta$

As force and velocity are in the some direction ,so $\theta = 0^\circ$

$P = Fv\cos 0^\circ$

$P = Fv$

Putting values, we get

$P = 400 \times 22.22$

$P = 8880$ watt

Or $P = 8.88 \times 10^3$ watt

Or $\boxed{P = 8.9\ kW}$

4.8. How large a force is required to accelerate an electron ($m=9.1\times10^{-31}kg$) from rest to a speed of $2.0\times10^7 ms^{-1}$ through a distance of 5.0 cm?

Given data:

Mass of an electron = $m = 9.1\times10^{-31}kg$

Initial velocity = $v_i = 0$

Final velocity = $v_f = 2\times10^7 m/s$

Distance = $d = 5cm = 0.05m$

To find:

Force required = F = ?

Calculation:

As work energy principle is

Work done = change in the K.E. of the object

$$Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$Fd = \frac{1}{2}mv_f^2 \qquad \text{As } v_i = 0$$

Putting values, we get

$$F \times 0.05 = \frac{1}{2}\times 9.1\times10^{-31}\times(2\times10^7)^2$$

$$F \times 0.05 = \frac{1}{2}\times 9.1\times10^{-31}\times 4\times10^{14}$$

$$F \times 0.05 = 18.2\times10^{-17}$$

$$F = \frac{18.2}{0.05}\times10^{-17}$$

$$\boxed{F = 3.6\times10^{-15}\ N}$$

4.9 A diver weighing 750 N dives from a board 10m above the surface of a pool of water. Use the conservation of mechanical energy to find his speed at a point 5.0 m above the water surface neglecting air friction.

Given Data:

Weight of the diver = mg = 750 N

Height of a point above the based = h_1 = 10m,

Height of a point above the water surface = h_2 = 5 m

To find:

Speed of diver at h_2 = v = ?

Calculations:

Gain of K.E. = loss of P.E. (for neglecting air friction)

$$\frac{1}{2}mv^2 = mg(h_1 - h_2)$$

Or $v^2 = 2g(h_1 - h_2)$

Putting values, we get

$$v^2 = 2\times9.8(10-5)$$

$$v^2 = 19.6\times5$$

$$v^2 = 98$$

$$\boxed{v = 9.9 m/s}$$

4.10 A child starts from rest at the top of a slide of height 4.0 m.

(a) What is his speed at the bottom if the slide is frictionless?

(b) If he reaches the bottom, with a speed of $6ms^{-1}$, what percentage of his total energy at the top of the slide is lost as a result of friction?

Given Data:

Initial velocity = v_i = 0

Height = h = 4m

To find:

(a) speed of child at the bottom if slide is frictionless = v= ?

(b) % age of total energy lost = ?

if velocity at bottom is $v' = 6ms^{-1}$

Calculations:

(a) Speed of child = v = ?

For frictionless system

Gain of K.E = loss of P.E.

$$\frac{1}{2}mv^2 = mgh$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

Putting values, we get

$$v = \sqrt{2\times9.8\times4}$$

$$v = \sqrt{78.4}$$

$$\boxed{v = 8.8 m/s}$$

(b) % age loss of Energy = ?

In the presence of friction

Speed = $v' = 6m/s$

P. E. at the top $= mgh$

$= m \times 9.8 \times 4$

$= 39.2\ m$

K. E. at the bottom $= \frac{1}{2} mv'^2$

$= \frac{1}{2} \times m \times (6)^2$

$= \frac{1}{2} \times m \times 36$

$= 18$

Loss of energy $= 39.2m - 18m$

$= 21.2m$

% loss of energy $= \frac{\text{Loss of energy}}{\text{Total Energy}} \times 100$

% loss of energy $= \frac{21.2m}{39.2m} \times 100$

$= 54\%$

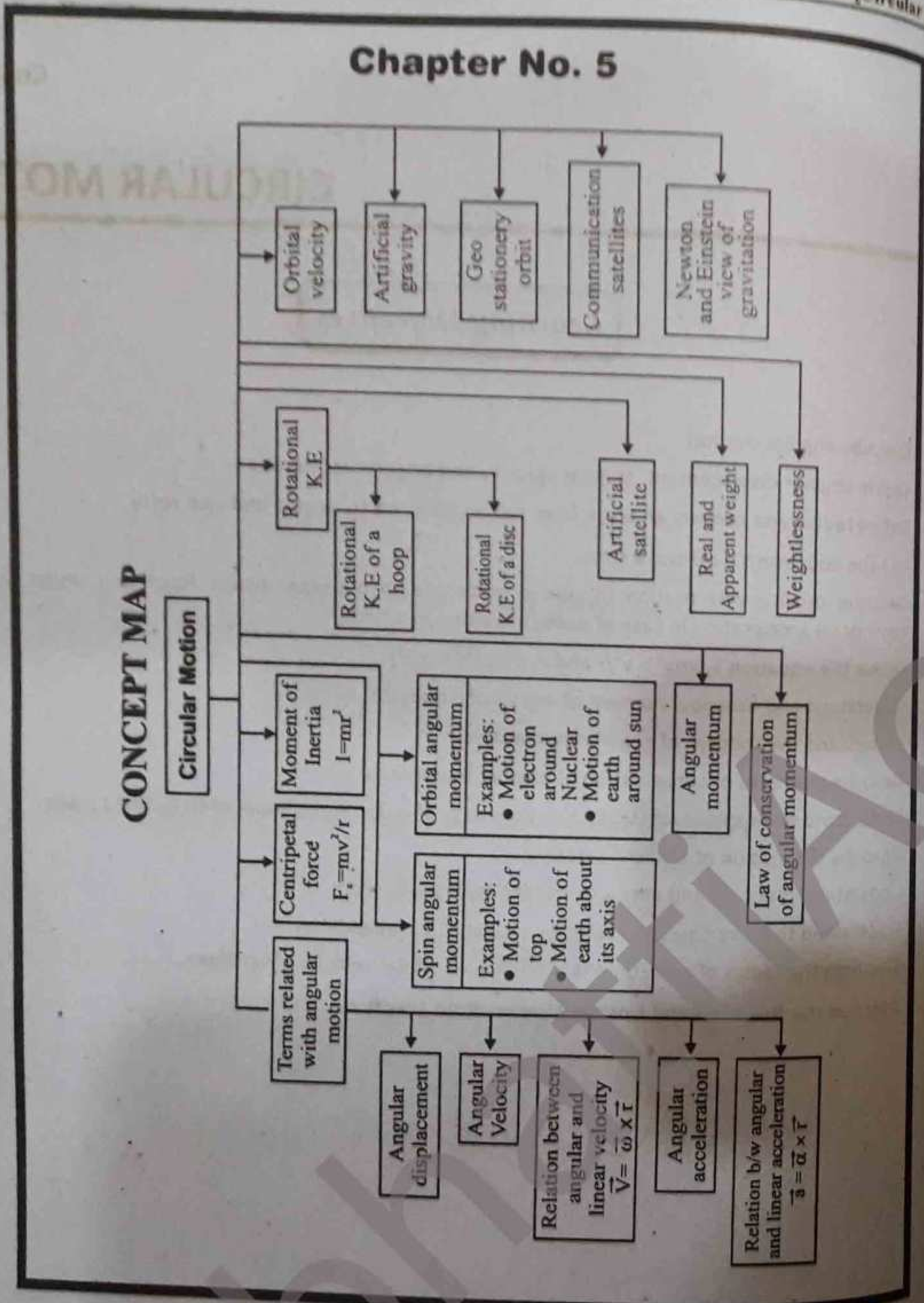
****

Chapter

CIRCULAR MOTION

Learning Objectives

1. Describe angular motion.
2. Define angular displacement, angular velocity and angular acceleration.
3. Define radian and convert an angle from radian measure to degree and vice versa.
4. Use the equation $S = r\theta$ and $v = r\omega$.
5. Describe qualitatively motion in curved path due to a perpendicular force and under stand th centripetal acceleration in case of uniform motion in a circle.
6. Derive the equation $a_c = r\omega^2 = v^2/r$ and $F_c = m\omega^2 r = mv^2/r$.
7. Understand and describe moment of inertia of a body.
8. Understand the concept of angular momentum.
9. Describe examples of conservation of angular momentum.
10. Understand and express rotational kinetic energy of a disc and a hoop on an inclined plane.
11. Describe the motion of artificial satellites.
12. Understand that how and why artificial gravity is produced.
13. Understand that the objects in satellite appear to be weightless.
14. Calculate the radius of geo-stationary orbits and orbital velocity of satellites.
15. Describe the Newton's and Einstein's views about gravity.



Circular Motion

The motion of an object in a circular path is called circular motion. In circular motion, the direction of the body continuously changes.

Examples

1) A stone whirled around by a string
2) A satellite in orbits around the earth.
3) Motion of electron in atoms
4) Motion of CD's.
5) Motion of circular saw blade
6) Motion of ceiling fan
7) Motion of merry go round etc.

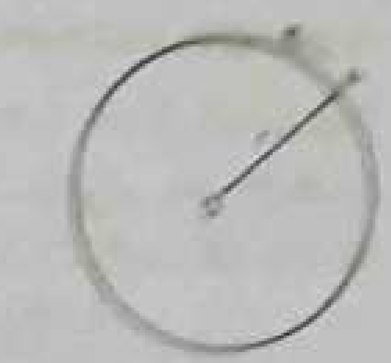


Fig. 5.1(a)

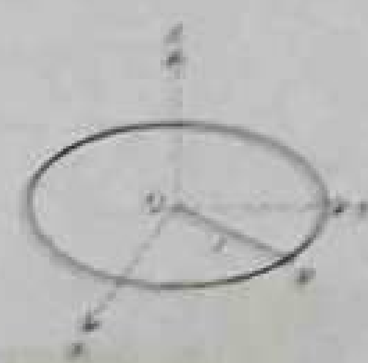


Fig. 5.1(b)

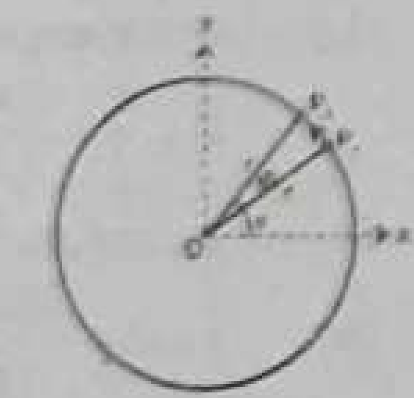


Fig. 5.1(c)

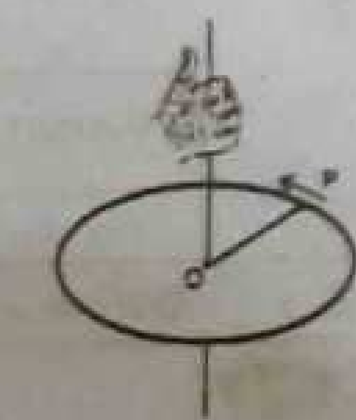


Fig. 5.1(d)

Q.1 Define and explain angular displacement. Show that 1 radian =57.3°

Ans.

Angular Displacement

The angle subtended at the center of a circle by a body moving along the circumference in a given time is called angular displacement. It is denoted by θ.

Explanation

Consider a body of mass m moving in a circular path of radius r. Suppose this motion is taking place by attaching the particle P at the end of massless rigid rod of length r whose other end is pivoted at the center O. The rod OP rotates in the plane of circle due to motion of the particle P. The axis of rotation passes through the center of the circle, which is perpendicular to the plane of circle while OP is rotating.

Suppose at any instant t the position of OP_1 makes an angle θ with x-axis. After some time $t+\Delta t$ its position be OP_2 making angle $\theta+\Delta\theta$ with x-axis. Thus the angular displacement can be defined as *the angle 'Δθ' of OP during time interval Δt. For very small values of Δθ, the angular displacement is a vector quantity.*

Degree
In one complete rotation, a rotating object subtends an angle of 360 degree. If the circular path is divided into 360 equal parts then the angle subtended by each part at the center of the circle is equal to one degree.
Revolution
A complete round trip of the body along the circumference of the circle is called one revolution.
Radian
It is the angle subtended by an arc at the center of the circle whose length is equal to the radius of circle.

Sigh Convention

For anticlockwise rotation of OP, the angular displacement Δθ is positive while for clock-wise rotation the angular displacement Δθ is negative.

Direction of Angular Displacement

In order to determine the direction of angular displacement, we use the *'right hand rule'*

Right hand Rule

Grasp the axis of rotation in right hand with fingers curling in the direction of rotation then the erect thumb indicates the direction of angular displacement.

Units

The SI unit of angular displacement is radian. It is the angle subtended by an arc at the center of the circle whose length is equal to the radius of circle.
Other units are degrees and revolution. Angular displacement has *no* dimensions.

Relation between linear displacement and angular displacement

Suppose S is the length of the circle of radius r which makes an angle 'θ' at the center of the circle. Then in terms of radian, it can be expressed as,

$$\theta = \frac{\text{Length of arc}}{\text{Radius of circle}}(\text{rad})$$

$$\theta = \frac{S}{r}(\text{rad})$$

Or $\boxed{S = r\,\theta}$

This is the relation between linear displacement and angular displacement.

Relation between radian and degree

In one complete revolution the linear distance covered by a particle is equal to the circumference of the circle i.e., $S = 2\pi r$ and the angle in radian would be

$$\theta = \frac{S}{r} = \frac{2\pi r}{r} = 2\pi \text{ radian}$$

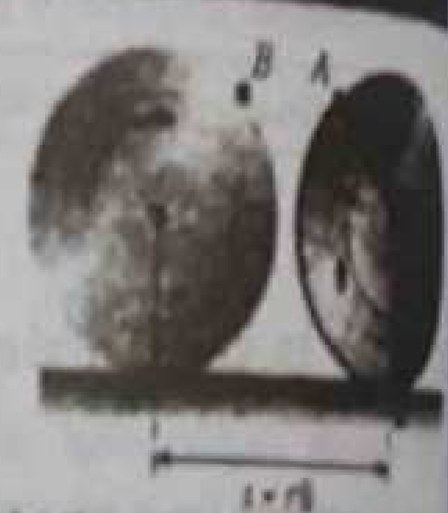
So, 2π radian $= 360° = 1$ revolution

Or $1 \text{ rad} = \frac{360°}{2\pi}$

Or $1 \text{ rad} = \frac{360°}{2\times 3.14}$

Or $\boxed{1\text{radian} = 57.3°}$

Do You Know?



As the wheel turns through angle θ, it lays out a tangential distance $S = r\theta$

Q.2 Define and explain the term angular velocity.

Ans

Angular Velocity

Time rate of change of angular displacement is called as angular velocity.

Suppose Δθ is the angular displacement during the time Δt. So the average angular velocity can be expressed as,

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$

The instantaneous angular velocity can be defined as the limiting value of Δθ/Δt as the time interval Δt, following the time t, approaches to zero.

$$\text{So, } \omega_{ins} = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}$$

Direction

Angular velocity is a vector quantity. Its direction is along the axis of rotation and can be determine by right hand rule.

Unit

The SI unit of angular velocity is rad/sec. It is also measured in revolution/min and degree/sec. The dimension of angular velocity is $[T^{-1}]$.

FOR YOUR INFORMATION

In pure translational motion every particle moves in straight line and covers the same distance while in translational motion every particle moves in circle and covers the same angular displacement.

Q.3 Define and explain the angular acceleration?

Ans

Angular Acceleration

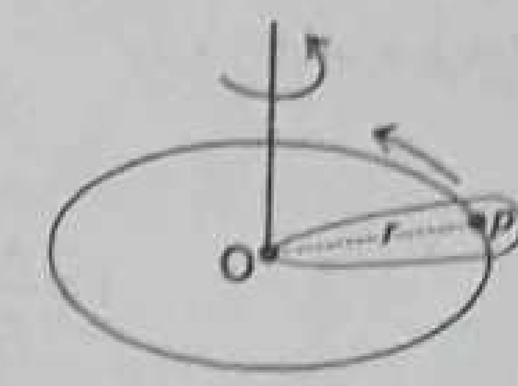
The time rate of change of angular velocity is called angular acceleration.

Average Angular Acceleration

When we switch on dm electric fan, the angular velocity goes on increasing. If ω_i is the initial angular velocity and ω_f is the final angular velocity at time t_i and t_f respectively. Then the average angular acceleration during time t_f-t_i is can be defined as *"the ratio of total change in angular velocity to the total time interval"*

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i}$$

Or $$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$$



FOR YOUR INFORMATION

When angular velocity of the body is increasing then angular acceleration is along the direction of angular velocity and if angular velocity is decreasing then 'α' is opposite to the direction of angular velocity.

Instantaneous angular acceleration

The instantaneous angular acceleration can be defined as *the limiting value of $\frac{\Delta\omega}{\Delta t}$ as the time interval Δt approaches to zero, is called instantaneous angular acceleration.*

So, $$\alpha_{ins} = \lim_{\Delta t\to 0}\frac{\Delta\omega}{\Delta t}$$

Direction

It is a vector quantity. The direction of angular acceleration is along the axis of rotation.

Unit

The SI unit of angular acceleration is rad/sec². Its dimension is $[T^{-2}]$

Axis of Rotation

All particles of a rotating body moves in circles. The line joining the centres of these circles is called axis of rotation.

Q.4 Derive the relation between angular and linear velocities.

OR

Show that v = rω

Ans

Relation between Angular and Linear Velocities

Consider a rigid body rotating about z-axis with an angular velocity ω. Suppose a point P in a rigid body at a perpendicular distance r from axis of rotation with linear velocity v. As the axis of rotation is fixed, so the direction of angular velocity ω always remains the same.

Suppose point P moves through a distance $P_1P_2 = \Delta S$ in time Δt during its circular motion. If reference line OP has an angular displacement Δθ during time interval Δt Then we can write,

$$\Delta S = r\Delta\theta$$

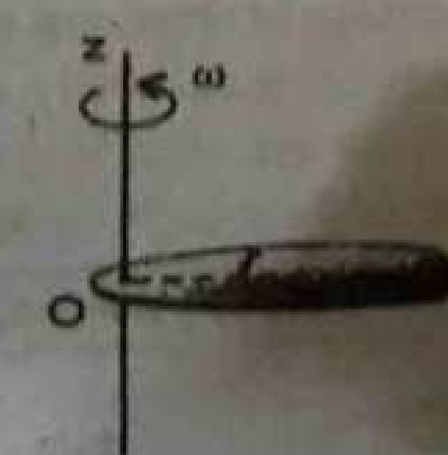


Fig. 5.4(a)

Dividing both sides by Δt

So,

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta \theta}{\Delta t}$$

Applying limit $\Delta t \to 0$

$$\lim_{\Delta t \to 0}\frac{\Delta S}{\Delta t} = \lim_{\Delta t \to 0} r\frac{\Delta \theta}{\Delta t}$$

$$\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t} = r\lim_{\Delta t \to 0}\frac{\Delta \theta}{\Delta t}$$

Or $\boxed{v = r\omega}$

In vector form $\vec{v} = \vec{\omega} \times \vec{r}$

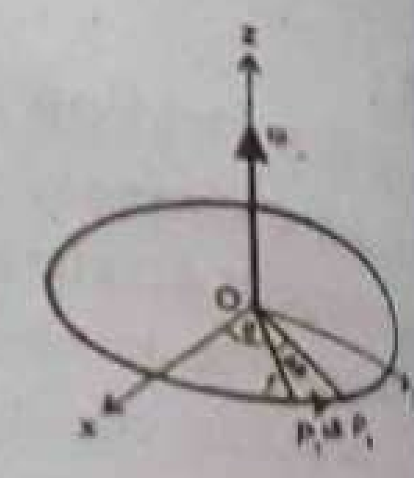


Fig. 5.4(b)

For limit $\Delta t \to 0$ the length of the arc P_1P_2 becomes very small and its direction represents the direction of the circle at point P_1. Thus the direction of velocity v at any point P is always tangent to the circular path. So the linear velocity at point P is called tangential velocity.

Q.5 Derive the relation between linear and angular acceleration.

Ans.

Relation between Linear and Angular Acceleration

As the reference line OP is rotating with angular acceleration α. The point P also has a linear or tangential acceleration. As we know,

$$\Delta v = r\Delta\omega$$

Dividing both sides by Δt

$$\frac{\Delta V}{\Delta t} = r\frac{\Delta \omega}{\Delta t}$$

On both sides, we have

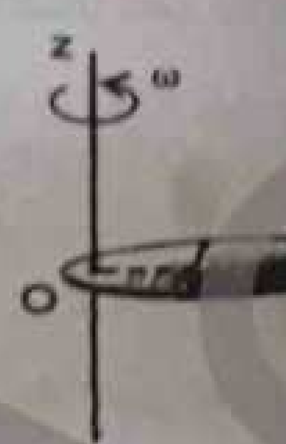


Fig. 5.4(a)

Applying limit as $\Delta t \to 0$

So, $$\lim_{\Delta t \to 0}\frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} r\frac{\Delta \omega}{\Delta t}$$

$$\lim_{\Delta t \to 0}\frac{\Delta v}{\Delta t} = r\lim_{\Delta t \to 0}\frac{\Delta \omega}{\Delta t}$$

Or $\boxed{a = r\alpha}$

In vector form $\vec{a} = \vec{\alpha} \times \vec{r}$

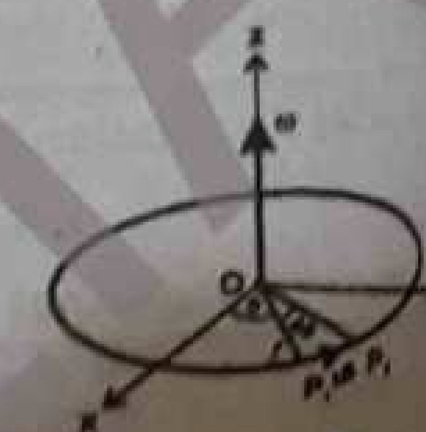


Fig. 5.4(b)

Where a is the tangential acceleration

Note:-

Advantage of rotational motion over translational motion

On a rotating rigid body, the points at different distances have the same angular displacement, angular speed and angular acceleration. While they have different linear displacement, velocity and acceleration.

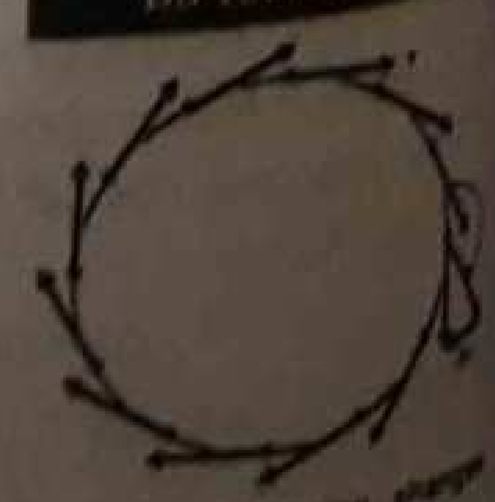
Q.6 How can we write the equations of motion in case of angular motion?

Ans.

Equation of Angular Motion

Equation of angular motion are similar to those in linear motion except that S, v

Do You Know?


Direction of motion changes continuously in circular motion

and a have been replaced with θ, ω and α respectively. Thus

Equations for linear motion

$$v_f = v_i + at$$

$$S = v_i t + \tfrac{1}{2} at^2$$

$$2aS = v_f^2 - v_i^2$$

Equation for angular motion

$$\omega_f = \omega_i + \alpha t$$

$$\theta = \omega_i t + \tfrac{1}{2}\alpha t^2$$

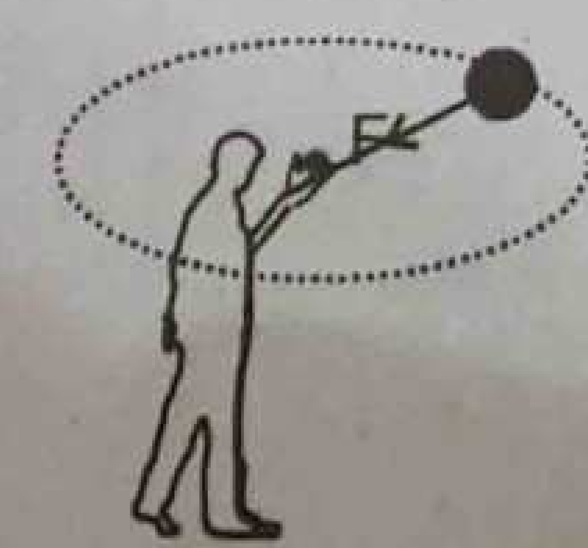
$$2\alpha\theta = \omega_f^2 - \omega_i^2$$

Equation of angular motion hold only if axis of rotation is fixed. In this case all the angular vectors have the same direction. So they can be treated as scalars.

Q.7 Define and explain the centripetal force and derive the relation for it?

Ans.

Centripetal Force



The force which bends the normally straight path of a particle into circular path is called centripetal force.

OR

A force which compels a body to move in a circular path is called centripetal force

Explanation

Consider a body attached with a string moving in a circular path. If the string is snapped (broken) then it would not continue to move in circle. Observation shows, if the string snaps, and when ball is at A then it will follow the straight line path AB. So in the absence of some force which pulls it toward the center of circle, the ball will not continue to move along circular path. It will move along the tangent at that point.

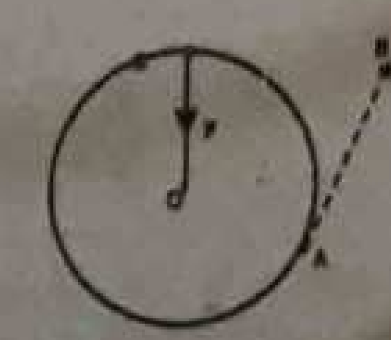


Fig. 5.5(b)

Examples of centripetal force:

(1) Force acting on electrons in fixed orbits around the nucleus.
(2) Force acting on artificial and natural satellites.
(3) Force acting on earth around the sun.

Expression for centripetal acceleration and centripetal force

Consider a particle that moves from point A to B with uniform speed v in a circle of radius 'r'. If Δv is the change in velocity then the acceleration of the particle is

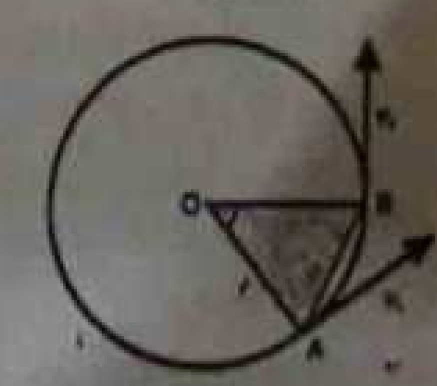


Fig. 5.5(a)

$$a = \frac{\Delta v}{\Delta t} \quad \text{(1)}$$

Where Δt is time required by particle to move from A to B. Let $\vec{v_1}$ and $\vec{v_2}$ are velocities at points A and B respectively. Since the speed of the particle is v, So time taken to travel distance S is,

$$\Delta t = \frac{S}{v}$$

Thus equation (1) becomes

$$a = \frac{\Delta v}{S/v}$$

Or $a = v\frac{\Delta v}{S}$ ______ (2)

Now we draw a triangle ΔPQR such that PQ is parallel and equal to $\vec{v}_1$ and PR is parallel and equal to $\vec{v}_2$. As radius of the circle is perpendicular to its tangent, so $\vec{v}_1$ and $\vec{v}_2$ are perpendicular to OA and OB respectively. So the angle ∠AOB is equal to the angle <QPR.

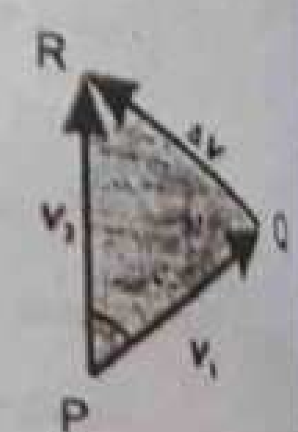


Fig. 5.6(b)

Moreover $|\vec{v}_1| = |\vec{v}_2| = v$ and OA = OB (radii of same circle)

So from isosceles triangles OAB and PQR, we can write,

$$\frac{QR}{PR} = \frac{AB}{OB}$$

Or $\frac{\Delta v}{v} = \frac{AB}{r}$ ______ (3)

when Δt→o, point 'B' is very close to 'A', then length of arc $\widehat{AB}$ nearly equal line AB. i.e. AB= S

Equation (3) becomes,

$$\frac{\Delta v}{v} = \frac{S}{r}$$

Or $\Delta v = S\frac{v}{r}$ ______ (4)

Putting this value in equation (2), we get

$$a = v\frac{Sv}{rS}$$

$\boxed{a = \frac{v^2}{r}}$ ______ (5)

Where a is the instantaneous acceleration, as this acceleration is caused by centripetal force, so it is called centripetal acceleration a_c.

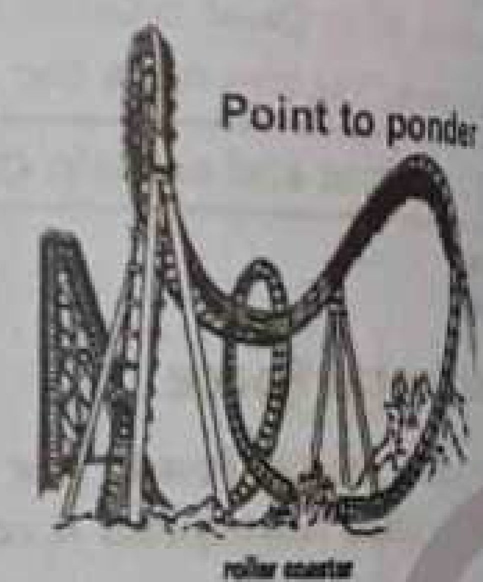
Direction of acceleration

Since PQ is perpendicular to OA and PR is perpendicular to OB. So QR is parallel to the perpendicular bisector of AB. As acceleration is parallel to Δv when AB → o, So direction of centripetal acceleration is along the radius, towards the center of the circle. So,

The instantaneous acceleration of an object traveling with uniform speed in a circle is directed towards the center of the circle and is called centripetal acceleration.

The direction of centripetal force is in the direction of centripetal acceleration.

Point to ponder



You may feel scared at the top of roller coaster ride in the parks but you never fall down even when

Explanation
You do not fall out of your seat when you go upside down on a roller coaster because of many forces on the in going loop on a roller coaster the centrifugal and centripetal forces, the force of drag, gravity, and friction. Due to centrifugal force, your body is pressed with the outer rim.

Expression for centripetal force

As $F_c = ma_c$

And $a_c = \frac{v^2}{r}$

so $\boxed{F_c = \frac{mv^2}{r}}$

In angular measurements, this equation becomes

So,

$$F_c = \frac{mr^2\omega^2}{r} \qquad (v = r\omega)$$

Or $\boxed{F_c = mr\omega^2}$

Unit of centripetal force is Newton and dimensions are [MLT^{-2}].

Q.8 Define and explain the moment of inertia?

Ans.

Moment of Inertia (Rotational Inertia)

Moment of inertia of a particle is defined ***as the product of mass of particle and square of its perpendicular distance from (pivot) the axis of rotation.***

It is denoted by I and is given by

$$I = mr^2$$

Explanation

Consider a mass attached to a massless rod which can rotate about a frictionless pivot O. Let the system be in horizontal plane. A force F acts on the mass perpendicular to the rod. So,

$F = ma$ ______ (1)

This force rotates the mass m about O. As the angular acceleration α can be expressed as

$$a = r\alpha$$

So, equation (1) becomes,

$F = mr\alpha$ ______ (2)

Multiplying both sides by r

$rF = mr^2\alpha$ ______ (3)

Since rF represents the torque.

So equation (3) becomes

$$\tau = mr^2\alpha$$

Here F is replaced by τ, a by α and m by mr^2. The quantity mr^2 is known as moment inertia I. It plays the same role in angular motion as mass plays in linear motion. *It determines the angular acceleration of a body.*

So, $\boxed{I = mr^2}$

Tid - Bits

Banked tracks are needed for turns that are taken so quickly that friction alone cannot provide energy for centripetal force.

EXPLANATION:
In circular motion, centripetal force is provided by force of friction. When speed of a car is increased at a turn, then frictional force is not sufficient to provide centripetal force. Therefore, the car will move away from the track. To avoid the side-slip, the outer edge of the road is raised through some angle, called the angle of inclination. The speed of the car at the turn is $v = \sqrt{rg\tan\phi}$ where ϕ is angle of inclination or banking.

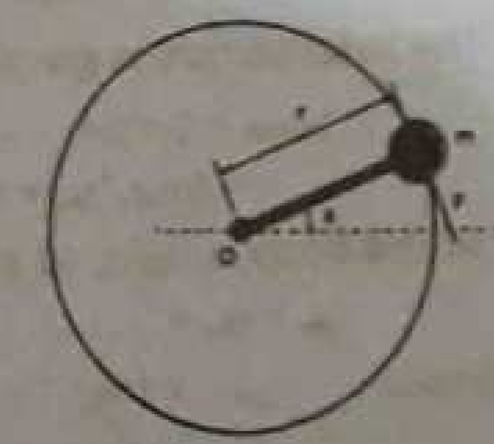
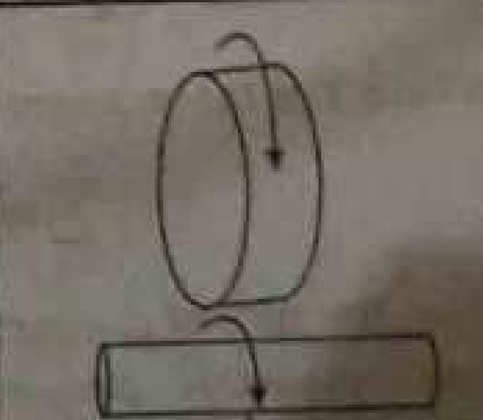


Fig 5.8



Two cylinders of equal mass. The one with he larger diameter has the greater rotational inertia.

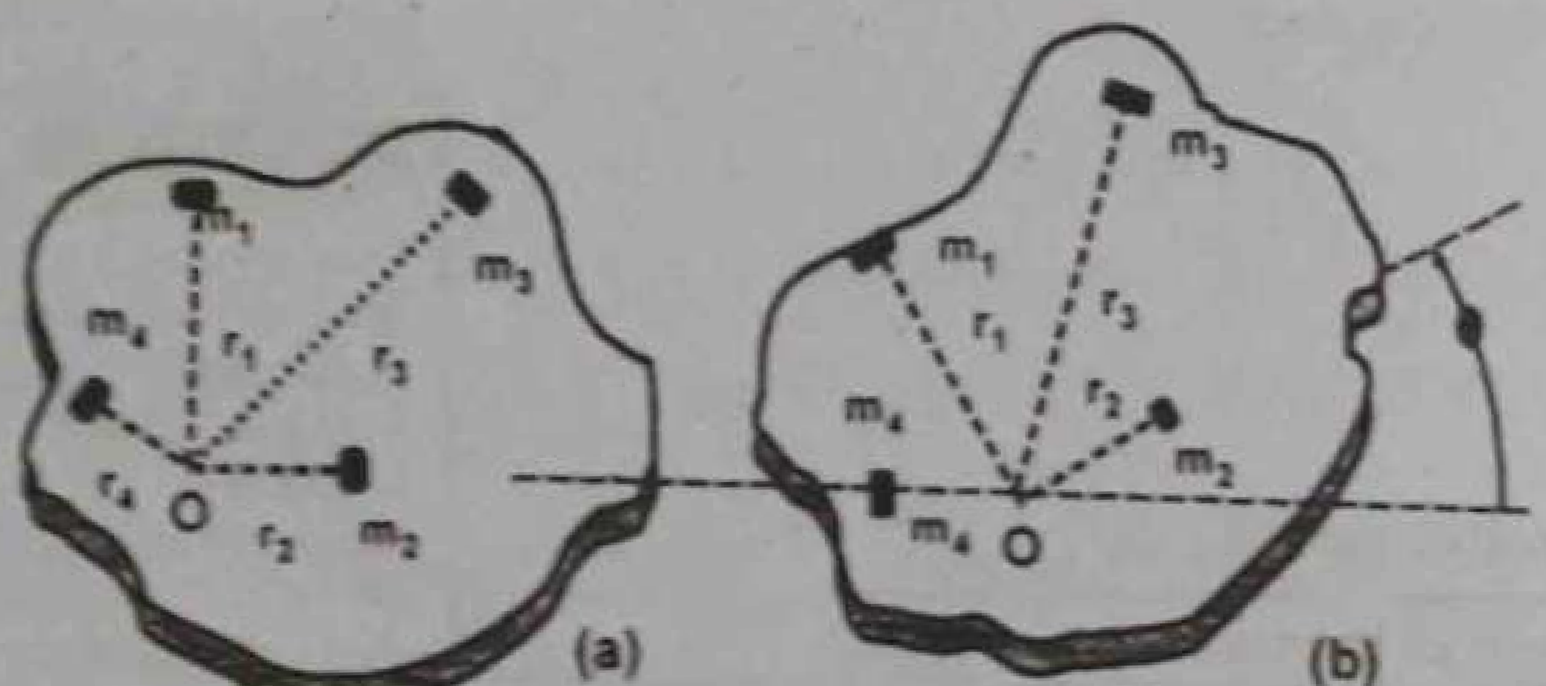
DO YOU KNOW
Moment of inertia represent the tendency to maintain its state of rest or state of uniform angular motion.

Dependence of moment of inertia

This shows that moment of inertia depends upon the mass m and the square of perpendicular distance from axis of rotation r. Practically, it also depends upon distribution of mass and position of axis of rotation.

Moment of inertia of a rigid body

Mostly the bodies have non-uniform mass distribution. Consider a rigid body made up of 'n' small pieces of masses m_1, m_2.......at distance r_1, r_2 .... from axis of rotation O



Suppose the body be rotating with angular acceleration α. So,

Magnitude of torque acting on m_1

$$\tau_1 = m_1 r_1^2 \alpha_1$$

For second mass

$$\tau_2 = m_2 r_2^2 \alpha_2$$

For nth mass

$$\tau_n = m_n r_n^2 \alpha_n$$

The total torque is given by

$$\tau_{total} = \tau_1 + \tau_2 + \_\_\_\_\_\_ + \tau_n$$
$$= (m_1 r_1^2 \alpha_1 + m_2 r_2^2 \alpha_2 + \_\_\_\_\_\_ m_n r_n^2 \alpha_n)$$

As the body is rigid, so all the masses rotate with same angular acceleration i.e.,

$$\alpha_1 = \alpha_2 = \_\_\_\_\_\_ = \alpha_n = \alpha \text{ (say)}$$

Thus, $\tau_{total} = (m_1 r_1^2 + m_2 r_2^2 + \_\_\_\_\_\_ m_n r_n^2)\alpha$

$$\tau_{total} = \left(\sum_{i=1}^{n} m_i r_i^2\right)\alpha \qquad \text{Or} \qquad \tau = I\alpha$$

Where 'I' is the moment of inertia and it can expressed as,

$$I = \left(\sum_{i=1}^{n} m_i r_i^2\right)$$

Thus, the sum of products of masses of the particles in a body and the squares of their respective perpendicular distance from the axis of rotation is called moment of inertia.

Unit

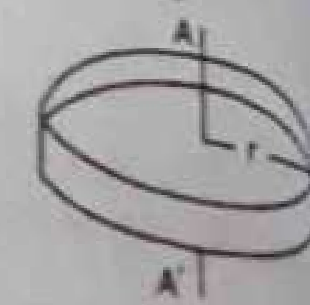
SI unit of moment of inertia is kgm^2 and dimension is $[ML^2]$.

(a)

Thin Rod

$I = \frac{1}{12} mL^2$

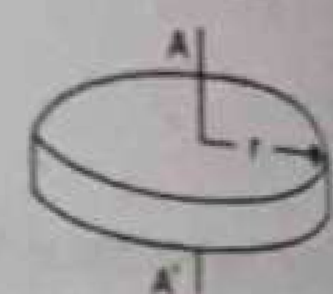
(b)



Thin ring or Hoop

$I = mR^2$

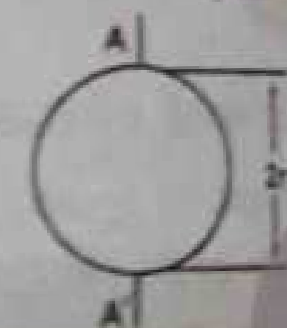
(c)



Solid disc or cylinder

$I = \frac{1}{2} mr^2$

(d)



Sphere

$I = \frac{2}{5} mr^2$

For Your Information

The sphere in (a) is rotating in the sense given by the arrow. Its angular velocity and angular momentum are taken to be upward along the rotational axis, as shown by the right-hand rule in (b).

Q.9 Define angular momentum? How can we relate it with moment of inertia?

Ans.

Angular Momentum (Moment of linear momentum)

The cross-product of position vector $\vec{r}$ with respect to axis of rotation and linear momentum $\vec{p}$ of a rotating particle is called angular momentum.

OR

A particle is said to have angular momentum about a reference axis if it moves in such a way that its angular position changes relative to that reference axis.

Explanation

Consider a body of mass m moving with velocity $\vec{v}$ and linear momentum $\vec{P}$ relative to origin O Then its angular momentum can be expressed as

$$\vec{L} = \vec{r} \times \vec{P}$$

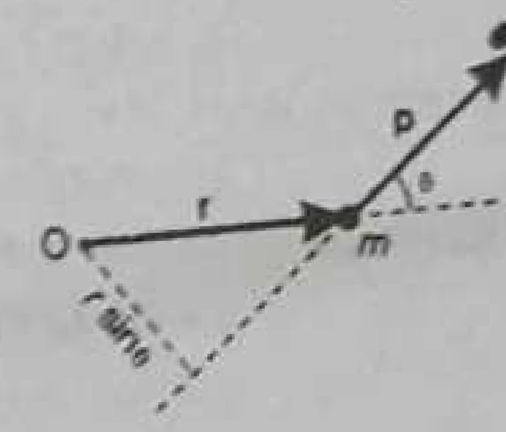


Fig. 5.10

Where $\vec{r}$ is the position vector at that instant with respect to origin.

The magnitude of $\vec{L}$ is given by

$$L = r\,p \sin\theta$$
$$= r(mv) \sin\theta$$
$$L = mrv\sin\theta \qquad \text{(since } p = mv)$$

Where θ is the angle between $\vec{r}$ and $\vec{P}$.

Direction

The direction of angular momentum is perpendicular to the plane containing $\vec{r}$ and $\vec{P}$. it can be determined by right hand rule.

Unit

SI unit of angular momentum is kgm^2/sec or J-sec and dimensions are $[ML^2T^{-1}]$.

$\boxed{\text{Show that } kgm^2/sec = Js}$

R.H.S $= J\ sec$

$= Nms \qquad (\because J = 1\ Nm)$

$= kg\frac{m}{s^2}.ms \qquad (\because 1\ Nm = 1\ kg\frac{m}{s^2})$

$= kg\frac{m}{s^2}$

R.H.S = L.H.S.

Angular momentum of a particle moving in a circle

If a particle is moving in a circle of radius 'r' with uniform angular velocity ω. The angle between r and tangential velocity v is 'θ' So,

Fig. 5.11

$$L = mrv\sin 90^\circ \ (\theta = 90^\circ)$$
$$L = mrv(1)$$
$$\boxed{L = mrv} \qquad [\text{As } v = r\omega]$$
$$L = mr(r\omega)$$
$$L = mr^2\omega \qquad [\text{But } mr^2 = I]$$
$$\boxed{L = I\omega}.$$

Another definition of angular momentum

The product of moment of inertia and angular velocity of a rotating body is called angular momentum.

Angular momentum of a rigid body

Consider a symmetrical rigid body rotating about a fixed axis through center of mass as shown in figure... particle of the rigid body rotates about the same axis in a circle with same angular velocity 'ω'. The an... momentum of a particle of mass m, particle,

$$L_1 = m_1 r_1^2 \omega$$

For second mass

$$L_2 = m_2 r_2^2 \omega$$

For nth mass

$$L_n = m_n r_n^2 \omega$$

The total angular momentum can be expressed as,

$$L = L_1 + L_2 + L_3 + \ldots\ldots\ldots + L_n$$

$$L = m_1 r_1^2 \omega + m_2 r_2^2 \omega \ldots\ldots\ldots + m_n r_n^2 \omega$$

$$L = (m_1 r_1^2 \omega + m_2 r_2^2 \ldots\ldots\ldots + m_n r_n^2)\omega$$

$$L = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega \qquad [\because I = \sum_{i=1}^{n} m_i r_i^2]$$

So, $L = I\,\omega$

Where I is moment of inertia of the rigid body about axis of rotation.

There are two types of angular momentum

Spin angular momentum

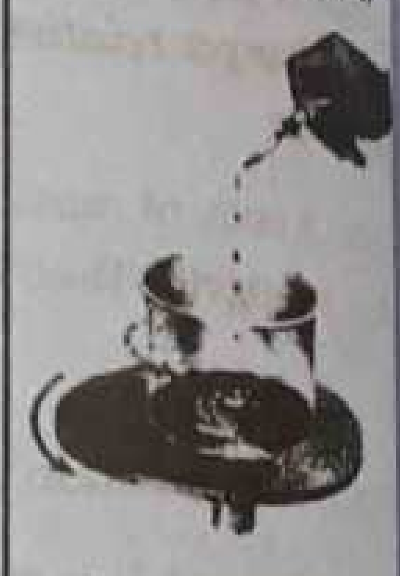
Angular momentum of a spinning body (i.e., rotating about its own axis) is called spin angular momentum (L_s)

Orbital angular momentum

Angular momentum of a body orbiting in a circular path is called orbital angular momentum (L_o)

So the total angular momentum of a body is equal to the sum of its spin and orbital angular momentum.

Point to Ponder



Why does the coasting ... system slow down as water ... into the beaker?

EXPLANATION: When water drips into the beaker, the mass of the contents in the beaker increases which increases the moment of inertia. Due to increase in moment of inertia, the angular velocity decreases according to the law of conservation of ... momentum.

Fig 5.12

Point object:
When the orbital radius is larger as compared to the size of the body, the body may be considered as a p... object.

Q.10 Define and explain the law of conservation of angular momentum?

Ans.

Law of Conservation of Angular Momentum

If no external torque acts on a system then the total angular momentum of the system remains constant.

$$L_{total} = L_1 + L_2 + \ldots\ldots\ldots\ldots = \text{constant}$$

Explanation

The effect of law of conservation of angular momentum is apparent if a single isolated spinning body changes its moment of inertia. **For example**, a diver pushes off the board with a small angular velocity. Upon lifting off from the board, the diver's legs and arms are fully extended. So the diver has a large moment of inertia 'I' about this axis. But when the diver's legs and arms are in closed tuck position, the moment of inertia reduces to I_2.

Fig. 5.13
A man diving from a diving board.

According to law of conservation of angular momentum.

$$L_1 = L_2$$

Or $\quad I_1\omega_1 = I_2\omega_2$

So diver must spin faster for small value of moment of inertia to conserve angular momentum.

This enables the diver to take extra somersaults. The direction of angular momentum is along the axis of rotation which remains fixed. *The axis of rotation of an object will not change its orientation unless an external torque causes it to do so.* Earth rotates about the sun experiences no sizeable external torque act on it, so Earth's axis of rotation remains fixed.

Q.11 Define rotational Kinetic Energy and show that $(K.E)_{rot} = \frac{1}{2} I\omega^2$. Also write down its practical use.

Ans.

Rotational Kinetic Energy

The energy possessed by the body due to its rotation about an axis is called rotational kinetic energy.

Expression for rotational K.E.

Suppose the body is spinning or rotating about an axis with constant velocity 'ω'. In order to find the total K.E of spinning body, we consider that it is composed of small pieces of masses $m_1, m_2 \ldots\ldots\ldots$

If mass 'm_1' has distance 'r_1' from axis of rotation.

K.E. of mass m_1 is given by

$$(K.E_1) = \frac{1}{2} m_1 v_1^2$$

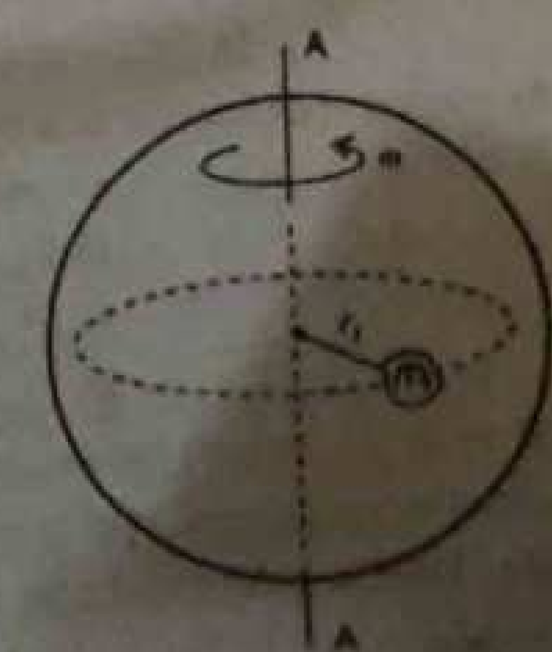


Fig. 5.14

By putting $v_1 = r_1\omega$, we get rotational K.E.

$$(K.E_1)_{rot} = \frac{1}{2} m_1 (r_1\omega)^2$$

$$(K.E_1)_{rot} = \frac{1}{2} m_1 r_1^2 \omega^2$$

Similarly $(K.E_2)_{rot} = ½\, m_2 r_2^2 \omega^2$

$$(K.E_n)_{rot} = ½\, m_n r_n^2 \omega$$

Hence, rotational K.E. of the whole body is

$$(K.E)_{rot} = (½\, m_1 r_1^2 \omega^2 + ½\, m_2 r_2^2 \omega^2 + \_\_\_\_ + ½ m_n r_n^2 \omega^2)$$

$$= ½\,(m_1 r_1^2 + m_2 r_2^2 + \_\_\_\_ + m_n r_n^2)\,\omega^2$$

$$= ½\left[\sum_{i=1}^{n} m_i r_i^2\right]\omega^2$$

$$(K.E)_{rot} = ½\, I\omega^2$$

Which is the rotational K.E. of the body.

Use of rotational K.E.

Practically, rotational K.E. is used by fly wheels, which are essential parts of many engines. *A fly wheel stores energy between the power strokes of the pistons, so that energy is uniformly distributed over the full revolution of the crank shaft and hence, the rotation remains smooth.*

Q.12 Find the rotational K.E. of the disc and hoop. Also derive the relations for the velocities of disc and hoop moving down an inclined plane.

Ans.

Rotational K.E. of a Disk

As we know

$$(K.E)_{rot} = ½\, I\omega^2$$

As moment of inertia of a disc is $I = ½\, mr^2$

So $(K.E)_{rot} = ½\,(½\, mr^2)\,\omega^2$

$$(K.E)_{rot} = ¼\, mr^2\, \omega^2$$

As we know $v = r\omega$

$$\boxed{(K.E)_{rot} = ¼\, mv^2}$$

Rotational K.E. of a Hoop:

As we know

$$(K.E)_{rot} = ½ I\omega^2$$

As moment of inertia of a hoop is

$$I = mr^2$$

So, $(K.E)_{rot} = ½\,(mr^2)\,\omega^2$

$$(K.E)_{rot} = ½\, mr^2\, \omega^2 \qquad [\because v = r\omega]$$

$$\boxed{(K.E)_{rot} = ½\, mv^2}$$

When both moves downward on an inclined plane from height h. Then it has both the rotational and translational motions. If no energy is lost in friction the total kinetic energy of the disc or hoop on reaching the bottom of the incline must be equal to the potential energy at the top.

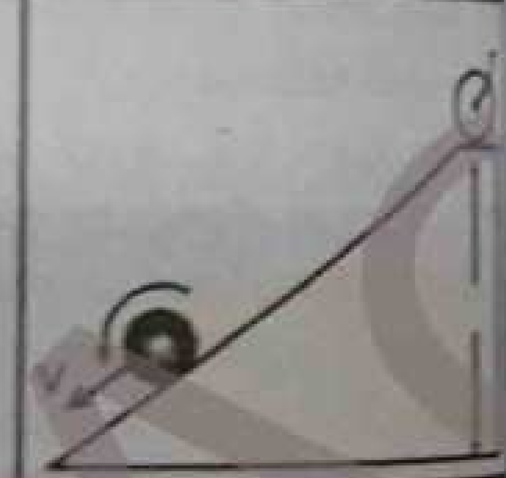
$$P.E = (K.E)_{trans} + (K.E)_{rot} \_\_\_\_\_\_ (1)$$

Tid bits

As the wheel rolls, it has bo[th] rotational and translational [motion]

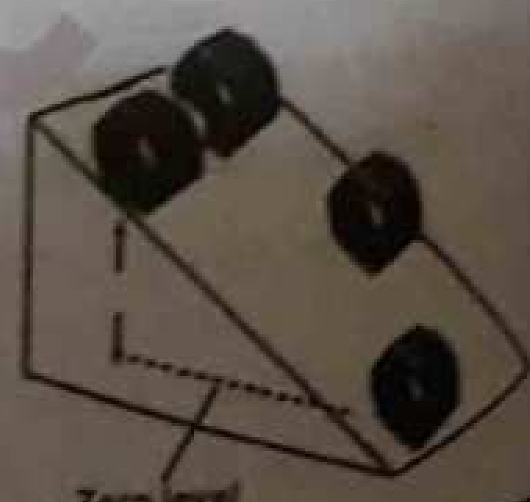
EXPLANATION:
$E_{Total} = K.E_{trans} + K.E_{rot}$

For Your Information



As the sphere rolls to the bottom of the incline, its gravitational potential energy is changed to kinetic energy of rotation and translation.

EXPLANATION
$P.E = K.E_{trans} + K.E_{rot}$



Do You Know?
Rolling motion is the combination of translational and rotational motion.

Speed of a Disc:

As for disc

$$(K.E)_{rot} = ¼ mv^2$$

Putting in equation . (1)

$$P.E = ½ mv^2 + ¼\, mv^2$$

$$mgh = (½ + ¼)\, mv^2$$

$$mgh = \left(\frac{2+1}{4}\right) mv^2$$

$$gh = \frac{3}{4} v^2$$

$$v^2 = \frac{4gh}{3}$$

$$\boxed{v = \sqrt{\frac{4gh}{3}}} \_\_\_\_\_\_ (2)$$

Speed of a Hoop:

As rotational K.E for disc is

$$(K.E)_{rot} = ½ mv^2$$

Putting in eq. (1)

$$P.E = ½\, mv^2 + ½\, mv^2$$

$$mgh = (½ + ½)\, mv^2$$

$$gh = v^2$$

$$\boxed{v = \sqrt{gh}} \_\_\_\_\_\_ (3)$$

Equation (2) and (3) shows that the velocity of the disc on reaching the bottom of the inclined plane is greater than hoop.

Q.13 What are artificial satellites? Find the expression for minimum velocity and period to put a satellite into the orbit?

Ans.

Artificial Satellite

Artificial satellites are the man made objects that orbit around the earth.

Satellite can be launched from earth's surface to circle the earth by mean of rocket. They are kept into their orbit by gravitational attraction of earth. The satellites which are near the earth have the acceleration $9.8 m/sec^2$. Other-wise they would fly off in the straight line tangent to earth.

Critical orbital velocity

The minimum velocity required to put a satellite into an orbit close to the earth is called critical velocity.

Expression

Consider a satellite of mass 'm' is moving with velocity v close to the earth in a circle of radius R.

The centripetal force acting on the satellite is

$$F = \frac{mv^2}{R} \_\_\_\_\_\_ (1)$$

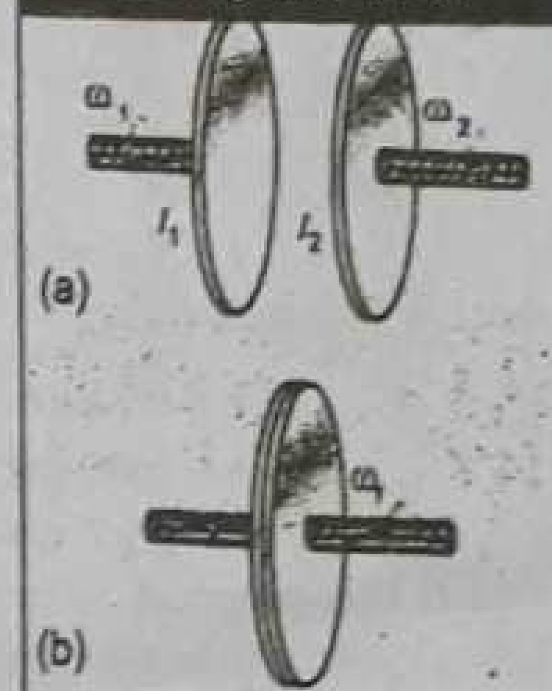
This force being provided by it weight. Hence

$$mg = \frac{mv^2}{R} \qquad [\because F = w]$$

Or

$$(\text{i.e., } R = 6.4 \times 10^6 \text{ m})$$

Intrusting Information



Rotational collision - the clutch

EXPLANATION:
The two discs with moment of inertia are spinning with initial angular velocities as shown in (Fig a). Rotational collision takes place between the discs and both of them combine as shown in (Fig b). As no torque is acting on the discs, therefore, by law of conservation of angular momentum, the combined discs rotate with angular velocity ω_f having total moment of inertia $(I_1 + I_2)$. Therefore,
$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$

Thus $g = \frac{v^2}{R}$ ______________(2)

Where v is the orbital velocity and R is the radius of earth

Thus from equation (2)

$$v^2 = gR$$

$$v = \sqrt{gR}$$

As

$$g = 9.8 m/sec^2, \qquad R = 6.4\times10^6 \, m$$

Putting values, we get

$$v = \sqrt{9.8\times6.4\times10^6}$$

$$v = 7900 \; m/sec$$

or $v = 7.9$ Km/sec

This is the minimum velocity necessary to move the satellite into orbit. This is called *critical velocity.*

Calculation of time period

The time period can be calculated as,

$$T = \frac{\text{circumference of the circle}}{\text{velocity of satellite}}$$

$$T = \frac{2\pi R}{v} \qquad (\text{As } t = \frac{S}{v})$$

As $\pi = 3.14$, $R = 6.4\times10^6$ m, $v = 7900$ m/sec

Putting values, we get

$$T = \frac{2(3.14)(6.4\times10^6)}{7900}$$

$$T = 5060 \; sec$$

$$T = \frac{5060}{60} = 84 \text{ min (approx)}$$

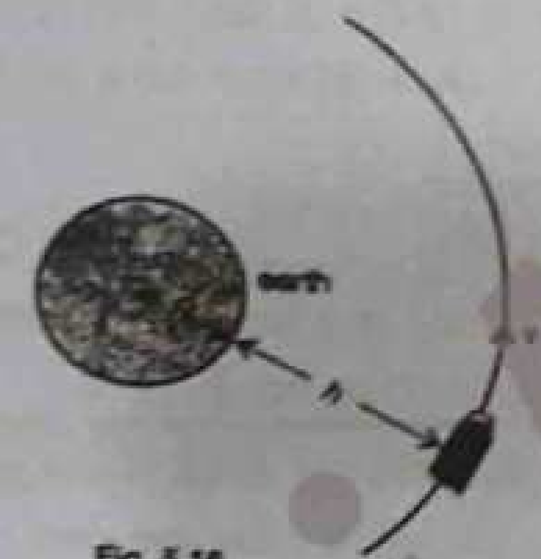


Fig. 5.16

Result

If the satellite moves at height 'h' from the surface of earth. Then the gravitational acceleration decrea inversely as the square of distance from center of earth. Thus, *higher the satellite, the slower will the requ speed and longer it will take to complete one revolution around the earth.*

Note

Closest orbiting satellites orbit the Earth at a height of about 400 km.

For Your Informa

Satellites Orbits

i. A satellite has escape ve of about 11kms^{-1} or 40,00
ii. A satellite with initial ve of about 8 kms^{-1} or 2700 moves in circular orbit.
iii. A satellite with velocity of 30,000 km/h in an elliptical orbit.
iv. Close orbiting satellite the Earth at a height of abo km
v. 24 close orbiting sat form the Global Positi System.

Q.14 What are real and apparent weight? Find the apparent weight in different cases for an ob suspended by a string and spring balance in an elevator?

Ans

Real and Apparent Weight

Real Weight

It is the gravitational pull of the earth on the object.

Similarly, the weight of an object on the surface of moon is the gravitational pull of the moon on the object.

Apparent Weight

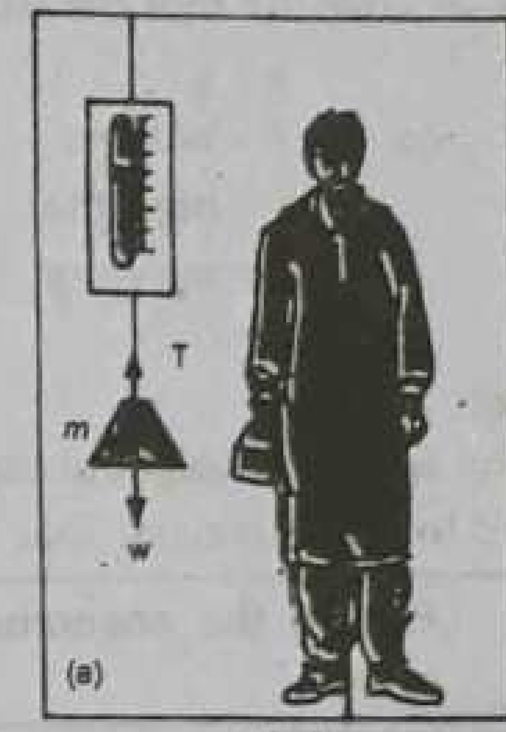
Generally weight of an object is measured by a spring balance.

The reading of the spring balance when the object is accelerating up or down is called apparent weight.

Apparent weight is equal and opposite to the force required to stop it from falling in the frame of reference.

Apparent weight of an object in a lift

Consider the apparent weight of an object of mass m suspended by a string and spring balance in a lift, as shown in figure. The tension 'T' in the string can be measured with the help of spring balance.



at rest
$a = 0$
$T = w$
Fig. 5.17(a)

CASE 1: When the lift is at rest or moving with uniform velocity

When the lift is at rest, Newton's second law tells us that the acceleration of object is zero. So the net force becomes zero. If 'W' is the gravitational force acting on the object and 'T' is the tension in the string.

Then , $T-W = ma$

$T-W = 0 \qquad$ As $a = 0$

$T = W$

or $T = mg$

Result:

Hence the apparent weight of an object is equal to the real weight for observer inside the lift.

acceleration downward
$w - T = ma$
$T = w - ma$
Fig. 5.17(b)

CASE II: When the lift is moving upward with acceleration 'a'.

When the lift is moving upwards with an acceleration a. So the upward force of tension T is greater than downward force of weight W then

Then the net firce acting on the body is

$T - W = ma$

OR $T = W + ma$

$T = mg + ma$

$T = m(g+a)$

Result:

Which shows that the apparent weight of object is increased by an amount of 'ma' than its actual weight.

CASE III: When the lift is moving downward with acceleration 'a'.

Consider the object is accelerating downward along the weight. So the weight 'W' is greater than the tension T.

So, $W - T = ma$

Or $-T = -W + ma$

$T = W - ma$

$T = mg - ma$

$T = m(g - a)$

Result:

Which shows that the apparent weight is less than the actual weight by a amount equal to ma.

Do you know?

Your apparent weight differs from your true weight when the velocity of the elevator changes at the start and end of a ride, not during the rest of the ride when that velocity is constant.

CASE IV: When a lift is falling freely under gravity:

Now we consider that the lift is falling freely under gravity. Then

$$a = g$$

As $T = W - ma$

$$T = mg - ma$$

$$T = mg - mg$$

$$T = 0$$

Result:

So, the apparent weight of object shown by the spring balance is zero. The object seems to be weightless. Thus it is state of weightlessness.

Q.15 Explain the phenomenon of weightless in satellites and gravity free system.

Ans.

Weightlessness in Satellite and Gravity Free System

When a satellite is falling freely in space under the action of force of attraction of earth, the sun or some distance star then the every thing within the satellite will be in state of weightlessness.

To show that the earth's satellite is a freely falling object, we consider the behavior of projectile thrown parallel to horizontal surface of the earth in the absence of air friction.

If the projectile is thrown successively at larger speeds, then during its free fall to the earth, the curvature of the path decreases with increasing horizontal speeds.

If the projectile is thrown fast enough parallel to the earth. The curvature of the path will match the curvature of the earth as shown in figure (i).

In this case projectile will start orbiting around the earth. The spaceship is accelerating towards the center of the earth at all times because it orbits around the earth. Its radial acceleration is simply 'g' the free fall acceleration. In fact the space-ship is falling towards the center of earth at all times, but the curvature of the earth prevents the spaceship from hitting the surface of earth. As spaceship is like a free fall object so all the objects inside it appears to be weightless, thus *No force is required to hold an object falling in the frame of reference of the space satellites. Such a system is called gravity free system.*

Q.16 What is orbital velocity and derive an expression for orbital velocity?

Ans.

Orbital Velocity

Orbital velocity is the tangential velocity to put satellite in orbit around the earth.

OR

The velocity of satellite with which it revolves round the earth is called orbital velocity.

The earth and some other planets revolve round the sun in nearly circular orbits. This type of motion is called orbital motion. Artificial satellites also revolve around the sun.

In 1984, at a height of 100 km above Hawaii island with a speed of 29000 kmh^{-1} Bruce McCandless stepped into space from a space shuttle and became the first human satellite of the Earth.

Expression for Orbital Velocity

Consider a satellite of mass m_s moving with orbital velocity v around the earth of mass M. If r is the radius of the orbit then centripetal force F can be expressed as

$$F_c = \frac{m_s v^2}{r} \quad \text{————} (1)$$

This force is provided by gravitational force of attraction between earth and satellite and is given by

$$F = G\frac{Mm_s}{r^2} \quad \text{————} (2)$$

Equating equation (1) and (2), we get

$$\frac{m_s v^2}{r} = G\frac{Mm_s}{r^2}$$

Or

$$v^2 = \frac{GM}{r}$$

Or

$$v = \sqrt{\frac{GM}{r}} \quad \text{————} (4)$$

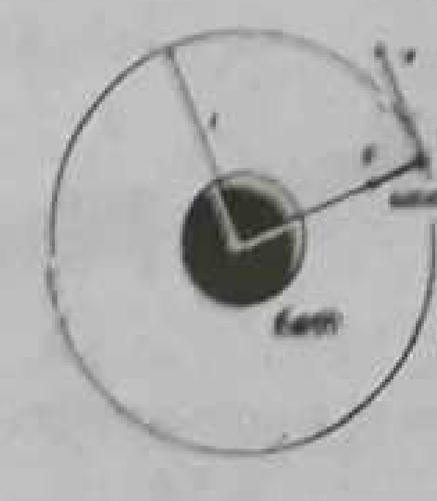


Fig. 5.19

Where $r = R + h$

r = Radius of the orbit

R = Radius of the earth

H = Height of the orbit above from the equator

Thus the mass of satellite is not important in describing the satellite orbit. So if the speed of satellite is less than the orbital speed then it will not be able to revolve around the earth and fall back to the earth.

Q.17 What is "Artificial Gravity"? Derive expression for frequency of spaceship required to provide the artificial gravity?

Ans.

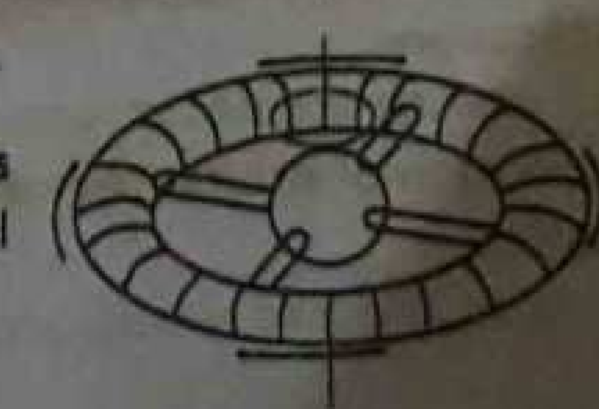
Artificial Gravity

Artificial gravity is the gravity like effect produced in an orbiting satellite by spinning it around its own axis.

Explanation:

There is no force on the space craft in a gravity free space. If the spaceship is to stay in the orbit for a longer time, this weightlessness creates a lot of problems for astronauts present in the spaceship. To overcome this difficulty, an artificial gravity is created in the spaceship to perform the experiments normally by astronauts. The astronaut then is pressed towards the outer rim and exerts a force on the floor of spaceship in much the same way as on the earth.

Expression for frequency:



Consider a space craft of ring shape, having 'R' as its outer radius. It rotates around its own central axis with angular speed 'ω'. Then its centripetal acceleration a_c is

$$a_c = \frac{v^2}{R} \qquad [\text{As } v = R\omega]$$

So

$$a_c = \frac{R^2\omega^2}{R}$$

Or $a_c = R\omega^2$

But $\omega = \frac{2\pi}{T}$, where 'T' is the period of revolution of spaceship.

So, $a_c = R\left(\frac{2\pi}{T}\right)^2$

$$a_c = R\frac{4\pi^2}{T^2}$$

$$a_c = 4\pi^2 \times R\left(\frac{1}{T^2}\right) \qquad \left[\text{As } \frac{1}{T} = f\right]$$

$$a_c = 4\pi^2 \times Rf^2$$

Or $f^2 = \frac{a_c}{4\pi^2 R}$

$$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

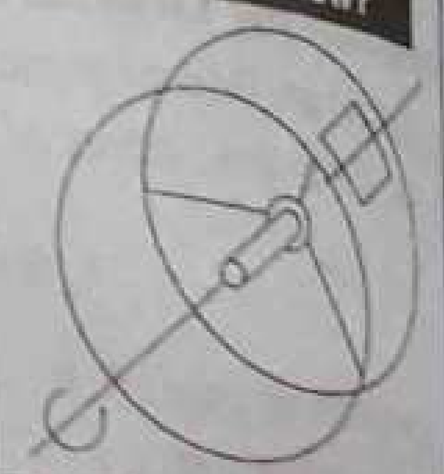
As force of gravity provides the necessary centripetal acceleration So,

$$a_c = g$$

Hence $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}}$

When the spaceship rotates with this frequency, then artificial gravity helps the astro nauts perform the work easily.

Do You Know?



The surface of the rotating space ship pushes on an object with which it is in contact and thereby provides the centripetal force needed to keep the object moving on a circular path.

EXPLANATION:
To Create artificial gravity on the space ship, the space ship is rotated with about its own axis. This centripetal force is acting on the astronaut. He exerts a force of reaction (centrifugal force) on the outer rim of the space ship and presses its floor in the same manner as we do on Earth.

Q.18 What are geo-stationary orbits and geo-stationary satellites? Find the orbital radius of geo-stationary satellites?

Ans

Geostationary Orbit

The orbit in which the period of rotation of satellite is exactly equal to the period of rotation of earth about its axis, is called geo-stationary orbit.

Geostationary Satellites

The satellite which completes its one revolution around earth in 24 hours is called geo-stationary satellite.

OR

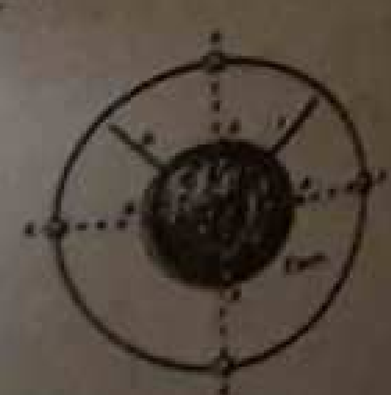
A satellite whose orbital motion is synchronized with the rotation of the earth about its own axis is called geo-stationary satellite.

This type of satellite is the one whose orbital motion becomes equal to the period of rotation of earth. So the satellite remains always over the same point on the equator as the earth spins on its axis.

Applications

Such satellites are used in communication system, weather observation and other military uses.

Expression of orbital radius of geo-stationary satellite:



As the orbital speed necessary for circular orbit is given by

$$v = \sqrt{\frac{GM}{r}} \quad \_\_\_\_\_\_ (1)$$

But this speed must be equal to average speed of satellite in one day. So,

$$v = \frac{S}{t}$$

Or $v = \frac{2\pi r}{T}$ ______ (2)

Where 'T' is period of revolution of satellite that is equal to one day. So the satellite also complete one rotation in exactly one day.

Equating equations (1) and (2), we get

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

Squaring, both sides, we get

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

Or $\boxed{r = \left[\frac{GMT^2}{4\pi^2}\right]^{1/3}}$

This equation gives the orbital radius of the geostationary satellite.

Substituting the values,

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$, $M = 6 \times 10^{24}$ Kg

$T = 1$ day $= 24$ hours $= 24 \times 60 \times 60$ s $= 86400$ s

So $r = \left[\frac{6.67\times10^{-11}\times6\times10^{24}\times(86400)^2}{4\times(3.14)^2}\right]^{1/3}$

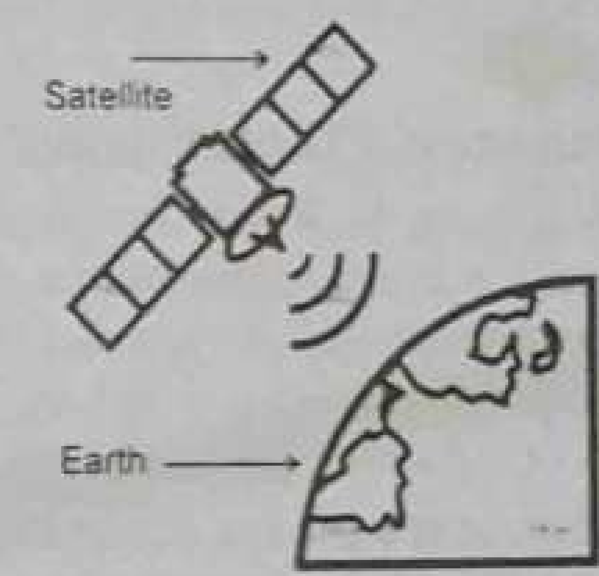
$$= 0.423 \times 10^8 \text{ m}$$

$$= 4.23 \times 10^7 \text{ m}$$

$$\boxed{r = 4.23 \times 10^4 \text{ km}}$$

Which is the orbital radius measured from center of the earth for geo-stationary satellite. A satellite at this height will always stay directly above a particular point on the surface of earth. This height above the equator comes out to be 36000 km.

For Your Information
A geostationary satellite orbits the Earth once per day over the equator so it appears to be stationary. It is used now for international communications.



Do You know?
1 GHz = 10^9 Hz

Q.19 Write a short note on communication satellites

Ans

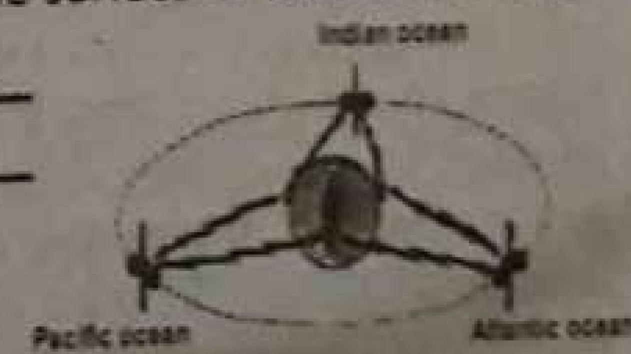
Communication Satellite

A communication satellite system can be set up by placing several geo-stationary satellites in orbit over different point on the surface of earth. Such a satellite covers **120°** of longitude. So the whole of the populated earth's surface can be covered by **three** correctly positioned satellites.

Use of waves for communication

Microwaves are used in communicational network because they travel in a narrow beam, in straight line and can pass easily through atmosphere of the earth.

Solar cells provide the energy to amplify and retransmit the signal. About 200 earth stations transmit and receive the signals via satellite from other countries.



The whole Earth can be covered by just three geo-stationary satellites.

Communications satellite INTELSAT VI

Satellite Organization (INTELSAT)

The largest satellite system, managed by 126 countries is called INTELSAT VI "INTELSAT" means international telecommunication, satellite organization.

It works at the microwave frequencies of **4, 6, 11 and 14 GHz** and has the capacity of 30,000 two way telephone circuit plus three T.V channels.

Q.20 What are the Newton's and Einstein views about gravitation?

Ans.

Newton's Theory about Gravitation

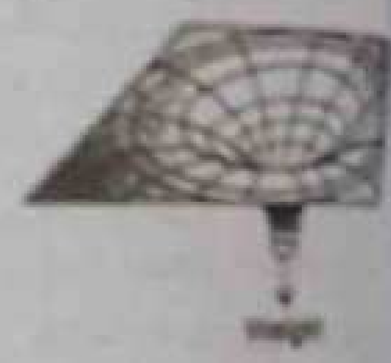
According to Newton's, *the gravitation is the intrinsic property of matter that every particle of matter attracts every other particle with a force that is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.*

In other words, *gravitation is due to the force between masses*

Einstein's theory about Gravitation:

According to Einstein's theory, *Gravity is due to the curvature of space and time caused by masses.*

To observe this we think the space as a thin rubber sheet. If a heavy weight is hung from it, it curves as shown in figure.



In Einstein theory we don't speak of force of gravity acting on bodies but we say that bodies and light rays move along geodesics (equivalent to the straight lines in plane geometry) in curved space time.

Differences between views

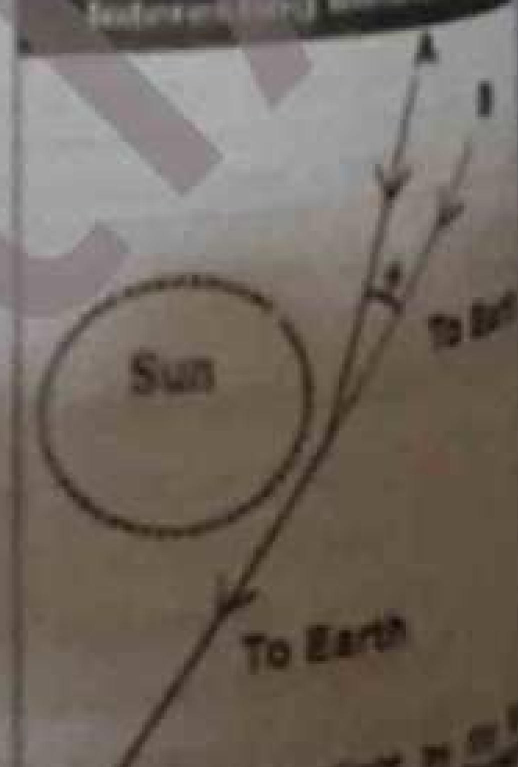
Einstein's theory gives a physical picture of how gravity works. Newton discovered inverse square law of gravity but give no explanation of why this should be so. However Einstein's theory tells us the reason of it (except in strong gravitational fields), but it fully explains the reason for obeying the inverse square law.

So Einstein's theory gives us that if gravity and acceleration are exactly equivalent, gravity must bend light by a definite amount that could be calculated. Newton's theory based on idea of light as a stream of tiny particles also suggested that light beam would be deflected by gravity. But in Einstein's theory the deflection of light is twice as great as it is according to Newton's theory.

Bending of starlight by the Sun:

When the bending of straight starlight caused by the gravity of the sun was measured during a solar eclipse in 1919. This measurement found to match Einstein's prediction rather than Newton.

Therefore, Einstein's theory was considered as scientific triumph.



INTERESTING INFORMATION:
Bending of starlight by the Sun. Light from the star A is deflected as it passes close to the Sun on its way to Earth. We see the star in the apparent direction B, shifted by angle Φ. Einstein predicted that Φ = 1.745 seconds of angle which was found to be the same during the solar eclipse on 1919

FORMULAE

1	Relation between S, r and θ	$S = r\theta$		$\theta = \frac{S}{r}$
2	Average angular velocity	$\vec{\omega}_{av} = \frac{\Delta\vec{\theta}}{\Delta t}$		
3	Instantaneous angular velocity	$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t}$		
4	Average angular acceleration	$\vec{\alpha}_{av} = \frac{\vec{\omega}_f - \vec{\omega}_i}{\Delta t}$		$\vec{\alpha}_{av} = \frac{\Delta\vec{\omega}}{\Delta t}$
5	Instantaneous angular acceleration	$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t}$		
6	Relation between v, r and ω	$v = r\omega$		$\vec{v} = \vec{\omega} \times \vec{r}$
7	Relation between a_t, r and α	$a_t = r\alpha$		$\vec{a}_t = \vec{\alpha} \times \vec{r}$
8	Equations of angular motion	$\omega_f = \omega_i + \alpha t$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$
9	Centripetal acceleration	$a_c = \frac{v^2}{r}$		
10	Centripetal force	$F_c = \frac{mv^2}{r}$		
11	Centripetal force (in angular measures)	$F_c = mr\omega^2$		
12	Moment of inertia of a particle	$I = mr^2$		
13	Torque on a particle rotating about a fixed point	$\tau = mr^2\alpha$		
14	Moment of inertia of a thin rod of length L	$I = \frac{1}{12}mL^2$		
15	Moment of inertia of a hoop	$I = mr^2$		
16	Moment of inertia of a disc	$I = \frac{1}{2}mr^2$		
17	Moment of inertia of a sphere	$I = \frac{2}{5}mr^2$		
18	2nd law of motion for rotational motion	$\tau = I\alpha$		
19	Angular momentum	$\vec{L} = \vec{r} \times \vec{P}$	$L = rp\sin\theta$	$L = mvr\sin\theta$
20	Angular momentum (in angular measures)	$L = mr^2\omega$		$L = I\omega$

21	Rotational K.E.	$K.E._{rot} = \frac{1}{2}I\omega^2$	$K.E._{rot} = \frac{1}{2}mr^2\omega^2$
22	Rotational K.E. of disc	$K.E._{rot} = \frac{1}{4}mr^2\omega^2$	$K.E._{rot} = \frac{1}{4}mv^2$
23	Rotational K.E. of hoop	$K.E._{rot} = \frac{1}{2}mr^2\omega^2$	$K.E._{rot} = \frac{1}{2}mv^2$
24	Velocity of hoop falling from an inclined plane of height h	$v = \sqrt{gh}$	
25	Velocity of disc falling from an inclined plane of height h	$v = \sqrt{\frac{4}{3}gh}$	
26	Velocity of sphere falling from an inclined plane of height h	$v = \sqrt{\frac{10}{7}gh}$	
27	Critical orbital velocity	$v = \sqrt{gR}$	$v = \sqrt{\frac{GM}{R}}$
28	Time period of close orbiting satellite	$T = \frac{2\pi R}{v}$	
29	Apparent weight of an object at rest or moving up with uniform velocity	$T = mg$	
30	Apparent weight of an object moving up with uniform acceleration a	$T = mg + ma$	
31	Apparent weight of an object moving down with uniform acceleration a	$T = mg - ma$	
32	Apparent weight of an object falling freely	$T = 0$	
33	Orbital velocity of a satellite	$v = \sqrt{\frac{GM}{r}}$	
34	Spinning frequency of a satellite about its own axis	$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$	
35	Orbital radius of a geo-stationary satellite	$r = \left(\frac{GmT^2}{4\pi^2}\right)^{\frac{1}{3}}$	

Multiple Choice Questions

Four possible answers to each statement are given below. Tick (✓) the correct answer:

1. Right hand rule for rotating bodies is used to find direction of
   (a) Angular velocity (b) Torque
   (c) Angular acceleration (d) All
2. The mud flies off the tyre of a fast moving car in the direction of:
   (a) Parallel to moving tyre (b) Antiparall to tyre
   (c) Tangent to moving tyre (d) None of these
3. The correct SI unit of angular momentum is:
   (a) $kgsm^{-2}$ (b) $kgms^{-1}$
   (c) kgm^2s^{-1} (d) kgm^2s^{-2}
4. A 30kg fly wheel is moving with uniform angular acceleration. If radius of flywheel is 2m, then its moment of inertia is:
   (a) $120kgm^2$ (b) $30kgm^2$
   (c) $2kgm^2$ (d) $60kgm^2$
5. Moment of inertia of 100kg sphere and having radius 5cm is:
   (a) $0.1kgm^2$ (b) $5kgm^2$
   (c) $500kgm^2$ (d) $2.5kgm^2$
6. The hoop and disc have same mass and radius their rotational K.E. are related by equation:
   (a) $K.E_{hoop} = K.E_{disc}$ (b) $K.E_{hoop} = 2K.E_{disc}$
   (c) $K.E_{hoop} = \frac{1}{2}K.E_{disc}$ (d) None
7. Critical velocity of an artificial satellite is:
   (a) $7.9miles\ s^{-1}$ (b) $7.9kms^{-1}$
   (c) $7.9kmh^{-1}$ (d) $790ms^{-1}$
8. In angular motion, Newton's 2nd law of motion is:
   (a) $F = ma$ (b) $F = \frac{\Delta P}{\Delta t}$
   (c) $\tau = I\alpha$ (d) All of above
9. A man of weight W is standing in an elevator which is ascending with uniform acceleration a then its apparent weight is
   (a) mg (b) mg − ma
   (c) mg + ma (d) ma − mg
10. If a body of mass 10kg is allowed to fall freely, its apparent weight becomes.
   (a) Zero (b) 89N
   (c) 9.8N (d) 10N

11. If external torque is zero, then which of these quantity is constant
(a) Angular momentum (b) Force
(c) Linear momentum (d) None of these

12. The acceleration due to gravity on moon is $\frac{1}{6}$ th of that on earth, what will be the mass of the bo... on moon, if its mass on earth is m?
(a) $\frac{m}{6}$ (b) 6m
(c) m (d) $\frac{m}{3}$

13. The value of angular momentum of a body is maximum if θ is equal to
(a) 0° (b) 45°
(c) 90° (d) 180°

14. The diver spins faster when moment of inertia becomes
(a) Smaller (b) greater
(c) Remains the same (d) None of these

15. When a body moves in a circle, then its linear and angular velocity are ________
(a) Parallel (b) Perpendicular
(c) Antiparallel (d) None

16. Torque per unit moment of inertia is equivalent to
(a) Angular velocity (b) Angular acceleration
(c) Inertia (d) Radius of gyration

17. The moment of linear momentum is called
(a) Torque (b) Couple
(c) Impulse (d) Angular momentum

18. The ratio of angular velocities of the hour hand and minute hand of a watch is ________
(a) 1 : 1 (b) 720 : 1
(c) 43200 : 1 (d) 1 : 12

19. If the earth shrinks to half of its radius without change in mass, the duration of the day will b... ________
(a) 6hrs (b) 12hrs
(c) 24hrs (d) 48hrs

20. The ratio of angular frequency and linear frequency is:
(a) 2π (b) π
(c) $\frac{1}{2\pi}$ (d) $\frac{\pi}{2}$

ANSWERS

1. d	2. c	3. c	4. d	5. a	6. b	7. b	8. c	9. c	10. a
11. a	12. c	13. c	14. a	15. b	16. b	17. d	18. d	19. a	20. a

Short Questions of Exercise

Q.5.1 Explain the difference between tangential velocity and the angular velocity, if one of these is given for a wheel of known radius, how will you find the other?

(Lhr 2010-2011)

Ans. Difference between tangential and angular velocities

- Angular velocity ω is directed along *axis of rotation* while tangential velocity v is *directed along tangent* to the circle.
- Unit of ω is rad/s while unit of v is m/s.

Determination

We can find the other velocity by using the following equation.

$$v = r\omega$$

Q.5.2 Explain what is meant by centripetal force and why it must be furnished to an object if the object is to follow a circular path?

(Rwp 2006, Grw 2006, Lhr 2009)

Ans. The force which bends the normally straight path into circular path is called centripetal force.

Or

The force which compels a body to move along a circular path is called centripetal force.

Mathematically, $F_c = \frac{mv^2}{r}$

Significance:

It is perpendicular to the tangential velocity and directed towards the center of the circular path. Without centripetal force body will *move along the tangent*.

Q.5.3 What is meant by moment of inertia? Explain its significance.

(Bwp 2006-2008, Mtn 2005, Lhr 2010-2011)

Ans. The moment of inertia of a particle is defined as the product of its mass and the square of its perpendicular distance from axis of rotation.

Mathematically, $I = mr^2$ (for a point mass)

Physical Significance:

The moment of inertia plays same role during angular motion which is played by mass during linear motion.

Mass is the measure of linear inertia while moment of inertia is the measure of rotational inertia of a body. Moment of inertia determine the *angular acceleration* while mass determine the linear acceleration.

Q.5.4 What is meant by angular momentum? Explain the law of conservation of angular momentum?

(Mir Pur 2004, Bwp 2006-2008, Mtn 2005)

Ans. The product of moment of inertia and angular velocity of a rotating body is called angular momentum.

It is denoted by $\vec{L}$. Mathematically

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L = rp \sin\theta$$

$$L = mvr \sin\theta$$

Law of conservation of angular momentum

The total angular momentum of the system remains constant, when no external torque acts on it.

Mathematically,

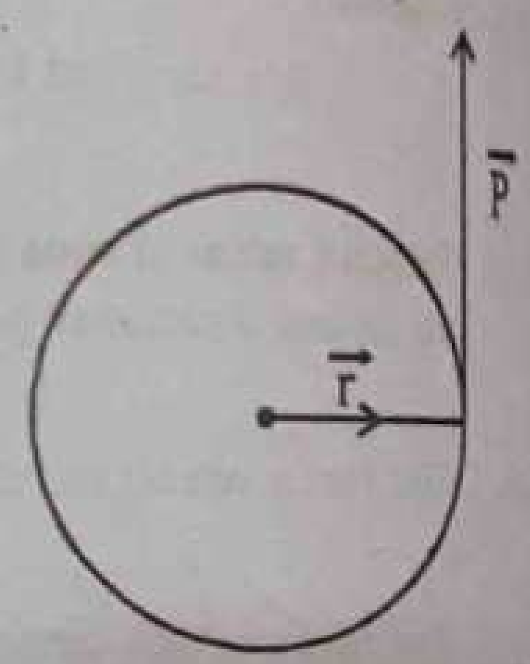
$$L_{total} = L_1 + L_2 + L_3 + ... = I\omega = \text{constant}$$

Q.5.5 Show that orbital angular momentum $L_o = mvr$

(Federal 2005, Mtn 2006, Fsd 2008, Grw 2008, Mir Pur 2009, Grw 2010)

Ans. Proof:

Let us consider particle of mass m moving in a circle of radius r as show in figure



As $\vec{L_o} = \vec{r} \times \vec{p}$

or $L_o = rp\sin\theta$

(where θ is the angle between r and p)

So $L_o = r(mv)\sin\theta$

Since $p = mv$

$L_o = mvr\sin\theta$

As the angle between r and v is 90°

Hence $L_o = mvr\sin 90°$

$= mvr\ (1)$

$L_o = mvr$ (hence proved)

Q.5.6 Describe what should be the minimum velocity, for a satellite, to orbit close to the earth around it.

(D.G.Khan 2005, Mtn 2009, Lhr 2010-2011)

Ans. Critical velocity:

The minimum velocity needed to orbit a satellite close to the earth is called critical velocity.

Calculation:

Consider a satellite of mass m is moving with velocity v in a circle of radius R (i.e radius of the Earth).

Since gravitational force provides the necessary centripetal force

So $mg = \frac{mv^2}{r}$

or $v^2 = gR$ r = R (radius of the Earth)

or $v = \sqrt{gR}$

$v = \sqrt{9.8 \times 6.4 \times 10^6}$

$v = 7900$ m/sec

or $v = 7.9$ km s^{-1}

Q.5.7 State the direction of the following vectors in simple situations; angular momentum and angular velocity.

(Grw 2009, Grw 2010)

Ans. Direction of angular momentum and angular velocity:

The direction of angular velocity and angular momentum is along the ***axis of rotation***, which can be determined by right hand rule.

Right hand rule

Grasp the axis of rotation in your right hand, curl the fingers along the direction of rotation then the erect thumb will represent the direction of angular quantity.

Q.5.8 Explain why an object, orbiting the earth, is said to be freely falling. Use your explanation to point out why objects appear weightless under certain circumstances.

(Fsd 2005)

Ans. Explanation:

When the object is thrown horizontally fast enough from a certain height, so that the curvature of its path will match with the curvature of the Earth then the object simply revolve round the Earth. Now, the motion of the object is under the constant acceleration due to gravity (equal to centripetal acceleration). Hence we can say the orbiting body is freely falling body.

Weightlessness of the body:

A freely falling body moves only under the action of gravitational force so that the object is said to be in state of weightlessness.

Q.5.9 When mud files off the tyre of a moving bicycle, in what direction does it fly? Explain it.

(Lhr 2005, Rwp 2005, Mtn 2005, Mir Pur 2006-2009, Bwp 2008, Fsd 2008, Grw2011)

Ans. The mud flies off along the *tangent to the tyre*.

Reason:

When speed of bicycle increases then adhesive force (sticking force) between the mud and the tyre is not sufficient to provide the necessary centripetal force so the mud leaves the tyre and moves along ***tangent*** to tyre.

Q.5.10 A disc and a hoop start moving down from the top of an inclined plane at the same time. Which one will be moving faster on reaching the bottom?

(Federal 2005-2006, Lhr 2006, Lhr 2009)

Ans. ***Disc*** will be moving faster on reaching the ground.

Proof:

Speed of hoop at the bottom of inclined plane is,

$$v_{hoop} = \sqrt{gh}$$

Speed of the disc moving down the inclined plane is,

$$v_{disc} = \sqrt{\frac{4}{3}gh}$$

or $$= \sqrt{\frac{4}{3}}\sqrt{gh}$$

$$v_{disc} = 1.15 v_{hoop}$$

Hence $$v_{disc} > v_{hoop}$$

Physical reason

Since the moment of inertia of disc is smaller than the moment of inertia of the hoop. So ang... velocity of disc is greater than hoop.

Q.5.11 Why does a diver change his body positions before and after diving in the pool?

(Federal 2005-2006, Sgd 2005, Bwp 2007, Grw 200...)

Ans. The diver changes his body position to make ***extra somersaults***.

Explanation

When a diver lifts off from the diving board, his legs and arms are full extended. In this case moment of inertia is large (I_1) but angular velocity (ω_1) is small.

When the legs and arms of the diver are drawn into the closed tuck position, its moment of inertia reduced considerably so that its angular velocity will increase to conserve the angular momentum.

$$I_1\omega_1 = I_2\omega_2$$

Q.5.12 A student holds two dumb-bells with stretched arms while sitting on a turn table. He is given a p... until he is rotating at certain angular velocity. The student then pulls the dumb-bells towards h... chest. what will be the effect on rate of rotation?

Fig. 5.25

Ans. Rate of rotation ***increases*** when student pulls the dumb-bells towards his chest.

Reason:

According to law of conservation of angular momentum.

$$I\omega = \text{constant}$$

When student holds two dumb-bells by stretching his arms, then value of angular inertia is increased. So the angular velocity is decreased. When the student pulls the dumb-bells towards his chest, the angular inertia decreases. So the angular velocity is increased. But during this whole process the total angular momentum remains constant.

Q.5.13 Explain how many minimum number of geo-stationary satellites are required for global coverage of T.V transmission.

(Fsd 2005, D.G.Khan 2005-2006, Mir Pur 2009, Mtn 2009)

Ans. Minimum three correctly positioned geo-stationary satellites are required for the global coverage of T.V transmission.

The whole Earth can be covered by just three geo-stationary satellites.

Explanation:

As each satellite in geo-stationary orbit covers ***120° of longitude*** so the whole populated Earth's surface can be covered by three correctly positioned geo-stationary satellites

Solved Exercises

Example 5.1

An electric fan rotating at 3 rev s^{-1} is switched off. It comes to rest in 18.0 s. Assuming deceleration to be uniform, find its value. How many revolutions did it turn before coming to rest?

Given Data:

Initial angular velocity = ω_i = 3.0 rev s^{-1}

Final angular velocity = ω_f = 0

Time = t = 18.0 sec.

To Find:

(i) Angular acceleration = α = ?

(ii) Number of revolution = Angular displacement = θ = ?

Calculation:

(i) As angular acceleration is

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

Putting values, we get

$$\alpha = \frac{0-3}{18}$$

$$\alpha = -\frac{3}{18}$$

$$\boxed{\alpha = -0.167 \text{ rev s}^{-2}}$$

(ii) As angular displacement is

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

Putting values, we get

$$\theta = 3 \times 18 + \frac{1}{2}(-0.167) \times (18)^2$$

$$\theta = 54 + (-0.083)(324)$$

$$\theta = 54 - 26.4$$

$$\theta = 27 \text{ rev (Approx)}$$

Example 5.2

A 1000 kg car is turning round a corner at 10 ms^{-1} as it travels along an arc of a circle. If the radius of the circular path is 10 m, how large a force must be exerted by the pavement on the tyres to hold the car in the circular path?

Given Data:

Mass of car = m = 1000 kg

Velocity of car = v = 10 ms^{-1}

Radius of circular path = r = 10 m

To Find:

Force required = F = ?

Calculation:

The force required to keep the car in circular path will be centripetal force,

So, $F_c = \frac{mv^2}{r}$

Putting values, we get

$$F_c = \frac{1000 \times (10)^2}{10}$$

$$F_c = \frac{1000 \times 100}{10}$$

$$F_c = 10000\ N$$

Or $\boxed{F_c = 1 \times 10^4 N}$

Example 5.3

A ball tied to the end of a string, is swung in a vertical circle of radius r under the action of gravity as shown in Fig. 5.7. What will be the tension in the string when the ball is at the point A of the path and its speed is v at this point?

Given Data:

As the ball is moving in a circle, thus the force acting on the ball must provide the required centripetal force.

At point A two forces are acting on the ball.

(i) Tension in the string T

(ii) Weight of the ball W

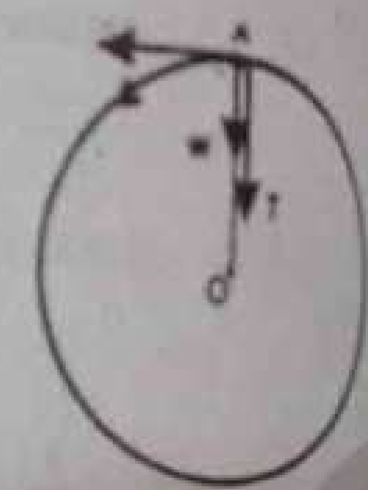


Fig 5.7

To Find:

Tension in the string = T = ?

Calculation:

These forces (i.e. T and W) act along the radius at Point A, so their vector sum must furnish the required centripetal force.

$$T + W = \frac{mv^2}{r}$$

or $$T = \frac{mv^2}{r} - W$$ But W = mg

$$T = \frac{mv^2}{r} - mg$$

$$T = m\left(\frac{v^2}{r} - g\right)$$

If $\frac{v^2}{r} = g$, then tension T will be zero and the centripetal force is just equal to the weight.

Example 5.4

The mass of Earth is 6.00×10^{24} kg. The distance r from Earth to the Sun is 1.50×10^{11} m. As seen from the direction of the North Star, the Earth revolves counter-clockwise around the Sun. Determine the orbital angular momentum of the Earth about the Sun, assuming that it traverses a circular orbit about the Sun once a year (3.16×10^7s)

Given Data:

Mass of earth = m = 6.00×10^{24} Kg

Distance = r = 1.50×10^{11} m

Time = T = 1 year = 3.16×10^7 s

To Find:

Orbital angular momentum of earth = L_o = ?

Calculation:

Using the formula

$$L_o = mv_o r \quad \text{.......... (1)}$$

Where v_o = orbital speed and $v_o = \frac{S}{t}$ (as $S = 2\pi r$)

When $S = 2\pi r$ = distance travelled in one year then t becomes the time period T.

Thus $$v_o = \frac{2\pi r}{T}$$

Putting values, we get

$$v_o = \frac{2 \times 3.14 \times 1.5 \times 10^{11}}{3.16 \times 10^7}$$

$$v_o = \frac{9.42}{3.16} \times 10^4$$

$$v_o = 2.981 \times 10^4\ ms^{-1}$$

Putting that value in equ. (1), we get

$$L_o = 6.0 \times 10^{24} \times 2.981 \times 10^4 \times 1.5 \times 10^{11}$$

$$L_o = 6 \times 2.981 \times 1.5 \times 10^{39}$$

$$L_o = 26.7 \times 10^{39}$$

$$\boxed{L_o = 2.67 \times 10^{40}\ kgm^2s^{-1}}$$

Example 5.5

A disc without slipping rolls down a hill of height 10.0 m. If the disc starts from rest at the top of the hill, what is its speed at the bottom?

Given Data:

Height of hill = h = 10 m

To Find:

Speed of disc at bottom = v = ?

Calculation:

Using the formula

$$v = \sqrt{\frac{4gh}{3}}$$

Putting values, we get

$$v = \sqrt{\frac{4 \times 9.8 \times 10}{3}}$$

$$v = \sqrt{130.67}$$

$$\boxed{v = 11.4\ ms^{-1}}$$

Example 5.6

An Earth satellite is in circular orbit at a distance of 384,000 km from the Earth's surface. What is its period of one revolution in days? Take mass of the Earth $M = 6.0 \times 10^{24}$ kg and its radius R = 6400km.

Given Data:

Height of satellite from earth = h = 384,000 km

Mass earth = $M = 6.0 \times 10^{24}$ kg

Radius of earth = R = 6400 km

To Find:

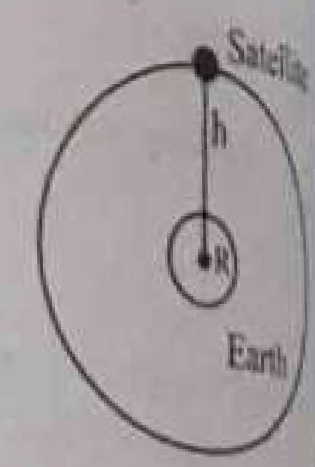
Period of one revolution in days = T = ?

Calculation:

Total distance of satellite from earth center = r = R + h

$$r = 6400 + 384000 = 390400 \text{ km}$$
$$r = 3.904 \times 10^8 \text{m}$$

As $$v = \sqrt{\frac{GM}{r}}$$



Putting values, we get

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{3.904 \times 10^8}}$$
$$v = \sqrt{10.25 \times 10^5}$$

Or $$v = \sqrt{102.5 \times 10^4}$$
$$v = 10.1 \times 10^2 \text{ ms}^{-1}$$

Now $$T = \frac{2\pi r}{v}$$

Putting value, we get

$$T = \frac{2 \times 3.14 \times 3.904 \times 10^8}{10.1 \times 10^2}$$
$$T = \frac{24.51}{10.1} \times 10^6 = 2.427 \times 10^6 \text{ seconds}$$
$$T = \frac{2.427 \times 10^6}{60 \times 60 \times 24} \text{ days}$$
$$\boxed{T = 28 \text{ days}}$$

Example 5.7

Radio and TV signals bounce from a synchronous satellite. This satellite circles the Earth once in 24 hours. So if the satellite circles eastward above the equator, it stays over the same spot on the earth because the Earth is rotating at the same rate. (a) What is the orbital radius for a synchronous satellite? (b) What is its speed?

Given Data:

Mass of earth = M = 6×10^{24}kg

Time period = T = 24 hours = 24 × 60 × 60 s

Gravitational constant = G = $6.67 \times 10^{-11}\text{Nm}^2\text{ kg}^{-2}$

To Find:

(a) Orbital radius of satellite = r = ?

(b) Speed of satellite = v = ?

Calculation:

(a) As $$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$$

Putting values, wet get

$$r = \left(\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times (60 \times 60 \times 2)^2}{4 \times (3.14)^2}\right)^{1/3}$$

on simplification, we get

$$\boxed{r = 4.23 \times 10^7 \text{m}}$$

(b) For orbital speed we know that

$$v = \frac{2\pi r}{T}$$

Putting values, we get

$$v = \frac{2 \times 3.14 \times 4.23 \times 10^7}{24 \times 60 \times 60}$$
$$v = \frac{26.56 \times 10^7}{864 \times 10^2}$$
$$v = 0.03074 \times 10^5$$

Or $v = 3.074 \times 10^3 \text{ ms}^{-1}$

Or $v = 3.074 \text{ kms}^{-1}$

Or $\boxed{v = 3.1 \text{ kms}^{-1}}$

Exercise Problems

5.1 A tiny laser beam is directed from the Earth to the Moon. If the beam is to have a diameter of 2.50 m at the Moon, how small must divergence angle be for the beam? The distance of Moon from the Earth is $3.8 \times 10^8 m$.

Given Data:

Diameter of beam = length of arc = S = 2.50 m

Distance of moon from the earth = r = $3.8 \times 10^8 m$

To find:

Divergence angle = θ = ?

Calculations:

As $S = r\theta$

Or $\theta = \frac{S}{r}$

Putting values, we get

$$\theta = \frac{2.50}{3.8 \times 10^8}$$
$$\boxed{\theta = 6.6 \times 10^{-9} \text{rad}}$$

5.2 A gramophone record turntable accelerates from rest to an angular velocity of 45.0 rev/min in 1.60s. What is its average angular acceleration?

Given Data:

Initial angular velocity = ω_i = 0

Final angular velocity = ω_f =45.0 rev/ min $= \frac{45\times2\pi}{60} = \frac{45\times2\times3.14}{60}$ = 4.71rad/sec

Time = t = 1.60sec

To find:

Average angular acceleration $= \alpha$ =?

Calculations:

As $\alpha = \frac{\omega_f - \omega_i}{t}$

putting values, we get

$$\alpha = \frac{4.7-0}{1.6}$$

$$\boxed{\alpha = 2.95 rad/sec^2}$$

5.3 A body of moment of inertia $I = 0.80$ kg m^2 about a fixed axis, rotates with a constant angular velocity of 100 rad/sec. Calculate its angular momentum L and the torque to sustain this motion.

Given Data:

Moment of inertia = $I = 0.80 kgm^2$

Angular velocity = ω=100 rad/ sec

To find:

Angular momentum = L =?

Torque = τ = ?

Calculations:

As L = Iω

putting values, we get

L = 0.80x100

L = 80 kgm²/sec

$\boxed{L = 80 \text{ Js}}$

Now torque = $\tau = I\alpha$

As angular velocity is constant, So $\alpha = 0$

Thus

$\tau = (0.80)(0)$

$\boxed{\tau = 0}$

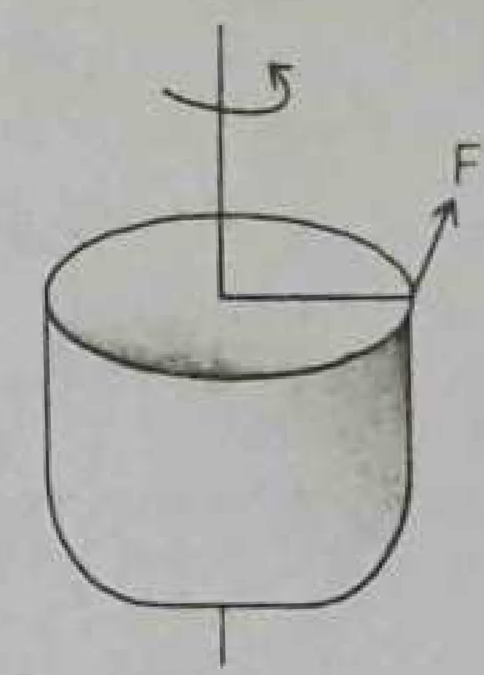
5.4: Consider the rotating cylinder shown in fig. 5.26
Suppose that m = 5.0kg, F = 0.60 N and r = 0.20 m.
Calculate (a) the torque acting on the cylinder, (b) the angular acceleration of the cylinder.

(Moment of inertia of cylinder $= \frac{1}{2}mr^2$)

Given Data:

Force acting on cylinder = F = 0.60 N

Radius = r = 0.20 m



To find:

(a) Torque acting on cylinder = τ = ?

(b) Angular acceleration of cylinder = α = ?

Calculations:

(a) As $\tau = rF\sin\theta$

As r and F are perpendicular to each other So $\theta = 90°$

$\tau = (0.20)(0.60)\sin 90°$

$\boxed{\tau = 0.12\text{Nm}}$

Now for α:

As $\tau = I\alpha$

$\tau = \frac{1}{2}mr^2\alpha$ As for cylinder, moment of inertia = $I = \frac{1}{2}mr^2$

Or

Or $\alpha = \frac{2\tau}{mr^2}$

Putting values, we get

$$\alpha = \frac{2\times0.12}{5\times0.20\times0.20}$$

$\boxed{\alpha = 1.2 \text{ rad/sec}^2}$

5.5 Calculate the angular momentum of a star of mass $2.0\times10^{30} kg$ and radius $7.0\times10^5 km$. If it makes one complete rotation about its axis once in 20 days, what is its kinetic energy?

Given Data:

Mass of star = m = $2\times10^{30} kg$

Radius of star = r = $7.0\times10^5 km = 7.0\times10^8 m$

Time for one rotation = T=20 days = $20\times24\times60\times60 = 1728\times10^3$ sec

To find:

Angular momentum =L= ?

Kinetic energy = K.E =?

Calculations:

As L = Iω

For sphere (star) = I = $\frac{2}{5}mr^2$ and $\omega = \frac{2\pi}{T}$

Thus

$$L = \frac{2}{5}mr^2 \times \frac{2\pi}{T}$$

putting values, we get

$$L = \frac{2}{5}\times2\times10^{30}\times(7\times10^8)^2\times\frac{2\times3.14}{1728\times10^3}$$

$L = 0.1424\times10^{43}$

$L = 1.4\times10^{42}$Js

$$\boxed{L = 1.4\times10^{42}\,Js}$$

Now

$$K.E = \frac{1}{2}I\omega^2$$

Or $$K.E = \frac{1}{2}\times\frac{2}{5}mr^2\times\left(\frac{2\pi}{T}\right)^2$$

Putting values, we get

$$K.E = \frac{1}{2}\times\frac{2}{5}\times2\times10^{30}\times(7\times10^{8})^2\times\left(\frac{2\times3.14}{1728\times10^{3}}\right)^2$$

$$K.E = 25.48\times10^{35}\,J$$

$$\boxed{K.E = 2.5\times10^{36}\,J}$$

5.6 A 1000 kg car travelling with a speed of 144 km h^{-1} rounds a curve of radius 100 m. Find the necessary centripetal force.

Given Data:

Mass of car = m = 1000 kg

Speed of car = v = 144 kmh^{-1} $\frac{144\times1000}{3600}$ = 40m/sec

Radius of curved path = r = 100 m

To find:

centripetal force = F_c = ?

Calculations:

As $$F_c = \frac{mv^2}{r}$$

Putting values, we get

$$F_c = \frac{1000(40)^2}{100}$$

$$F_c = 16000N$$

$$F_c = 1.6\times10^4 N$$

Or $$\boxed{F_c = 1.6\times10^4 N}$$

5.7 What is the least speed at which an aero plane can execute a vertical loop of 1.0 km radius so that there will be no tendency for the pilot to fall down at the highest point?

Given Data:

Radius of loop = r = 1.0 km = 1000 m

Acceleration due to gravity = g = 9.8 m/sec^2

To find:

Speed of aeroplane = v = ?

Calculations:

As $$a_c = \frac{v^2}{r}$$

As an aeroplane executes a circular loop then centripetal force is supplied by gravity.

Thus $$g = \frac{v^2}{r}$$

Or $$v^2 = rg$$

Or $$v = \sqrt{rg}$$

Putting values, we get

$$v = \sqrt{1000\times9.8}$$

$$\boxed{v = 99ms^{-1}}$$

5.8 The Moon orbits the Earth so that the same side always faces the Earth. Determine the ratio of its spin angular momentum (about its own axis) and its orbital angular momentum.(In this case, treat the Moon as a particle orbiting the Earth). Distance between the Earth and the Moon is $3.85\times10^8\,m$. Radius of the Moon is $1.74\times10^6\,m$

Given Data:

Distance between Earth and Moon = r_o = $3.85\times10^8\,m$

Radius of moon = $r_s = 1.74\times10^6$m

To find:

Ratio of spin and orbital angular momentum $= \frac{L_s}{L_o} = ?$

Calculation:

The spin angular momentum of the Moon about its own axis is

$$L_s = I_s\omega$$

$$L_s = \frac{2}{5}mr_s^2\omega ....(1)$$ As for Moon (sphere) $I_s = \frac{2}{5}mr^2$

The orbital angular momentum is given by

$$L_o = I_o\omega$$

$$L_o = mr_o^2\omega ....(2)$$ As for Moon (point mass) $I_o = mr^2$

Diving equation (1) by equation (2), we get

$$\frac{L_s}{L_o} = \frac{\frac{2}{5}mr_s^2\omega}{mr_o^2\omega}$$ Angular speed ω is same for both

$$\frac{L_s}{L_o} = \frac{2}{5}\frac{r_s^2}{r_o^2}$$

putting values, we get

$$\frac{L_s}{L_o} = \frac{2}{5}\frac{(1.74\times10^6)^2}{(3.85\times10^8)^2}$$

$$\frac{L_s}{L_o} = \frac{6.05\times10^{12}}{7.37\times10^{17}}$$

$$\boxed{\frac{L_s}{L_o} = 8.2\times10^{-6}}$$

5.9 The Earth rotates on its axis once a day. Suppose, by some process the Earth contracts so that its radius is only half as large as at present. How fast will it be rotating then?

(For sphere $I = 2/5MR^2$)

Given Data:

Time period = T_1 = 24 hours

Moment of inertia of sphere $= I_1 = \frac{2}{5}MR_1^2 = \frac{2}{5}MR^2$

Original radius = $R_1 = R$

Radius after contraction = $R_2 = \frac{1}{2}R$

Moment of inertia after contraction = $I_2 = \frac{2}{5}MR_2^2 = \frac{2}{5}M(\frac{R}{2})^2$

To find:

period of rotation after contraction = T_2 = ?

Calculations:

According to law of conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2$$

or

$$\frac{\omega_1}{\omega_2} = \frac{I_2}{I_1} \text{ .... (1)}$$

As $\omega_1 = \frac{2\pi}{T_1}$ and $\omega_2 = \frac{2\pi}{T_2}$

Putting values, in equ.(1)we get

$$\frac{\frac{2\pi}{T_1}}{\frac{2\pi}{T_2}} = \frac{\frac{2}{5}M(\frac{R}{2})^2}{\frac{2}{5}MR^2}$$

$$\frac{T_2}{T_1} = \frac{1}{4}$$

$$T_2 = \frac{T_1}{4}$$

or $T_2 = \frac{24}{4}$

$$\boxed{T_2 = 6 \text{ hours}}$$

Hence earth would complete its one rotation in 6 hours.

5.10 What should be the orbiting speed to launch a satellite in a circular orbit 900 km above the surface of the Earth? (Take mass of the Earth as 6.0×10^{24}kg and its radius is 6400 km).

Given Data:

Mass of earth = $M = 6.0 \times 10^{24}$ kg

Radius of earth = r = 6400 km

Height of circular orbit = h = 900 km

To find:

Orbital speed = v = ?

Calculations:

As $v = \sqrt{\frac{GM}{r}}$ ....(1)

Where r is the distance from the centre of earth.

$r = R + h$

Hence $r = 6400 + 900$

$r = 7300km$

$r = 7300 \times 10^3 m$

Putting values in equ. (1), we get

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{7300 \times 10^3}}$$

$$v = \sqrt{54.8 \times 10^6}$$

$$v = 7.4 \times 10^3 \text{m/sec}$$

Or $\boxed{v = 7.4 \text{km/sec}}$

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Chapter 6

FLUID DYNAMICS

Learning Objectives

Understand that viscous forces in a fluid cause a retarding force on an object moving through it.

Use Stoke's law to derive an expression for terminal velocity of a spherical body falling through a viscous fluid under laminar conditions.

Understand the terms steady flow, incompressible flow, non viscous flow as applied to the motion of an ideal fluid.

Appreciate that at a sufficiently high velocity, the flow of viscous fluid undergoes a transition from laminar to turbulence conditions.

Appreciate the equation of continuity Av = Constant for the flow of an ideal and incompressible fluid.

Appreciate that the equation of continuity is a form f principle of conservation of mass.

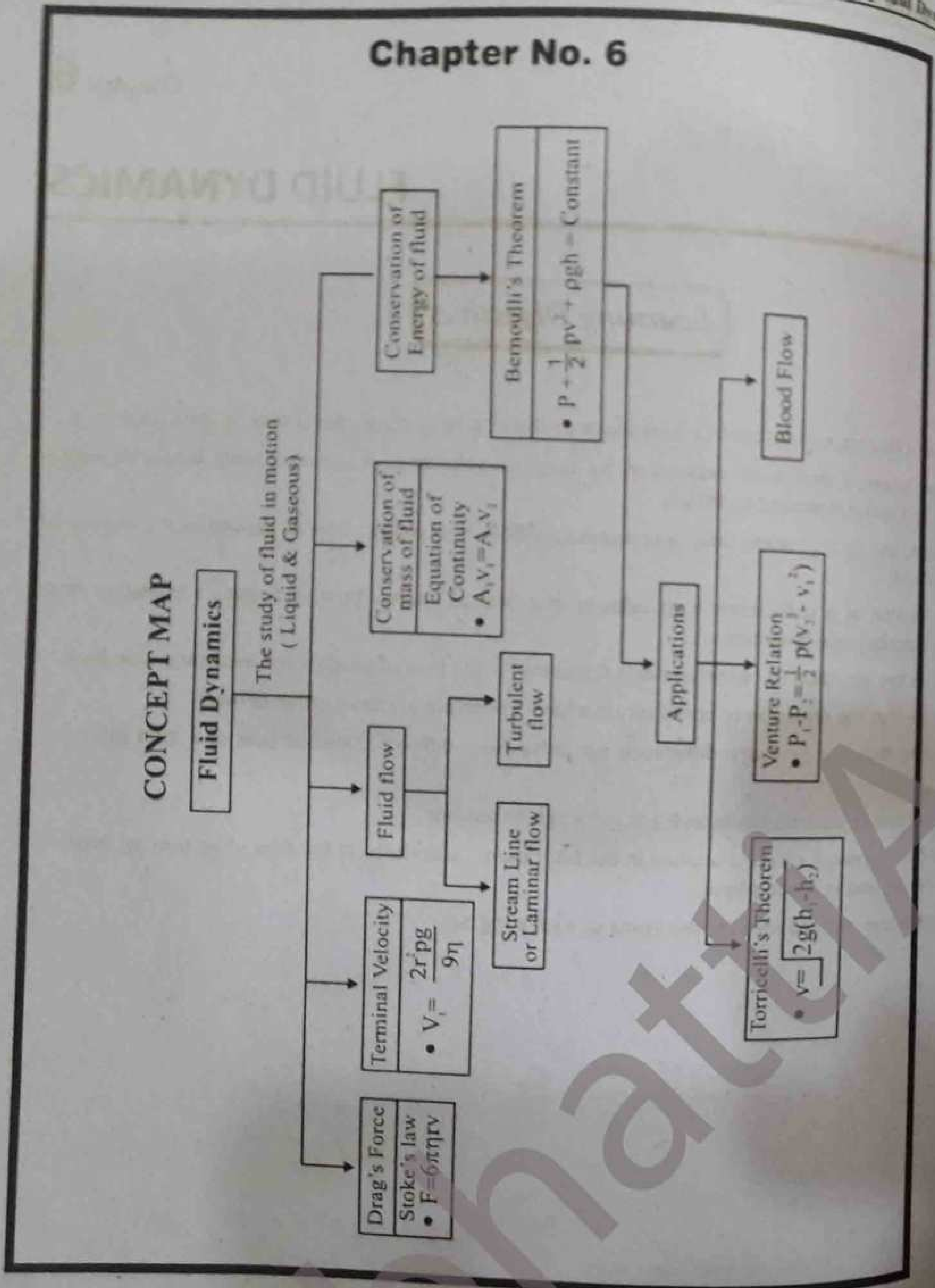
Understand that the pressure difference can arise from different rates of flow of a fluid (Bernoulli Effect).

Derive Bernoulli's equation in form $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

Explain how Bernoulli effect is applied in the filter pump, atomizers, in the flow of air over an aerofoil, Venturimeter and in blood physics.

Give qualitative explanations for the swing of a spinning ball.

Chapter No. 6



Fluid

Any substance that can ***flow*** from one place to another is called a fluid.

Examples

liquids and ***gases*** are classified as fluids.

Fluid dynamics

The branch of physics which deals with the study of ***fluids in motion*** is called fluid dynamics.

Conservation law in fluid dynamics

The study of dynamic fluid is relatively complicated, but analysis can be simplified by making a few assumptions. For this purpose we also use the following conservation laws.

- Law of conservation of ***mass*** which gives basis of the equation of continuity
- Law of conservation of ***energy*** gives the basis of Bernoulli's theorem.

Q.1 What is meant by the term viscosity? Explain. and fluid friction or drag force and state Stock's Law?

Ans.

Viscosity (η)

Viscosity: The frictional effect between different layers of a flowing fluid is described in terms of viscosity of the fluid. Viscosity measures, how much force is required to slide one layer of the liquid over another layer. It is denoted by η. The SI unit of viscosity is Ns/m^2.

Examples

- Substances like ***honey*** and thick ***tar*** have large coefficient of viscosity. So they can not flow easily.
- Substance like ***water and milk*** has small coefficient of viscosity η. So they can flow easily.

Unit

- The SI unit of coefficient of viscosity is ***$kgm^{-1}s^{-1}$ or $Nm^{-2}s$***
- Its dimension is $[ML^{-1}T^{-1}]$.

Viscosity of liquids and gases

- Liquids and gases have ***non-zero*** viscosity.
- Viscosity of gases ***increases*** with increase in temperature while for solids and liquids it deceases.

Q.2 What is meant by drag force?

Ans.

Drag Force

An object moving through a fluid experiences a retarding force called drag force.

OR

~~The force which **resists** the motion of a body moving through the fluid is called drag force or fluid friction.~~

For example

When we stick out our hand out of the window of a fast moving car, we feel a force opposite to our motion.

Factors on which depends upon

According to Stoke's law, drag force is given by,

$$F = 6\pi\eta rv$$

So drag force is a velocity dependant force. This equation shows that drag force depends upon

i) ***speed*** of sphere (v)
ii) ***size*** of sphere (r)
iii) ***viscosity*** of medium (η)

Q.3 State Stoke's Law. What are its limitations?

Ans.

Stoke's Law

This law states that the drag force F acting on a sphere of radius r moving slowly with velocity v in a fluid of viscosity η is given by

$$\boxed{F = 6\pi\eta r v}$$

Limitation

This law is valid only for spherical bodies moving slowly. For high speeds drag force is not simply proportional to velocity.

Q.4 What is terminal velocity? Show that terminal velocity of fog droplet is directly proportional to the square its radius?

Ans.

Terminal Velocity

The maximum and constant velocity of an object falling vertically downward is called terminal velocity.

Explanation

Consider a fog droplet falling vertically downward. The drag force of air increases as the velocity droplet increases. The net force on the droplet is,

Net force = weight – drag force

$$F = mg - 6\pi\eta rv$$

$$ma = mg - 6\pi\eta rv$$

When drag force becomes equal to the weight of droplet then it will start to move with uniform velocity, called terminal velocity (v_t). So its acceleration becomes zero. (i.e. a = o)

Thus the above equation becomes

$$m(0) = mg - 6\pi\eta rv_t$$

$$0 = mg - 6\pi\eta rv_t$$

OR $6\pi\eta r v_t = mg$

OR $$\boxed{v_t = \frac{mg}{6\pi\eta r}} \qquad (1)$$

Where v_t = the terminal velocity and

η = coefficient of viscosity.

Relation between terminal velocity and radius of droplet

Since $\text{density} = \dfrac{\text{mass}}{\text{volume}}$

OR $\rho = \dfrac{m}{V}$

OR $m = \rho V$

DO YOU KNOW

Drag force along acts opposite to the direction of motion of object

For Your Information

Viscosities of Liquids and Gases at 30°C

Material	Viscosity 10^{-3} (Nsm^{-2})
Air	0.019
Acetone	0.295
Methanol	0.510
Benzene	0.564
Water	0.801
Ethanol	1.00
Plasma	1.6
Glycerin	6.29

Can You Do That?

A table tennis ball can be made suspended in the stream of air coming from

EXPLANATION:

The air is ejected from the nozzle with high velocity, therefore, according to Bernoulli's equation, the pressure at the nozzle is decreased. The tennis ball is suspended due to greater pressure around it.

OR $$m = \left(\frac{4}{3}\pi r^3\right)\rho \qquad \left[\because V = \frac{4}{3}\pi r^3 \text{ for sphere}\right]$$

Hence equation (1) becomes,

$$v_t = \frac{\left(\frac{4}{3}\pi r^3\right)\rho g}{6\pi\eta r}$$

OR $$v_t = \frac{(4\pi r^3)\rho g}{3\times 6\pi\eta r}$$

$$\boxed{v_t = \frac{2gr^2\rho}{9\eta}} \qquad (2)$$

OR $$v_t = \left(\frac{2g\rho}{9\eta}\right)r^2 \qquad \left[\because \frac{2g\rho}{9\eta} = \text{constant}\right]$$

OR $v_t \propto r^2$

Thus terminal velocity is directly proportional to the ***square*** of the radius of droplet.

Q.5 What is the difference between steady and turbulent flow?

Ans.

Fluid Flow

Let us consider the flow of the fluid through the pipes. It may be either streamline or turbulent.

Laminar Flow

In laminar or stream line flow each particle along a stream line moves exactly along the same path as followed by the particle which passes through that point earlier.

The flow of a fluid in which every particle of the fluid moves along a smooth path is called laminar flow.

Turbulent Flow

The irregular or unsteady flow of the fluid is called turbulent flow.

When the speed of flowing fluid exceeds a certain ***critical*** value, the flow becomes extremely irregular and complex and it changes continuously with time.

Thus,

According to figure,

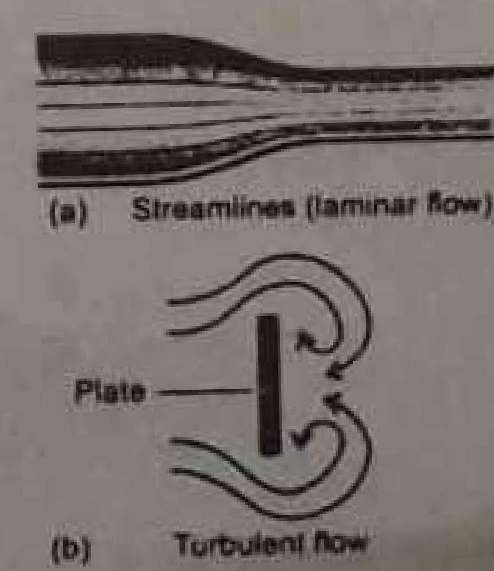


Fig. 6.1

Difference between Laminar Flow and Turbulent *Flow*

- In laminar flow, each particle of fluid moves along a ***smooth path*** which does not change with time.
- In turbulent flow the ***flow pattern is not smooth*** but continuously changes with time.
- In laminar flow, stream lines ***do not*** cross each other while in turbulent flow it does not happen.
- In turbulent flow, the velocity of fluid changes ***abruptly.***

Note (Steady flow condition)

For steady flow, different streamlines can ***never*** intersect each other. This condition is called steady flow condition.

Q.6 What is an ideal fluid?

Ans.

Ideal Fluid

A fluid which satisfies the following condition is called an ideal fluid

1. The fluid is ***non-viscous*** i.e., there is no internal frictional force between adjacent layers of fluid.
2. The fluid is ***incompressible***, i.e., its density is constant.
3. The fluid motion is ***steady***.
4. ***Irrotational*** flow.

Q.7 State and explain equation of continuity.

Ans.

For Your Information

Equation of Continuity: "The product of cross sectional area of the pipe and the fluid speed at any point along the pipe is a constant. This constant equals the volume flow per second of the fluid or simply flow rate."

Mathematical form:

$$A_1V_1 = A_2V_2$$

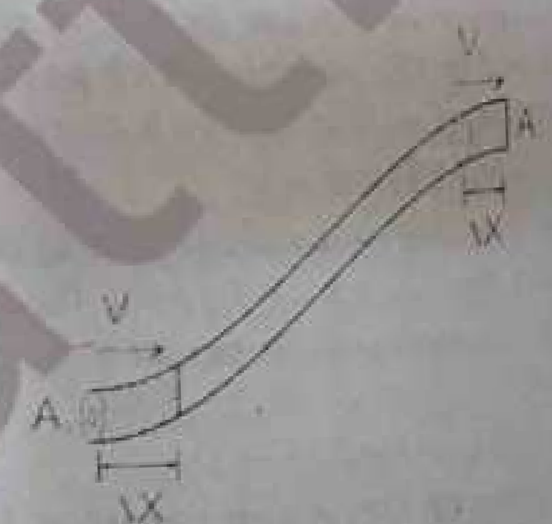
or

$$AV = \text{Constant} = \text{Flow rate}$$

$$AV = \frac{\text{Volume}}{\text{time}}$$

Explanation

Consider, a fluid is flowing through the non-uniform pipe. The particles in the fluid move along the streamlines in a steady state flow as shown in figure.



At lower end of pipe

Let

area of cross-section of pipe = A_1

distance moved by fluid in time $\Delta t = \Delta x_1$

velocity of fluid = v_1

density of fluid = ρ_1

volume of the fluid contained at lower end = V_1

mass of the fluid contained at lower end = Δm_1

Then

$$\Delta m_1 = \rho_1 V_1$$

OR $\Delta m_1 = \rho_1 A_1 \Delta x_1$ $[\because V_1 = A_1 \Delta x_1]$

OR $\boxed{\Delta m_1 = \rho_1 A_1 v_1 \Delta t}$ $[\because \Delta x_1 = v_1 \Delta t]$

At upper end of pipe

Let

area of cross-section of pipe = A_2

distance moved by fluid in time $\Delta t = \Delta x_2$

velocity of fluid = v_2

density of fluid = ρ_2

volume of the fluid contained at upper end = V_2

mass of the fluid contained at lower end = Δm_2

Then

$$\boxed{\Delta m_2 = \rho_2 A_2 v_2 \Delta t}$$

If the fluid is incompressible and flow is steady, the mass of fluid is conserved. i.e.

The mass that flows into the bottom of the pipe through A_1 in time Δt must be equal to mass of the liquid that flows out through A_2 in the same time. Therefore,

$$\Delta m_1 = \Delta m_2$$

$$\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$$

OR $\boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$

This equation is called the equation of continuity.

Since the fluid incompressible, so the density is constant i.e. $\rho_1 = \rho_2 = \rho$ (say).

Thus, the equation of continuity becomes

$$\rho A_1 v_1 = \rho A_2 v_2$$

OR $A_1 v_1 = A_2 v_2$

OR $\boxed{Av = \text{constant}}$

The product Av is called flow rate.

Tid - Bits

As the water falls, its speed increases and so its cross sectional area decreases as mandated by the continuity equation

EXPLANATION:

Continuity equation is $A_1v_1 = A_2v_2$

Or $Av = \text{constant}$

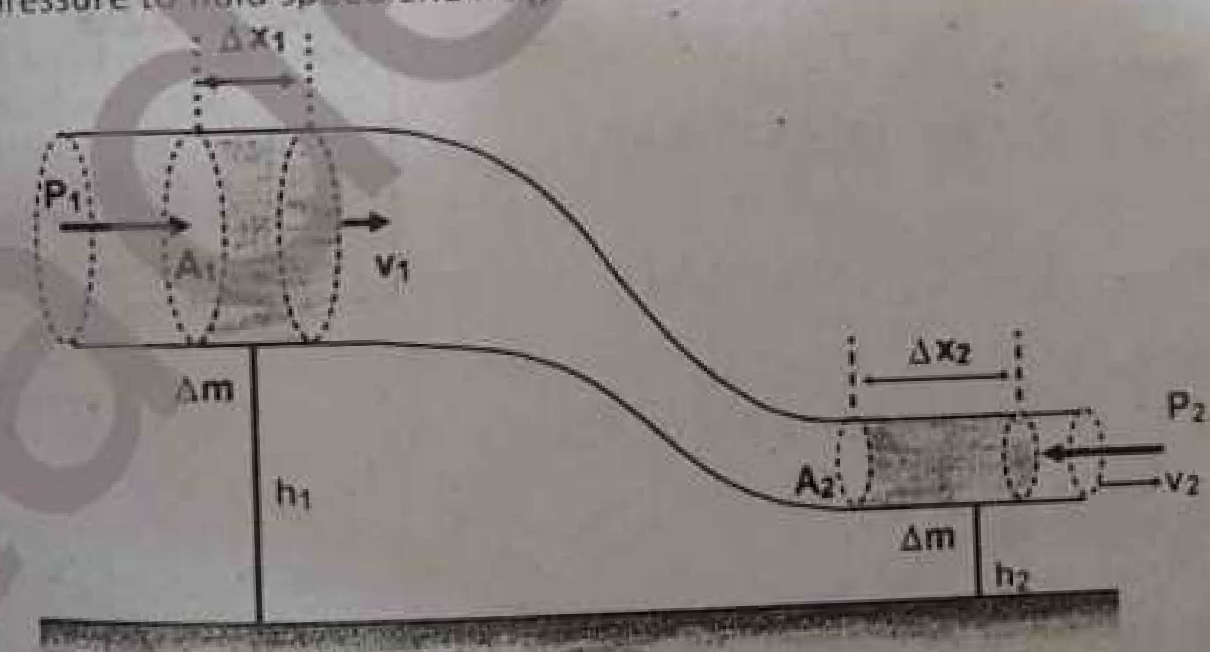
$$A = \frac{\text{const}}{v} \Rightarrow A \propto \frac{1}{v}$$

Q.8 State and explain Bernoulli's Equation.

Ans.

Bernoulli's Equation

Bernoulli's equation is the ***fundamental*** equation in fluid dynamics which relates the pressure to fluid speed and height.



Statement

The sum of pressure, K.E. per unit volume and P.E. per unit volume of an incompressible, non-viscous fluid flowing in steady state is constant at each point along a stream line.

Mathematically

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

Proof

Let us consider the steady flow of an incompressible, non-viscous fluid through a pipe in time Δt, as shown in figure below.

Work done at upper end

Let, at upper end

Area of cross section of pipe = A_1
The pressure of fluid = P_1
Force exerted on the fluid = F_1
Distance moved by fluid in time $\Delta t = \Delta x_1$
Velocity of fluid = v_1
Density of fluid = ρ
Volume of the fluid contained at upper end = V
The work done on the fluid by the fluid **behind** it is given by

$$W_1 = F_1 \Delta x_1$$

OR $W_1 = P_1 A_1 \Delta x_1 \qquad [\because P_1 = \frac{F_1}{A_1} \Rightarrow F_1 = P_1A_1]$

OR $W_1 = P_1 A_1 v_1 \Delta t$ ______ (1) $[\because \Delta x_1 = v_1\Delta t]$

Work done at lower end

Let at lower end

Area of cross section of pipe = A_2
The pressure of fluid = P_2
Force exerted on the fluid = F_2
Distance moved by fluid in time $\Delta t = \Delta x_2$
Velocity of fluid = v_2
Density of fluid = ρ
Volume of the fluid contained at lower end = V

Similarly, the work done on the fluid by the fluid **ahead** of it is given by

$$W_2 = -F_2 \Delta x_2$$

$W_2 = -P_2 A_2 \Delta x_2 \qquad [\because P_2 = \frac{F_2}{A_2} \Rightarrow F_2 = P_2A_2]$

$W_2 = -P_2 A_2 v_2 \Delta t$ ______ (2) $[\because \Delta x_2 = v_2\Delta t]$

The negative sign shows that this work is done **against** the fluid force.

Net work done

The net work done both at upper and lower end is

$$W = W_1 + W_2$$
$$W = P_1A_1\Delta x_1 \Delta t - P_2A_2\Delta x_2 \Delta t$$
$W = P_1A_1v_1\Delta t - P_2A_2v_2\Delta t$ ______ (3)

According to equation of continuity

$$A_1v_1 = A_2v_2$$

OR $A_1v_1\Delta t = A_2v_2\Delta t = V$

Hence, equation (3) becomes

$$W = P_1V - P_2V$$

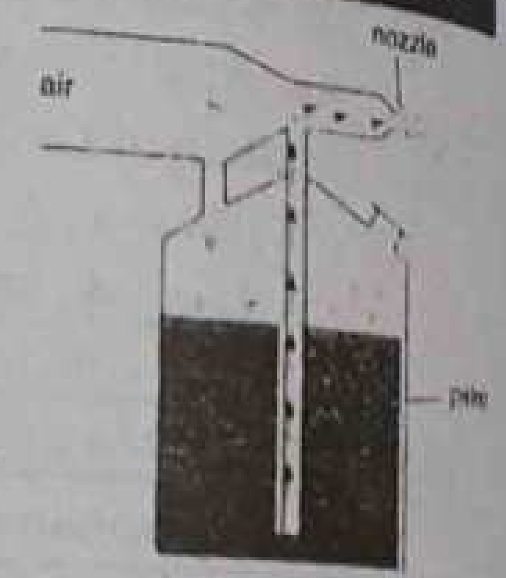
OR $W = (P_1 - P_2)V$ ______ (4)

$W = (P_1 - P_2)\frac{m}{\rho}$ ______ (5) $[\because \rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}]$

Change in K.E and change in P.E

A part of this workdone changes K.E. of fluid and a part changes its gravitational P.E. So,

Interesting Information



A stream of air passing over a tube dipped in a liquid will cause the liquid to rise in the tube as shown. This effect is used in perfume bottles and paint sprayers.

EXPLANATION:
Due to greater speed of air in narrow part of the tube, pressure is decreased. Therefore, the liquid in the tube rises up in the tube as shown in the figure.

Change in K.E. = Δ(K.E) $= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

Changing in P.E.= Δ(P.E) $= mgh_2 - mgh_1$

Where h_2 and h_1 are the heights of the upper and lower end respectively.

By law of conservation of energy

Work done = change in K.E. + change in P.E.

$$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$$

Multiplying both sides by $\frac{\rho}{m}$, we get,

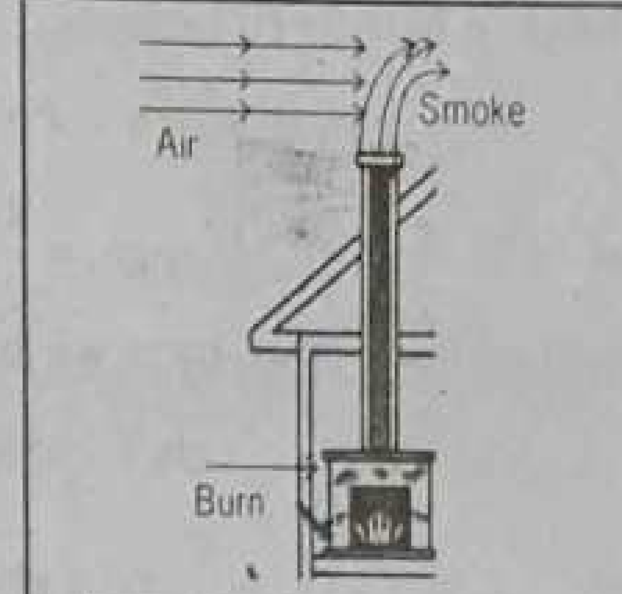
$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho gh_2 - \rho gh_1$$

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2}$$

This is the Bernoulli's equation and it can also be expressed as

$\boxed{P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}}$ ______ (6)

Do You Know?



A chimney works best when it is tall and exposed to air currents, which reduces the pressure at the top and force the upward flow of smoke.

In the upper region atmospheric pressure is small and the smoke in the tall chimney rises upward.

Q.9 State and explain Torricelli's Theorem.

Ans.

Application of Bernoulli's Equation

Torricelli's Theorem

Torricelli's Theorem is Bernoulli's equation with certain assumption made.

Statement

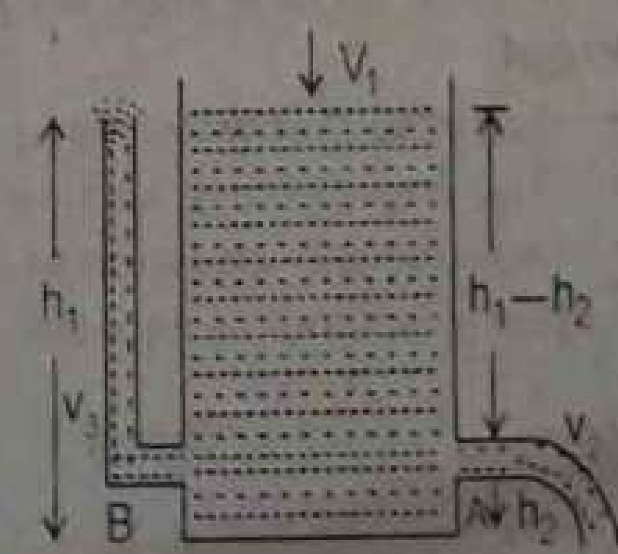
The speed of efflux is equal to the velocity gained by the fluid in falling through the distance $(h_1 - h_2)$ under the action of gravity.

Proof

Consider a large tank of fluid having small orifice (hole) A on it, as shown in figure.

Let

area of cross-section of upper end of tank = A_1
area of cross-section of lower small hole = A_2
speed of the fluid at upper surface of tank = v_1
Speed of the efflux = v_2
Pressure at the upper end of tank = P_1
Pressure at the lower small hole of tank = P_2
Height of fluid at the upper end = h_1
Height of fluid at the lower end = h_2
Density of fluid = ρ



According to Bernoulli's equation

$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$ ______ (1)

by equation of continuity

$$A_1v_1 = A_2v_2$$

OR $v_1 = \frac{A_2}{A_1}v_2 = 0$ ______ (2) $\qquad [\because A_1 >> A_2]$

Also, the pressure will be the same because they are open to the atmosphere. So

$$P_1 = P_2 = P \text{ (atmospheric pressure)}$$

Hence, equation (1) becomes

$$P + \frac{1}{2}\rho(0)^2 + \rho g h_1 = P + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

OR
$$\rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Dividing both sides by ρ, we get

$$g h_1 = \frac{1}{2} v_2^2 + g h_2$$

OR
$$\frac{1}{2} v_2^2 = g h_1 - g h_2$$

OR
$$\frac{1}{2} v_2^2 = g(h_1 - h_2)$$

OR
$$v_2^2 = 2g(h_1 - h_2)$$

OR
$$\boxed{v_2 = \sqrt{2g(h_1 - h_2)}} \quad \text{\_\_\_\_\_\_ (3)}$$

This is Torricelli's theorem.

Note

- The speed of efflux is ***independent of the direction of flow*** whether the opening is directed upward, downward, or horizontally.
- Speed of efflux of liquid is the ***same*** as the speed of the ball that falls freely under the force of gravity through a height $(h_1 - h_2)$.
- If the hole is pointed upward as at 'B' shown in figure, this K.E. would allow the liquid to rise to the level of the water tank.
- In practice, viscous energy losses would change the result to some extent.

Q.10 What is relation between pressure and speed of fluid in a horizontal pipe system?

Ans.

Relation between Speed and Pressure of the Fluid

Where the speed is high, the pressure will be low.

Proof

- Suppose that water flows through a horizontal pipe system as shown in figure.
- The area of cross- section of the pipe at A is greater than the area of cross-section at B
- According to equation of continuity. The speed of fluid at B is greater than at A.

Applying Bernoulli's equation at points A and B, we have

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_B \quad \text{\_\_\_\_\_\_ (1)}$$

Where

P_A = pressure of water at A
P_B = pressure of water at B
v_A = speed of water at A
v_B = speed of water at B
ρ = density of water = 1000 kg/m^3

As the pipe system is horizontal, therefore $h_A = h_B$. Hence average P.E. is same at both ends. So equation (1) becomes

For Your Information

A filter pump has a constriction in the centre so that a jet of water from the tap flows faster here. This causes a drop in pressure near it and air therefore, flows in from the side tube. The air and water together are expelled through the lower part of the pump.

EXPLANATION:
At the constriction velocity is greater and pressure is small therefore, the air flows in the tube.

$$P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2 \quad \text{\_\_\_\_\_\_ (2)}$$

Also, let v_A = 0.20 m/s and v_B = 2.0 m/s

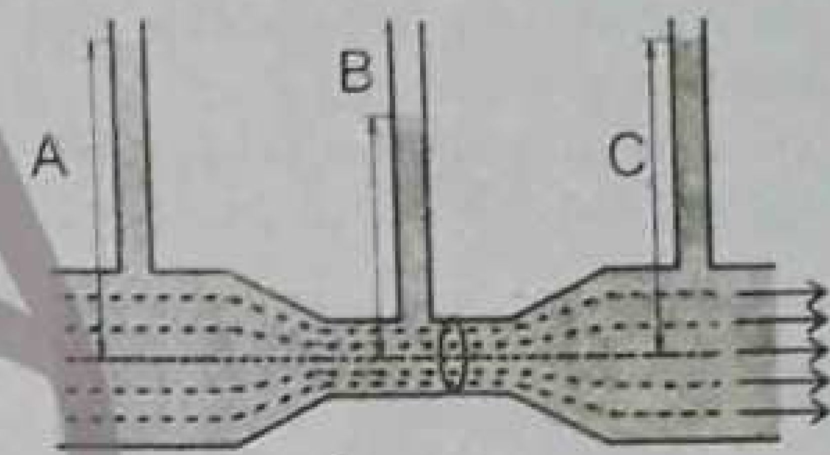
Using these values in equation (1), we have

$$P_A + \frac{1}{2}(1000)(0.20)^2 = P_B + \frac{1}{2}(1000)(2)^2$$

$$P_A + 20 = P_B\ 2000$$

OR
$$P_A - P_B = 2000 - 20$$

OR
$$P_A - P_B = 1980 \text{ N/m}^2$$



This shows that the pressure in narrow pipe where stream lines are closer together is smaller than wider pipe. Thus

Where speed is high, the pressure will be low.

Q.11 How a dynamic lift is produced in an aeroplane?

Ans.

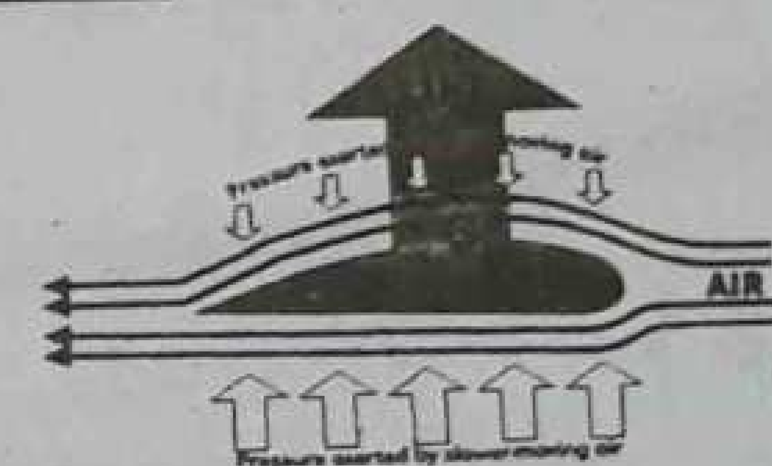
Lift on an aeroplane

The lift on an aeroplane is due to the effect, *where speed of fluid is high, its pressure will be low.*

Explanation

The design of wing deflects the air in such a way that

- Stream lines are ***closer*** together above the wing than lower.
- Air is moves faster at the ***upper side*** of the wing than the lower side.
- Pressure is ***lower*** at the top of the wing.
- Hence, the wing experiences a net ***upward*** force.



Q.12 How swing produced in a tennis ball?

Ans.

Swinging of a ball

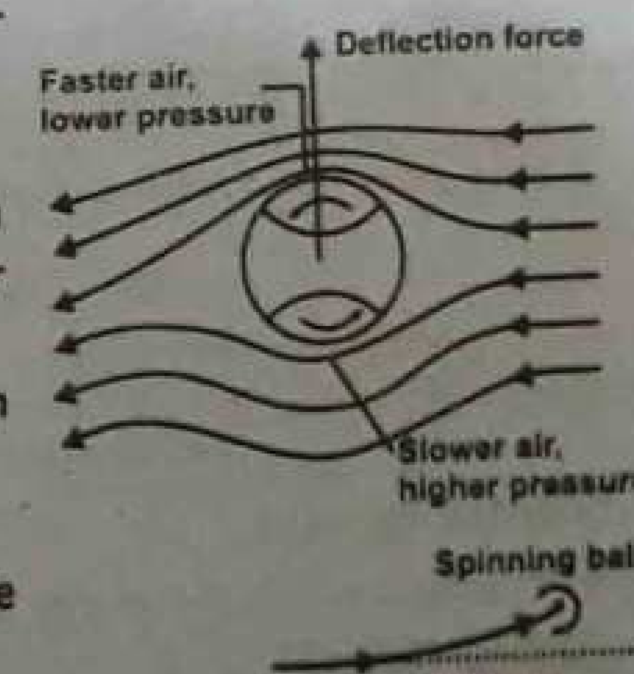
Consider a tennis ball moving through air from left to right (shown in figure) in such a way that it **spins as well as move forward**. The stream lines of air appear to move from right to left.

The layers of air near the surface of ball are pulled around in the direction of spin due to ***friction*** between the ball and the air

The speed of air at one side of the ball ***increases*** than at other side.

In this case, the pressure at one side of ball becomes ***smaller*** than that at the other side.

This gives an ***extra curvature*** to the ball known as swing, which deceives an opponent player.



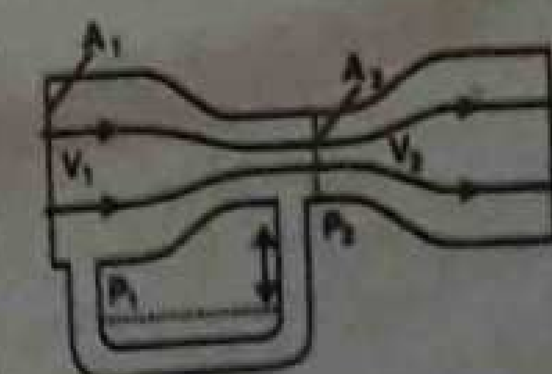
Q.13 What is venturi relation? Explain

Ans.

Venturi Relation

The effect of decrease of pressure with increase of velocity of the fluid in horizontal pipe is called venturi effect.

Suppose that an ***ideal*** fluid flows through a horizontal pipe system as shown in figure.



The area of cross- section of the pipe at A_1 is greater than the area of cross-section at A_2

The speed of fluid at A_2 is greater than at A_1. [By equation of continuity]

Applying Bernoulli's equation at points A and B, we have

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \_\_\_\_\_\_ (1)$$

Where

P_1 = pressure of fluid at wide part of the pipe

P_2 = pressure of fluid at narrow part (called throat) of the pipe

v_1 = speed of fluid at wide part

v_2 = speed of fluid at narrow part

ρ = density of fluid

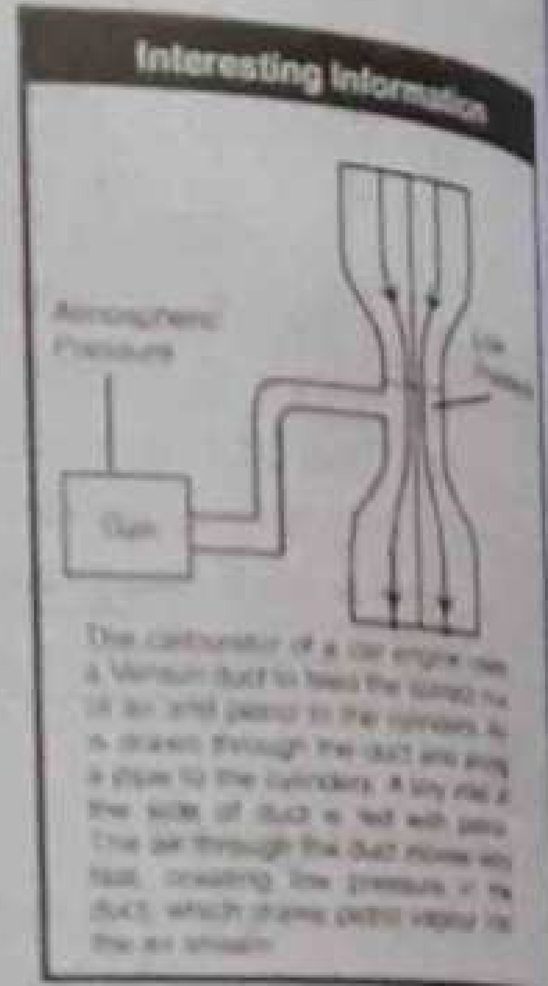
As the pipe system is horizontal, therefore $h_1 = h_2$.

Hence average P.E. is same at both ends. So equation (1) becomes

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

OR $$\boxed{P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)} \_\_\_\_\_\_ (2)$$



This is called venturi relation which used in venturi-meter to find the speed of fluid

Special case

If $A_1 >> A_2$ therefore, $v_1 << v_2$.

Then, according to equation of continuity

$$A_1 v_1 = A_2 v_2$$

OR $$v_1 = \left(\frac{A_2}{A_1}\right) v_2 = 0$$

Hence equation (2) becomes,

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - 0)$$

$$\boxed{P_1 - P_2 = \frac{1}{2}\rho v_2^2} \_\_\_\_\_\_ (3)$$

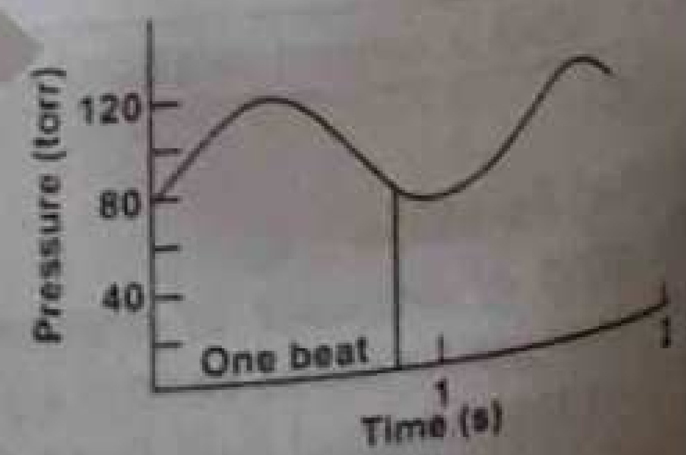
Venturi-meter

The device which measures the ***fluid speed*** is called venturi meter. The working principle of venturi meter is ***venturi relation***

Q.14 What do you know about blood pressure? How it can be measured dynamically?

Ans.

Blood Flow



Blood

- Blood is an ***incompressible*** fluid
- Density of blood is ***nearly equal*** to that of water.
- Viscosity of blood increases ***three to five times*** that of water due to high concentration of red blood cells (~50%)

Blood Pressure (BP)

The pressure exerted by the circulating blood on the walls of the blood vessels is called blood pressure.

- Blood vessels are not ***rigid***.
- Under normal conditions, the volume of the blood is sufficient to keep the vessels inflated at all times.
- So the pressure of the blood inside the vessels is ***greater*** than atmospheric pressure.
- During each heart beat, BP varies between a maximum (systolic) and a minimum (diastolic) pressure.
- A person's blood pressure is usually expressed in terms of the systolic pressure over diastolic pressure (mmHg), for example 120/80.

Do you know?

Blood pressure is a measurement of the force applied to the walls of the arteries as the heart pumps blood through the body. The pressure is determined by the force and amount of blood pumped, and the size and flexibility of the arteries.

Systolic Pressure

- It represents the ***maximum*** pressure exerted when the heart contracts.
- The value of high blood pressure (systolic pressure) is ***120 torr***

Diastolic Pressure

- It represents the minimum pressure in the arteries when the heart is at rest.
- The value of low blood pressure (diastolic pressure) is about ***75 – 80 torr***.

For your information

Blood pressure is usually measured while you are seated with your arm resting on a table. Your arm should be slightly bent so that it is at the same level as your heart. Your upper arm should be bare, with your sleeve comfortably rolled up.

Note

The value of blood pressure ***increases*** with age due to decrease in the flexibility of vessels.

Unit of blood pressure

The blood pressure is measured in ***torr or mm of Hg***.

Relation between torr and pascal

$$\boxed{1 \text{ torr} = 133.3 \text{Pa} = 133.3 \text{ N/m}^2}$$

Measurement of Blood Pressure

Sphygmomanometer is used to measure the pressure of blood dynamically in the vessels.

Steps to measure blood pressure

- An inflatable bag is ***wound*** around the arm.
- The bag is inflated to ***increase*** the external pressure on the arm to compress the blood vessels inside.
- When the external pressure becomes ***larger*** than systolic pressure, the vessels fall down.
- The flow of blood is ***cut off***.
- Head of a ***stethoscope*** is placed over the artery.

Systolic pressure

- ***Open*** the release valve to decrease the external pressure gradually.
- When external pressure becomes equal to ***systolic pressure***, the vessel opens a little bit.
- A first surge of blood flows out of the narrow opening of vessel with ***high speed***.
- As the flow speed is high, so the flow is ***turbulent***.
- At this instant, we hear a ***gurgle*** in the stethoscope.
- This is signal to record ***systolic*** pressure.

Diastolic pressure

- Now decrease the external pressure further till it becomes equal to ***diastolic pressure***.
- The vessel gets normal.
- The blood flow changes from turbulent to ***laminar***.
- The gurgle in the stethoscope ***disappears***.
- This is time to record ***diastolic*** pressure.

FORMULAE

1	**Drag force**	$F=6\pi r\eta v$	
2	**Terminal velocity of fog droplet**	$v_t=\frac{mg}{6\pi r\eta}$	$v_t=\frac{2g\rho}{9\eta}r^2$
3	**Equation of continuity**	$\frac{V}{t}$ constant Av =constant	$A_1v_1=A_2v_2$
4	**Bernoulli's equation**	$P_1+\frac{1}{2}\rho v_1^2+\rho gh_1=P_2+\frac{1}{2}\rho v_2^2+\rho gh_2$	$P+\frac{1}{2}\rho v^2+\rho gh=\text{constant}$
5	**Torricelli's Theorem (speed of efflux)**	$v_2=\sqrt{2g(h_1-h_2)}$	
6	**Venturi's relation**	$P_1-P_2=\frac{1}{2}\rho(v_2^2-v_1^2)$	

Multiple Choice Questions

♦ *Four possible answers to each statement are given below. Tick (✓) the correct answer:*

1. **If the radius of falling body is doubled, then what will be effect on its terminal velocity?**
   (a) Increase by 2 times (b) Decreases by 2 times
   (c) Increases by 4 times (d) Decreases by 4 times
2. **A ten meter high tank is full of water. A hole appears at its middle. The speed of efflux be:**
   (a) 5 ms^{-1} (b) 10 ms^{-1}
   (c) 100 ms^{-1} (d) 5.11 ms^{-1}
3. **The maximum drag force on a falling sphere is 9.8 N. Its real weight is:**
   (a) 1 N (b) 9.8 N
   (c) 4.9 N (d) 0.0 N
4. **The effect of decrease in pressure with increase in speed of the fluid in a horizontal tube is known as:**
   (a) Bernoulli's effect (b) Torricelli's theorem
   (c) Viscosity effect (d) Equation of continuity
5. **The SI unit of flow rate is:**
   (a) $m^2 s^{-1}$ (b) $m s^{-1}$
   (c) $m^3 s^{-1}$ (d) $m^3 s^{-2}$
6. **When the temperature increases, the viscosity of the gases**
   (a) Decreases (b) Increases
   (c) Remains constant (d) None of these
7. **"Dynamic lift" is related to:**
   (a) Bernoulli's theorem (b) Archimedes' principle
   (c) Equation of continuity (d) Pascal's law
8. **A gale [*i.e.*, very strong wind] blows over a house. The force due to the gale on the roof is:**
   (a) In the downward direction (b) In the upward direction
   (c) Zero (d) horizontal
9. **With increase in temperature, the viscosity of:**
   (a) A gas decreases and a liquid increases (b) A gas increases and a liquid decreases
   (c) Both gas's and liquid's decrease (d) Both gas's and liquid's increase
10. **The viscous drag on a small sphere moving with a speed v is proportional to:**
    (a) v (b) $\sqrt{v}$
    (c) $\frac{1}{\sqrt{v}}$ (d) v^2
11. **Two fog droplets have radius of 2 : 3, their terminal velocities are:**
    (a) 4 : 9 (b) 4 : 6

(c) 2 : 9 (d) 4 : 3

12. Which one is the venturi relation?

(a) $P_1 - P_2 = \frac{1}{2}\rho v_2^2$ (b) $V_2 = \sqrt{2g(h_1 - h_2)}$

(c) $A_1 v_1 = A_2 v_2$ (d) None of these

13. Laminar flow usually occurs at:

(a) High speed (b) Low speed

(c) Very high speed (d) None of these

14. The S.I. unit of co-efficient of viscosity is

(a) $Kgm^{-1}s^{-2}$ (b) $Kgm^{-1}s^{-1}$

(c) Nsm^{-2} (d) Both b & c

15. The relation between Nm^{-2} and torr is

(a) 1 torr = 13.33 Nm^{-2} (b) 1 torr = 133.3 Nm^{-2}

(c) 1 torr = 1333.0 Nm^{-2} (d) 1 torr = 1.333 Nm^{-2}

16. How the fluid speed changes if the diameter of a pipe is increased to double?

(a) Remain same (b) Increases to double

(c) Reduces to half (d) None of these

17. High concentration of red blood cells increases the viscosity of blood from

(a) 2-3 times that of water (b) 3-5 times that of water

(c) 2-4 times that of water (d) 5-7 times that of water

18. For which position, will the maximum blood pressure in the body have the smallest value

(a) Standing up right (b) Lying horizontally

(c) Standing on one's head (d) Sitting relaxed

19. The value of blood pressure ________ with age.

(a) Increases (b) Decreases

(c) Remain same (d) None of these

20. Sphygmomanometer is used to measure

(a) Fluid speed (b) Blood pressure

(c) Blood density (d) Both b & c

ANSWERS

1. c	2. b	3. b	4. a	5. c	6. b	7. a	8. b	9. b	10. a
11. a	12. a	13. b	14. d	15. b	16. d	17. b	18. b	19. a	20. b

Short Questions of Exercises

Q.6.1 Explain what do you understand by the term viscosity?

(Lhr 2004, Fsd 2005, Mir Pur 2004-2006, Lhr 2010-2011,Grw2011)

Ans. Viscosity

- Viscosity of a fluid is the measure of its ***resistance to flow***. It is the ***frictional effect*** between different layers of a flowing fluid. It measures that ***how much force*** is required to slide one layer of the liquid over another layer.

Q.6.2 What is meant by drag force? What are the factors upon which drag force acting upon a small sphere of radius r moving down through a liquid, depend?

Ans. Drag Force (Federal 2004, Mtn 2009)

An object moving through a fluid experiences a ***retarding force*** called drag force.

Factors

According to Stoke's law, drag force is given by,

$$F = 6\pi\eta r v$$

This equation shows that drag force depends upon

i) ***speed*** of sphere (v)

ii) ***radius*** of sphere (r)

iii) ***coefficient of viscosity*** of medium(η)

Q.6.3 Why fog droplets appear to be suspended in air?

(Federal 2003, Mir Pur 2003-2009, Sgd 2005, Mtn 2004, Rwp 2006, Fsd 2005-2008, Bwp 2007 Grw 2009, Lhr 2010-2011)

Ans. Reason

Terminal velocity of a fog droplet is

$$v_t = \frac{mg}{6\pi\eta r}$$

OR $v_t \propto mg$

As the weight of a fog droplet is ***very small***, the drag force become equal to the its weight very quickly. Thus, terminal velocity is ***very small*** and hence the droplet appears to be suspended.

Q.6.4 Explain the difference between laminar flow and turbulent flow?

(Federal 2004, Mir Pur 2004, Mtn 2005, Lhr 2008, Bwp 2008, Grw 2003-2008, Lhr 2009)

Ans. Difference between Laminar Flow and Turbulent Flow

- In laminar flow, each particle of fluid moves along a ***smooth path*** which does not change with time.
- In turbulent flow the ***flow pattern is not smooth*** but continuously changes with time.
- In laminar flow stream lines do not cross other while in turbulent flow it, does not happens.

Q.6.5 State the Bernoulli's relation for a liquid in motion and describe some of its applications?

(Lhr 2009, Grw 2009)

Ans. Bernoulli's relation

For an ideal fluid, the sum of pressure, kinetic energy per unit volume and the potential energy per unit volume at any point along a streamline always remains constant.

$$P+\frac{1}{2}\rho v^2+\rho gh= \text{ constant}$$

Applications

(i) The swing of ball (ii) Lift on an aeroplane
(iii) Working of carburetor (iv) Blood flow

Q.6.6 A person is standing near a fast moving train. Is there any danger that he will fall towards it?

(Sgd 2003-2004, Lhr 2005-2008, D.G,Khan 2005, Bwp 2004,Grw 2005, Fsd 2004)

Ans. *Yes*, there is danger that he will fall towards the train.

Reason

When fast moving train passes near the person, speed of air between train and the person ***increases***. According to Bernoulli's relation

where the speed of fluid is high, pressure will be low.

So pressure between train and the person ***decreases***. Hence large pressure behind the person pushes him towards the train.

Q.6.7 Identify the correct answer. What do you infer from Bernoulli's theorem?

(i) Where the speed of the fluid is high the pressure will be low?

(ii) Where the speed of the fluid is high the pressure is also high?

(iii) This theorem is valid only for turbulent flow of the liquid?

(Mtn 2004, Mir Pur 2004)

Ans. Statement (i) is the correct answer.

Q.6.8 Two row boats moving parallel in the same direction are pulled towards each other. Explain?

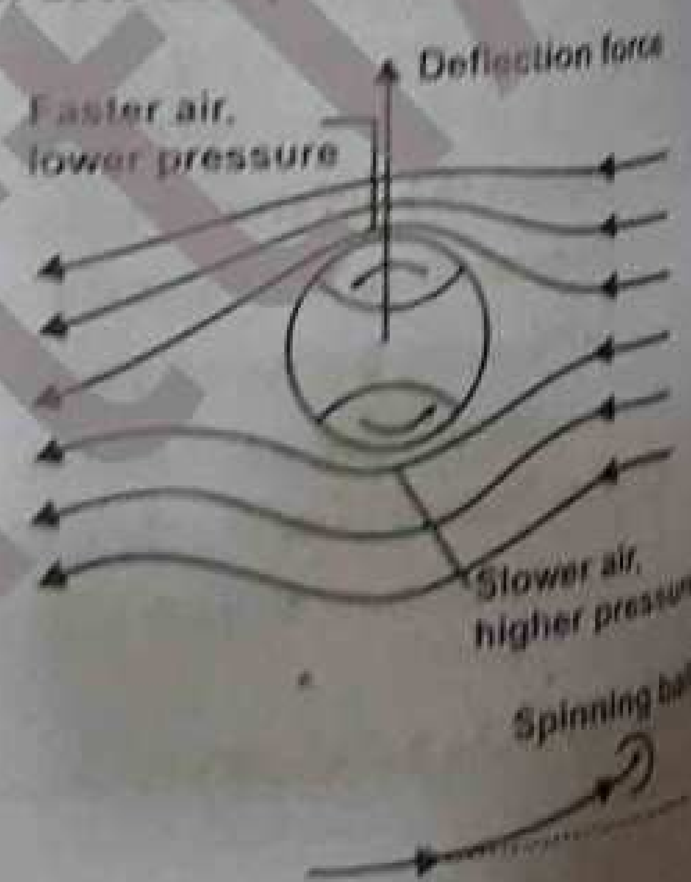
Ans. Reason

As speed of water between the two boats ***increases***. So, According to Bernoulli's relation

where the speed of fluid is high, its pressure will be low.

So pressure between the two boats ***decreases***. Hence pressure difference is produced which pulls the boats towards each other.

(Rwp 2003, Federal 2005, Mir Pur 2006-2009, Grw 2009-2010, Lhr 2010-2011,Grw2011)



Q.6.9 Explain, how the swing is produced in a fast moving cricket ball?
(Federal 2005, Fsd 2005, D.G.Khan 2005, Bwp 2007, Lhr 2009)

Ans. Swing in a cricket ball

When the ball moves forwards as well as spins, the speed of air on its one side becomes ***greater*** as compared to the other. According to Bernoulli's relation;

where the speed of fluid is high, its pressure will be low.

So at that side, the pressure of air decrease. ~~A net force acts on the ball~~ This gives an extra curvature to the ball called swing. This which deceives the batsman.

Q.6.10 Explain the working of a carburetor of a motorcar using Bernoulli's principle.

(Rwp 2005,Grw2011)

Ans. Working of carburetor

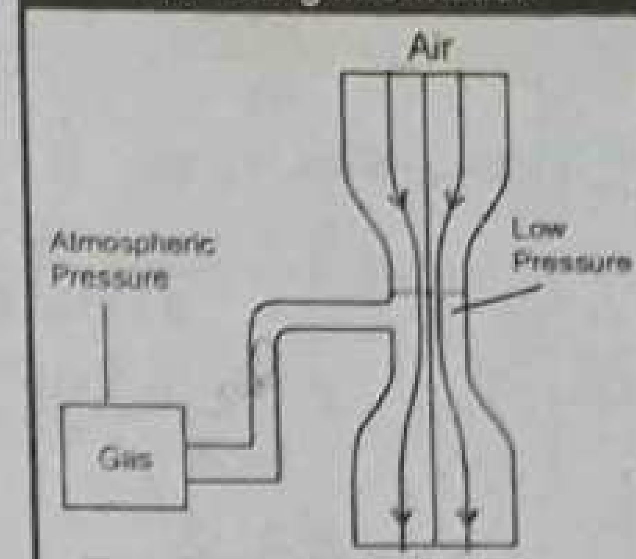
The carburetor of car engine uses a ***venturi duct*** to feed the correct mixture of air and petrol to the cylinders.

Air passes through the duct and along a pipe to the cylinders.

Petrol is mixed with air by a small valve at the side of duct.

The air through the duct moves very fast which produces ***low pressure*** in the duct It draws petrol vapours into the air stream.

Interesting Information



The carburetor of a car engine uses a Venturi duct to feed the correct mix of air and petrol to the cylinders. Air is drawn through the duct and along a pipe to the cylinders. A tiny inlet at the side of duct is fed with petrol. The air through the duct moves very fast, creating low pressure in the duct, which draws petrol vapour into the air stream.

Q.6.11 For which position will be maximum blood pressure (systolic pressure) in the body having the smallest value. (a) standing up right. (b) sitting (c) lying horizontally (d) Standing on one's head?

(Grw 2010)

Ans. The correct answer is (c)

(a) Standing upright, systolic pressure has maximum value in the neck

(b) Same as above

(d) In this case, the systolic pressure has maximum value in the legs.

Q.6.12 In an orbiting space station, would the blood pressure in major arteries in the leg ever be greater than the blood pressure in major arteries in the neck?

(D.G.Khan 2006)

Ans. Blood pressure would be ***same***.

Reason

In a orbiting space-station, everything is in state of ***weightlessness***. So, pressure will be same in major arteries of both in neck and legs.

Solved Exercises

Example 6.1

A tiny water droplet of radius 0.010cm descends through air from a high building. Calculate its terminal velocity. Given that η for air = 19×10^{-6} kg $m^{-1}s^{-1}$ and density of water ρ = 1000 kgm^{-3}

Given Data:

Radius of water droplet = r = 0.010 cm = 1.0×10^{-4}m

Density of water = ρ = 1000 kgm^{-3}

Viscosity of air = η = 19×10^{-6} $kgm^{-1}s^{-1}$

To Find:

Terminal velocity = v_t = ?

Calculation:

Terminal velocity is given by

$$v_t = \frac{2gr^2\rho}{9\eta}$$

Putting values, we get

$$v_t = \frac{2\times9.8\times(1.0\times10^{-4})^2\times1000}{9\times19\times10^{-6}}$$

$$v_t = \frac{2\times9.8\times10^{-8}\times10^3}{9\times19\times10^{-6}}$$

$$v_t = \frac{19.6\times10^{-5}}{171\times10^{-6}}$$

$$v_t = 0.11\times10\,ms^{-1}$$

Or $\boxed{v_t = 1.1 m/sec}$

Example 6.2

A water hose with an internal diameter of 20 mm at the outlet discharges 30 kg of water in 60s. Calculate the water speed at the outlet. Assume the density of water is 1000 kgm^{-3} and its flow is steady.

Given Data:

Internal diameter of water hose = d = 20mm = 0.02m

Internal radius of water hose = $r = \frac{d}{2} = \frac{0.02}{2} = 0.01m$

Mass of water = m = 30 kg

Time taken = t = 60s

Density of water = ρ = 1000 kgm^{-3}

To Find:

Speed of water = v = ?

Calculation:

Mass flow per second = $\frac{m}{t}$

$$= \frac{30}{60} = 0.5\ kg\ s^{-1}$$

Cross sectional area = A = πr^2

$$= 3.14\times(0.01)^2 = 3.14\times10^{-4}\ m^2$$

From equation of continuity

Av = Rate of flow

Or $Av = \frac{volume}{sec}$

Thus, $Av = \frac{mass}{sec\times density}$ As $volume = \frac{Mass}{Density}$

Or $v = \left(\frac{Mass}{Sec}\right)\times\frac{1}{area\times density}$

$$v = \frac{m}{t}\times\frac{1}{A\times\rho}$$

Putting values, wet get

$$v = 0.5\times\frac{1}{3.14\times10^{-4}\times1000}$$

$$v = \frac{0.5}{0.314}$$

$$\boxed{v = 1.6\ ms^{-1}}$$

Example 6.3

Water flows down hill through a closed vertical funnel. The flow speed at the top is 12.0 cms^{-1}. The flow speed at the bottom is twice the speed at the top. If the funnel is 40 cm long and the pressure at the top is 1.013 × 10^5 Nm^{-2}, what is the pressure at the bottom?

Given Data:

Flow speed of water at top = v_1 = 12 cms^{-1} = 0.12ms^{-1}

Flow speed of water at bottom = $v_2 = 2v_1 = 2\times0.12 = 0.24$ ms^{-1}

Length of the funnel = $h = h_1 - h_2$ = 40 cm = 0.40 m

Pressure at the top = $P_1 = 1.013\times10^5$ Nm^{-2}

Density of water = ρ = 1000 kgm^{-3}

To Find:

Pressure at the bottom = P_2 = ?

Calculation:

According to Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

or $$P_2 = P_1 + \rho g(h_1 - h_2) + \frac{1}{2}\rho(v_1^2 - v_2^2)$$

or $$P_2 = P_1 + \rho gh + \frac{1}{2}\rho(v_1^2 - v_2^2)$$

Putting values, we get

$$P_2 = (1.013\times10^5) + (1000\times9.8\times0.40) + \frac{1}{2}\times1000\times((0.12)^2 - (0.24)^2)$$

$$P_2 = 1.013\times10^5 + 3920 + 500\,(0.0144 - 0.076)$$

$$P_2 = 1.013\times10^5 + 3920 + 500\,(-0.0432)$$

$$P_2 = 1.013\times10^5 + 3920 - 21.6$$

$$P_2 = 1.013\times10^5 + 3898.4$$

$$P_2 = 1.013\times10^5 + 0.0389\times10^5$$

$$P_2 = (1.013 + 0.0389)\times10^5$$

$$\boxed{P_2 = 1.05\times10^5\ Nm^{-2}}$$

Exercise Problems

6.1 Certain globular protein particle has density of 1246 kgm^{-3}. It falls through pure water($\eta = 8.0\times10^{-4} Nm^{-2}s$) with a terminal speed of 3.0 cm h^{-1}. Find ther radius of the particle.

Given Data:

Density of protein particle $=\rho=1246\text{kgm}^{-3}$

Co- efficient of viscosity $=\eta=8.0\times10^{-4}\text{Nm}^{-2}\text{s}$

Terminal velocity $= v_t=3.0\text{cmh}^{-1}=\dfrac{3}{100\times60\times60}=8.33\times10^{-6}\text{m/s}$

To find:

Radius of particle = r =?

Calculations:

As $v_t=\dfrac{2gr^2\rho}{9\eta}$

Or $r^2=\dfrac{9\eta v_t}{2\rho g}$

Putting values, we get

$$r^2=\frac{9\times8\times10^{-4}\times8.33\times10^{-6}}{2\times1246\times9.8}$$

$$r^2=2.46\times10^{-12}$$

$$r=\sqrt{2.46\times10^{-12}}$$

$$r=1.567\times10^{-6}$$

$$\boxed{r=1.56\times10^{-6}\text{m}}$$

6.2 Water flows through a hose, whose internal diameter is 1 cm at a speed of 1m/s. What should be the diameter of the nozzle if the water is to emerge at 21m/s?

Given Data:

Internal diameter of hose $=d_1=1\text{cm}=1\times10^{-2}\text{m}$

Speed of water in the hose $=v_1=1\text{ms}^{-1}$

Speed of water emergence $=v_2=21\text{ms}^{-1}$

To find:

Diameter of the nozzle $=d_2=?$

Calculations:

According to equation of continuity,

$A_1v_1=A_2v_2$ and $r=\dfrac{d}{2}$

Or $\pi r_1^2v_1=\pi r_2^2v_2$

Or $r_1^2v_1=r_2^2v_2$

Or $\dfrac{d_1^2}{4}v_1=\dfrac{d_2^2}{4}v_2$

$d_1^2v_1=d_2^2v_2$

Or $d_2^2=\dfrac{d_1^2}{v_2}v_1$

Putting values, we get

$$d_2^2=\frac{(1\times10^{-2})^2(1)}{21}$$

$$d_2^2=4.76\times10^{-6}$$

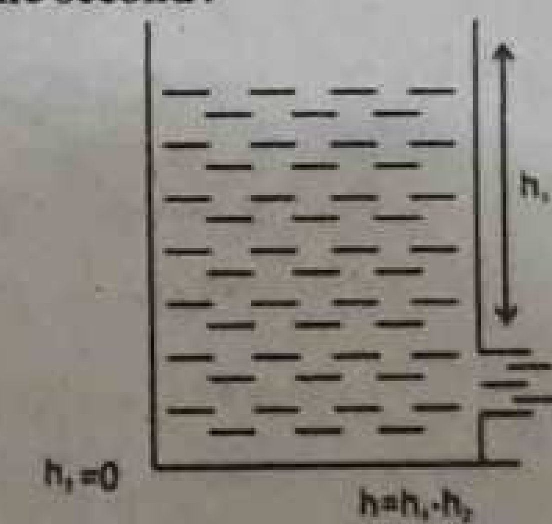
$$d_2=2.18\times10^{-3}\text{m}$$

Or $\boxed{d_2=0.2\text{cm}}$

6.3 The pipe near the lower end of a large water storage tank develops a small leak and a stream of water shoots from it. The top of water in tank is 15 m above the point of leak.

a) with what speed does the water rush from the hole?

b) If the hole has an area of 0.060 cm^2, how much water flows out in one second?



Given Data:

Height of water above leak point $=h=15m$

Area of the hole $= A = 0.06\text{cm}^2 = 0.06\times10^{-4}m^2$

To find:

Speed of water =v = ?

Volume of water flow per second $=\dfrac{V}{t}=?$ (i.e. flow rate)

Calculations:

(a) According to Torricelli's theorem

$$v=\sqrt{2g(h_1-h_2)}$$

$$v=\sqrt{2gh}$$

Putting values , we get

$$v=\sqrt{2\times9.8\times15}$$

$$v=\sqrt{294}$$

$$v=17.14\text{m/s}$$

Or $\boxed{v=17\text{m/s}}$

(b) Rate of flow $=\dfrac{V}{t}=$ volume of water flow per sec $=Av$

Rate of flow = Av

$$= 0.06\times10^{-4}\times17$$
$$= 102\times10^{-6}m^3s^{-1}$$

Hence volume of water flowing out per second i.e.

$$\boxed{\text{Rate of flow} = 102\text{cm}^3/\text{sec}}$$

6.4 Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3.0 ms^{-1} while at another point 3.0m higher, the speed is 4.0 ms^{-1}. If the pressure is 80 kPa at the lower point. What is pressure at the upper point?

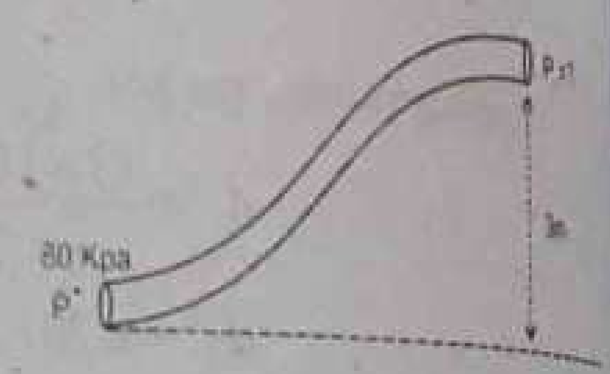
Given Data:

Speed of water at one point = $v_1 = 3ms^{-1}$

Speed of water at second point $= v_2 = 4ms^{-1}$

Difference of height between two points $h_2 - h_1 = 3$

Pressure at lower point = $P_1 = 80kPa = 80{,}000$ Pa



To find:

Pressure at upper point = $P_2 = ?$

Calculation:

According to Bernoulli's theorem

$$P_1+\frac{1}{2}\rho v_1^2+\rho gh_1=p_2+\frac{1}{2}\rho v_2^2+\rho gh_2$$

Or $$P_2=P_1+\frac{1}{2}\rho v_1^2-\frac{1}{2}\rho v_2^2+\rho gh_1-\rho gh_2$$

$$P_2=P_1+\frac{1}{2}\rho(v_1^2-v_2^2)-\rho g(h_2-h_1)$$

Putting values, we get

$$P_2 = 80000+\frac{1}{2}\times1000(3^2-4^2)-1000\times9.8\times3$$
$$= 80000+500(9-16)-29400$$
$$= 80000+4500-8000-29400=84500-37400$$
$$= 47100$$
$$= 47.1\times10^3 Pa$$
$$\boxed{P_2 = 47kPa}$$

6.5 An airplane wing is designed so that when the speed of the air across the top of the wing is 450 ms^{-1}, the speed of air below the wing is 410 ms^{-1}. What is the pressure difference between the top and bottom of the wings? (Density of air = ($1.29kgm^{-3}$)

Given Data:

Speed of air on the upper surface = $v_1 = 450ms^{-1}$

Speed of air on the lower surface = $v_2 = 410ms^{-1}$

Density of air = $\rho = 1.29kgm^{-3}$

$$h_1 \approx h_2 = h(\text{say})$$

To find:

Pressure difference = $P_2 - P_1 = \Delta P = ?$

Calculations:

According to Bernoulli's equation

$$P_1+\frac{1}{2}\rho v_1^2+\rho gh_1=p_2+\frac{1}{2}\rho v_2^2+\rho gh_2$$

$$P_1+\frac{1}{2}\rho v_1^2+\rho gh=p_2+\frac{1}{2}\rho v_2^2+\rho gh$$

$$P_2-P_1=\frac{1}{2}\rho(v_1^2-v_2^2)$$

$$=\frac{1}{2}\times1.29((450)^2-(410)^2)$$
$$=0.645(202500-168100)$$
$$=0.645(34400)$$
$$=22188\text{Pa}$$
$$=22.188\times10^3\text{Pa}$$
$$\boxed{P_2-P_1 = 22kPa}$$

6.6 The radius of the aorta is about 1.0 cm and the blood flowing through it has a speed of about 30cm^{-1}. Calculate the average speed of the blood in the capillaries using the fact that although each capillary has a diameter of about 8×10^{-4} cm, there are literally millions of them so that their total cross section is about 2000cm^2.

Given Data:

Radius of aorta = r_1 = 1 cm = 1×10^{-2} m

Speed of inflowing blood = v_1 = 30 cm/s $= 30\times10^{-2}$ m/s

Total cross sectional area = A_2 = 2000 cm^2 $= 2000\times10^{-4}m^2$ $= 0.2\ m^2$

Average diameter of capillary = $d_2 = 8\times10^{-4}$ cm $= 8\times10^{-6}$m

To find:

Average speed of blood $= v_2 = ?$

Calculations:

Applying equation of continuity

$$A_1v_1=A_2v_2$$

or $$v_2=\frac{A_1v_1}{A_2}$$

$$v_2=\frac{\pi r_1^2\times v_1}{A_2}$$

Putting values, we get

$$v_2 = \frac{3.14\times(1\times10^{-2})^2\times30\times10^{-2}}{0.2}$$

$$= \frac{3.14\times30\times10^{-6}}{0.2}$$

$$= \frac{942\times10^{-6}}{0.2} = 471\times10^{-6}$$

$$= 4.71\times10^{-4} = 5\times10^{-4}\,\text{ms}^{-1} \quad (\textit{Approx})$$

6.7 How large must a heating duct be if air moving 3.0 m/s along it can replenish the air in a room of 300 m^3 volume every 15 min? Assume the air's density remains constant.

Given Data:

Speed of air = v = $3ms^{-1}$

Volume of air = V = $300m^3$

Time = t = 15 min = 15x60 = 900 sec

To find:

Size of the duct = r = ? (i.e. radius of duct)

Calculation:

As Rate of flow $= \frac{\text{volume}}{\text{time}}$ = (area of cross - section) (speed of the fluid)

Rate of flow $= \frac{V}{t} = Av$

Rate of flow $= \frac{V}{t} = \pi r^2 v$

Or $r^2 = \frac{V}{t\times\pi\times v}$

Putting values, we get

$$r^2 = \frac{300}{900\times3.14\times3}$$

$$r^2 = \frac{300}{8478}$$

$$r^2 = 0.0354$$

$$r = 0.188\text{m} = 0.19\text{m}$$

$$r \approx 19\text{cm}$$

$$\boxed{r = 19\text{ cm}}$$

6.8 An airplane design calls for a lift due to the net force of the moving air on the wing of about 1000 Nm^{-2} of wing area. Assume that air flows past the wing of an aircraft with streamline flow. If the speed of flow past the lower wing surface is 160 ms^{-1}, what is the required speed over the upper surface to give a lift of 1000 Nm^{-2}? The density of air is 1.29kgm^{-3} and assumes maximum thickness of wing to be one metre.

Given Data:

Pressure difference on the wing = $P_1 - P_2 = 1000N/m^2$

Speed of air past the lower surface = $v_1 = 160m/s$

Density of air = ρ = 1.29kgm^{-3}

Thickness of wing = $h_2 - h_1 = 1m$

To find:

Speed of air over the upper surface wing = v_2 = ?

Calculations:

According to Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Or $$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$

Putting values, we get

$$1000 = \frac{1}{2}\times1.29(v_2^2 - (160)^2) + (1.29)(9.8)(1)$$

$$1000 = \frac{1}{2}\times1.29(v_2^2 - 25600) + 12.64$$

$$1000 - 12.46 = \frac{1}{2}\times1.29(v_2^2 - 25600)$$

$$987.54 = (0.645)(v_2^2 - 25600)$$

$$\frac{987.54}{0.645} = v_2^2 - 25600$$

$$1531.07 + 25600 = v_2^2$$

$$v_2^2 = 2.7\times10^4$$

$$v_2 = 164.71$$

Or $$\boxed{v_2 = 165\text{ms}^{-1}}$$

6.9 What gauge pressure is required in the city mains for a stream from a fire hose connected to the mains to reach a vertical height of 15.0m?

Given Data:

Vertical *height* = *h* = $h_1 - h_2 = \Delta h$ = 15 m

Density of water = $\rho = 1000kgm^{-3}$

To find:

Pressure difference = $P_2 - P_1 = \Delta P$ = ?

Calculations:

Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

We assume that the pipe is of uniform cross sectional area so the speed of stream remains same throughout its motion, So $v_1 = v_2 = v$(say)

Thus, $P_1+\rho gh_1=P_2+\rho gh_2$

$P_2-P_1=\rho gh_1-\rho gh_2$

Or $P_2-P_1=\rho g(h_1-h_2)$

Putting values, we get

Or $P_2-P_1=1000\times9.8(15)$

$P_2-P_1=147000Nm^{-2}$

Or $\boxed{\Delta P=1.47\times10^5 Pa}$

****

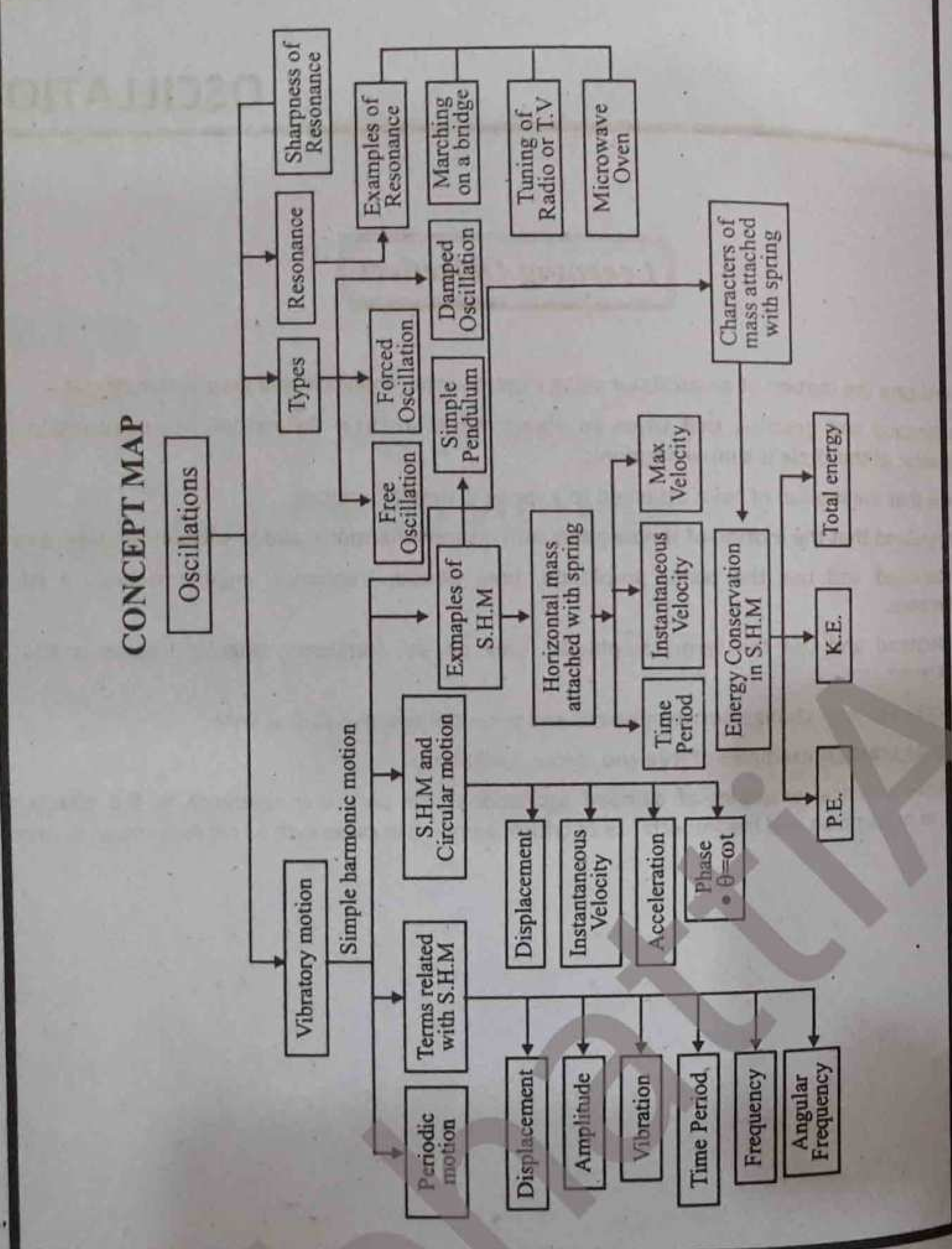
Chapter 7

OSCILLATIONS

Learning Objectives

1. Investigate the motion of an oscillator using experimental, analytical and graphical methods
2. Understand and describe that when an object moves in circle the motion of its projection on the diameter of the circle is simple harmonic.
3. Show that the motion of mass attached to a spring is simple harmonic.
4. Understand that the motion of simple pendulum is simple harmonic and to calculate its time period.
5. Understand and use the terms amplitude, time period, frequency, angular frequency ad phase difference.
6. Understand and use the terms amplitude, time period, frequency, angular frequency and phase difference.
7. Describe the inter change between kinetic and potential energies during SHM.
8. Describe practical examples of free and forced oscillations.
9. Describe practical examples of damped oscillations with particular reference to the effects of the degree of damping and the importance of critical damping in cases such as car suspension system.

Chapter No. 7



Oscillatory Motion

To and fro motion of a body about a mean position is called oscillatory or vibratory motion.

Periodic Motion

The oscillatory motion that repeats itself after equal intervals of time is called periodic motion

Examples of vibrating bodies

1- The motion of mass suspended from a spring
2- The motion of bob of a simple pendulum
3- A steel ruler clamped at one end to a bench oscillates when the free end is displaced sideways.
4- A steel ball rolling in a curved dish

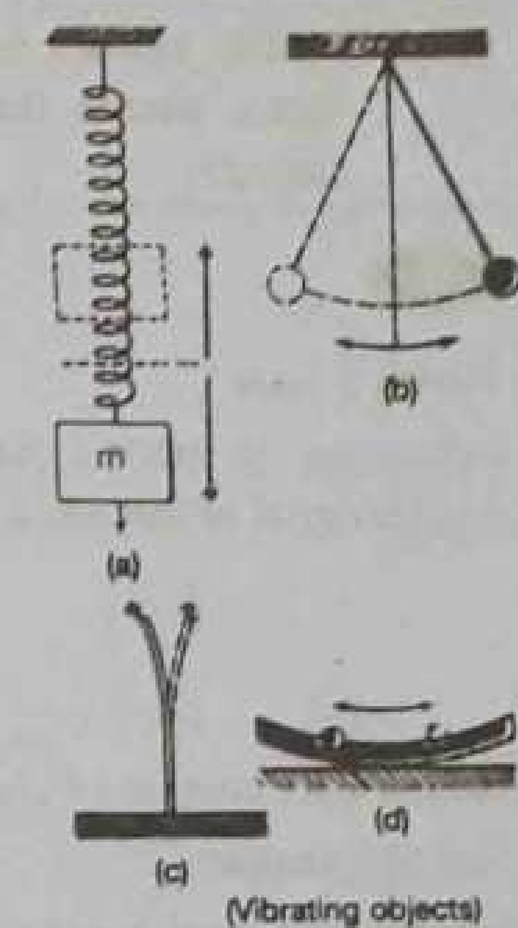


Fig. 7.1

Restoring force

The force which brings the system back to its stable equilibrium position is called elastic restoring force.

Mathematically

$$\vec{F} = -k\vec{x}$$

It is equal and opposite to the applied force

How to produce oscillation

In order to get oscillation;

- A body is **pulled** away on one side from its equilibrium position and then released.
- The body begins to oscillate (vibrate) due to **restoring force.**
- Under the action of this restoring force, the body **accelerates** and it passes over the rest position due to **inertia.**
- The restoring force pulls it back.
- Since restoring force is always directed towards the **mean position,** so the acceleration is also directed towards the mean position.

Requirements for oscillation

Two requirements for oscillations are

1- Oscillating system has ***restoring force.***
2- Oscillating system has ***inertia.***

Note

The vibrating bodies produce waves. e.g. A violin string produces waves.

There are many phenomena in nature, which are explained on the concept of vibration and waves. There are many large structures such as skyscrapers and bridges, which appears to be rigid. They actually vibrate in the light of these facts, so the architects and engineers take into account these vibrations, while designing and building of certain structure.

Q.1 Define Hook's law and simple harmonic motion? What is restoring force, derive the relation for acceleration of mass attached with a spring?

Ans.

Hook's Law

According to Hook's law, within elastic limit, the applied force is directly proportional to the displacement. Mathematically

$$\vec{F} \propto \vec{x}$$

OR $\vec{F} = k\vec{x}$ ______ (1)

Where k is constant of proportionality, known as spring constant.

Spring Constant

It spring constant is defined as *the force per unit extension*. Its SI unit is Nm^{-1} and dimension is $[MT^{-2}]$..

Simple Harmonic Motion

The oscillatory motion, in which acceleration of the body at any instant is directly proportional to displacement from the mean position and* is always *directed towards the mean position, is called simple harmonic motion (SHM).

Examples

1- Motion of simple pendulum
2- Motion of mass attached to a spring.
3- Motion of a swing.

Conditions for SHM

1. The system must have ***inertia***
2. The system must obey ***Hook's law***
3. The system should have ***elastic restoring force***
4. The system should be ***frictionless***

Q.2 Show that motion of mass attached with a spring is SHM.

Ans.

Motion of Mass attached to a spring

Consider a mass m attached with one end of the spring. The mass m can move freely on a frictionless horizontal surface as shown in figure.

When mass m is displaced through a distance x from mean position by a force F then,

According to Hook's law.

$$\vec{F} = k\vec{x}$$

Due to elasticity, spring opposes the applied force. This opposing force is called ***restoring force.***

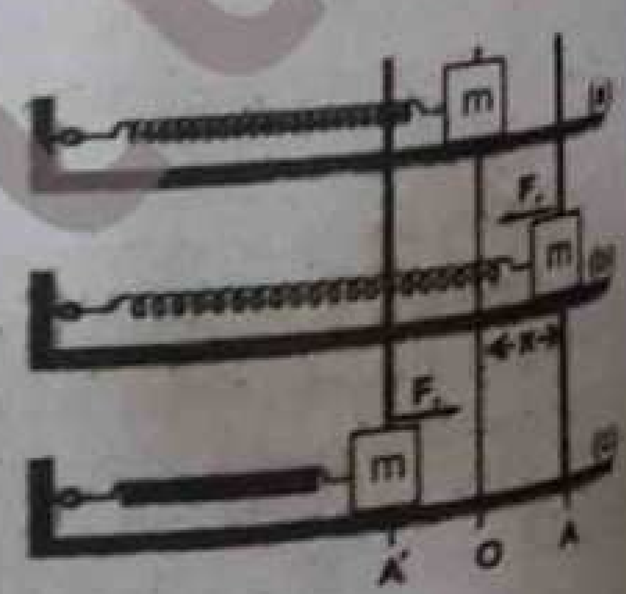


Fig. 7.2

Elastic Restoring Force

The force which brings the body back towards its mean position is called elastic restoring force.

The restoring force is represented a F_r is

$$\vec{F}_r = -k\vec{x} \text{ ______ (1)}$$

The negative sign shows that F_r is directed opposite to x.

When the mass is released, it begins to oscillate about the equilibrium position as shown in figure, such type of oscillations are due to restoring force and inertia. This type of oscillatory motion is called simple harmonic motion.

Expression for acceleration

The acceleration a produced in the mass m due to restoring force can be calculated using second law of motion

$$\vec{F} = m\vec{a} \text{ ______ (2)}$$

Comparing equations (1) and (2), we get

$$m\vec{a} = -k\vec{x}$$

$$\boxed{\vec{a} = -\frac{k}{m}\vec{x}} \text{ ______ (3)}$$

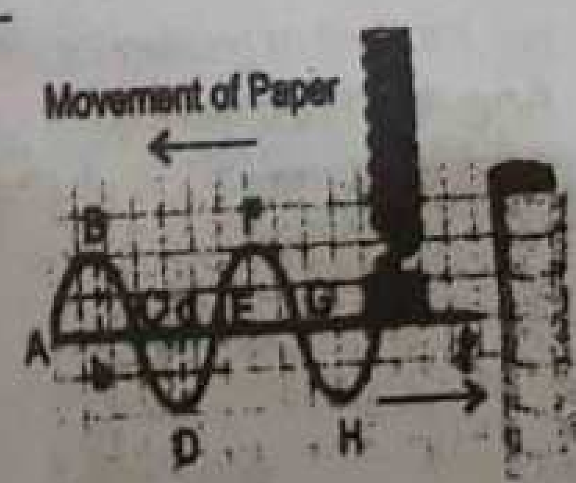
$$\vec{a} = -\text{constant } \vec{x}$$

$$\vec{a} \propto -\vec{x} \quad \text{(Hence proved)}$$

Q.3 Define the following terms related to SHM.

(a) Wave form of S.H.M. (b) Instantaneous displacement
(c) Amplitude (d) Vibration
(e) Time period (f) Frequency
(g) Angular Frequency

Ans.



Wave form of SHM

The curve representing the variation displacement with time is called wave from of SHM,

Explanation

Consider a mass spring system with vertical arrangement in such a way that pen attached with mass m form the trace on the strip of paper moving at constant speed from right to left.

So it provides a time scale on the strip. The ***sine curve*** is obtained which shows the variation of displacement with time.

It is called wave form of SHM.

The point A, C and E show its mean position while B and D represent the extreme position.

Instantaneous Displacement (x)

The distance of the vibrating body at any instant from its mean position is called instantaneous displacement

It is usually denoted by x. The value of instantaneous displacement is zero at mean position while it has maximum value at the extreme positions.

Amplitude (x_o)

The maximum value of displacement of the vibrating body on either sides from its mean position is called amplitude.

It is denoted by x_o. In figure, the amplitude is the measure of line Bb or Dd.

Vibration

One complete round trip of a body about its mean position is called one vibration.

The motion of body from mean position to upper extreme position, from upper extreme position to lower extreme position and back to its mean position is called one vibration. So according to the figure ABCDE shows one vibration of the body.

Time Period

The time required to complete one vibration is called time period.

It is represented by T .Its unit is second.

Frequency

The number of vibrations completed in one second by the body is called frequency. It is the reciprocal of the time period

It is represented by f. The unit of frequency is hertz or vib/sec or cycles /sec.

$$\boxed{f = \frac{1}{T}}$$

OR $f \times T = 1$

(i.e. product of frequency and time period equals one)

Angular Frequency

If T is the time period of a body executing SHM, its angular frequency (ω) is given as

$$\omega = \frac{2\pi}{T}$$

$$\omega = 2\pi\left(\frac{1}{T}\right)$$

$$\omega = 2\pi f$$

Note

Basically, angular frequency is the property of circular motion. In SHM, it provides an easy method to determine the instantaneous displacement and instantaneous velocity of body executing SHM.

Q.4 Show that the motion of projection of body moving along a circular path is SHM

Ans.

SHM and Uniform Circular Motion

Consider a mass m attached with the end of a vertically suspended spring. It vibrates simple harmonically with period T, frequency f and the amplitude x_o. The motion of the mass is displayed by a pointer P_1.

$0, T/4, T/2, 3T/4, T$ $\quad$ $0, T/4, T/2, 3T/4, T$

A, B, A, C, A $\quad$ O, D, O, E, O

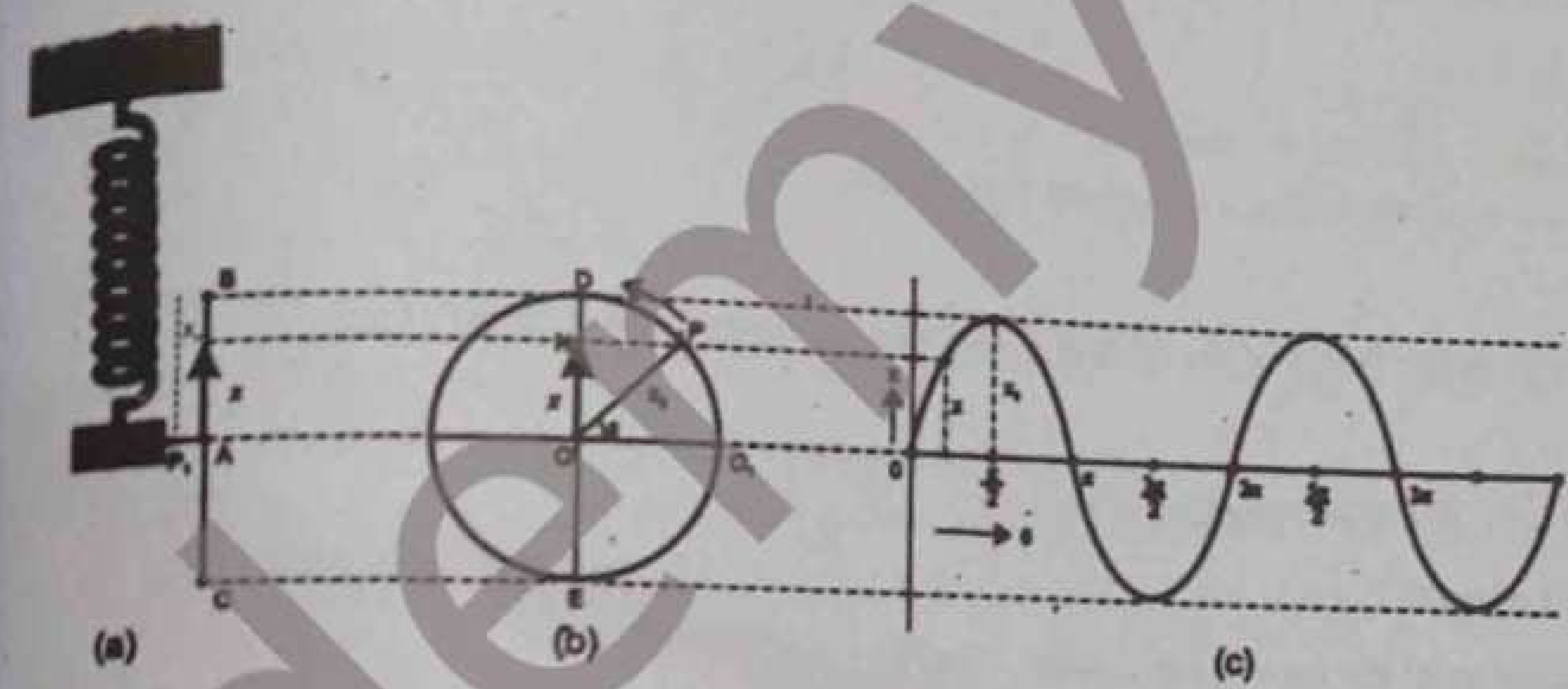


Fig. 7.4

At t = 0 pointer is at position A then at position B, A, C and back to A at instant T/4, T/2, 3T/4 and T respectively.

In circular motion point 'P' is moving in a circle of radius x_o with uniform angular frequency ω. Now consider the motion of point N, the projection of 'P' on diameter DE. The levels of D and 'E' are similar to points B and C.

With the motion of 'P' on the circle, the point N moves to and fro on DE. Let point P is at O_1 at t = 0, the projection N at instants 0, T/4, T/2, 3T/4 and T will be at O, D, O, E and O respectively.

Result

Hence the comparison of motion of N and P_1 shows that it is a copy of pointer's motion. Hence ***the motion of projection of particle P moving in a circle is SHM.***

Q.5 Derive the expressions for instantaneous displacement, instantaneous velocity and acceleration of the projection of a particle moving in a circle of radius x_o.

Let N be the projection of a particle P moving in a circle.

Angular frequency of P = ω

The angle subtended by OP at any time t = $\theta = \omega t$

Radius of the circle = x_o

(1) Instantaneous Displacement

From figure (in right angled triangle OPN)

$$\frac{ON}{OP} = \sin\theta$$

OR $ON = OP \sin\theta$ $\quad$ [$\because \angle OPN = \angle O_1OP = \theta$ (alternate angles)]

But ON = x and OP = x_o

So $\boxed{x = x_o \sin\theta}$

As $\theta = \omega t$

$\boxed{x = x_o \sin\omega t}$ ______ (1)

This equation shows the displacement of pointer N at instant t.

Phase angle (θ)

The angle θ which gives the states of the system during one complete cycle is called phase.

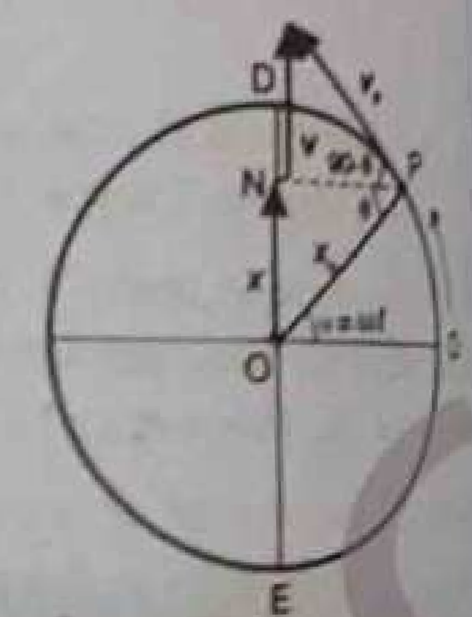
Values of θ

The wave form of SHM is shown in fig 1(c). In which

1. At the start of cycle, $\theta = 0$.
2. When 1st **quarter** of the cycle is completed, $\theta = \pi/2$.
3. When **half** of the cycle is completed, $\theta = \pi$.
4. When **three fourth** of the cycle is completed cycle, $\theta = 3\pi/2$.
5. For the **complete** cycle, $\theta = 2\pi$.

Note

For each quarter of the cycle, the phase of vibration is changed by $\pi/2$ radian.



(2) Instantaneous Velocity (v)

The linear velocity of point P at any instant t = v_p

Then $v_p = x_o\,\omega$

Since the motion of N on diameter DE is due to the motion of P on the circle.

The velocity of N is actually the vertical component of velocity v_p in the direction parallel to DE.

The component of velocity parallel to DE is

$$v = v_p \sin(90^\circ - \theta)$$

OR $v = v_p \cos\theta$ [Since ∵ $(90^\circ - \theta) = \cos\theta$]

OR $v = x_o\omega \cos\theta$

OR $v = x_o\omega \cos\omega t$ ______ (2)

From right angled triangle OPN

$$\cos\theta = \frac{PN}{OP} \quad \text{______ (3)}$$

Applying Pythagorean theorem, for calculating value of PN

$$(OP)^2 = (PN)^2 + (ON)^2$$

$$(PN)^2 = (OP)^2 - (ON)^2$$

OR $(PN)^2 = x_o^2 - x^2$

OR $PN = \sqrt{x_o^2 - x^2}$

FOR YOUR INFORMATION

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \sqrt{1-\sin^2\theta}$

Since $x = x_o \sin\theta$

So $\sin\theta = \frac{x}{x_o}$

Hence

$\cos\theta = \sqrt{1-\left(\frac{x}{x_o}\right)^2}$

$\cos\theta = \sqrt{1-\frac{x^2}{x_o^2}}$

$\cos\theta = \sqrt{\frac{x_o^2-x^2}{x_o^2}}$

$\cos\theta = \frac{\sqrt{x_o^2-x^2}}{x_o}$

Putting values of PN and OP in equation (3), we get

$$\cos\theta = \frac{\sqrt{x_o^2 - x^2}}{x_o}$$

Putting values of cos θ in equation (2), we get

$$v = x_o\omega\frac{\sqrt{x_o^2 - x^2}}{x_o}$$

OR $\boxed{v = \omega\sqrt{x_o^2 - x^2}}$ ______ (4)

Direction of Velocity of N

The direction of velocity depends upon the value of phase angle.

- When it varies from 0º to 90º then the direction of v is O to D.
- When it varies from 90º to 270º then the direction of v is D to E.
- When varies from 270º to 360º then the direction of v is E to O.

Special cases

- At mean position (i.e. x = 0) the velocity is maximum [i.e. $v_{max} = \omega x_o$]
- At extreme position (i.e. x = x_o), the velocity is zero [i.e. $v_{min} = 0$]

(3) Instantaneous Acceleration (a)

The acceleration at any point P moving along the circle can be expressed as,

$$a_p = x_o\,\omega^2$$

It is always directed towards the center O.

The acceleration of point N will be component of acceleration a_p along the diameter DE as shown in figure.

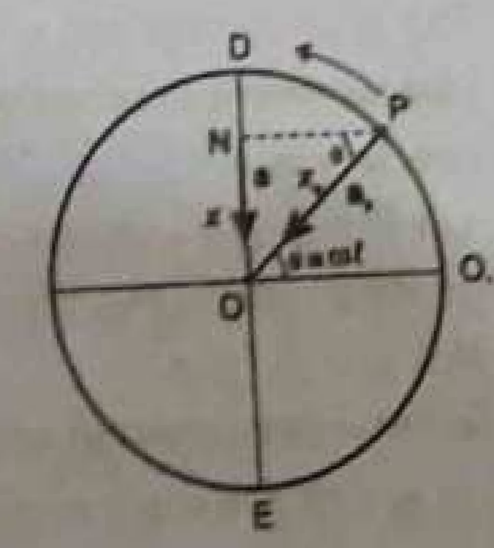


Fig. 7.5(b)

So,

$$a = a_p \sin\theta$$

OR $a = x_o\omega^2 \sin\theta$ ______ (5)

From figure

$$\sin\theta = \frac{ON}{OP}$$

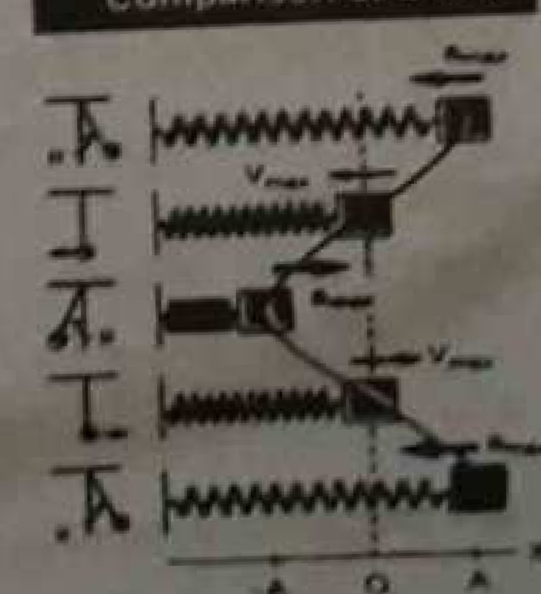
OR $\sin\theta = \frac{x}{x_o}$ ______ (6)

So equation (5) becomes

$$a = x_o\omega^2\left(\frac{x}{x_o}\right)$$

OR $a = \omega^2 x$

Comparison of SHMs



Special cases

- At mean position (i.e. x = 0) the acceleration is zero [i.e. $a_{min} = 0$]
- At extreme position (i.e. x = x_o), the acceleration is maximum [i.e. $a_{max} = \omega^2 x_o$]

The acceleration $\vec{a}$ is directed towards the mean position, so we get that direction of $\vec{a}$ is opposite to $\vec{x}$. So, above equation can be written as

$$\vec{a} = -\omega^2 \vec{x}$$

$$\vec{a} = -\text{constant}\ \vec{x} \quad [\because \omega^2 = \text{constant}]$$

OR $$\vec{a} \propto -\vec{x}$$

This equation shows that the acceleration is directly proportional to displacement and is directed towards the mean position which is the property of SHM.

So we can say that point N is performing the SHM with the same amplitude, time period and instantaneous displacement of pointer P_1.

Q.6 Define the phase angle.

Ans.

Phase

The angle ($\theta = \omega t$) which gives the displacement as well as the direction of motion of point executing SHM is known as phase.

OR

The angle ($\theta = \omega t$) which determines the state of motion of the vibrating point is called phase.

Note

This angle is obtained when SHM is related with circular motion.

Displacement in terms of phase

Special case

Let at t=0, the point P is at O_1 and N is at mean position. Then the initial phase of rotating radius OP is zero hence the displacement at t = 0 is zero. It is considered as a special case.

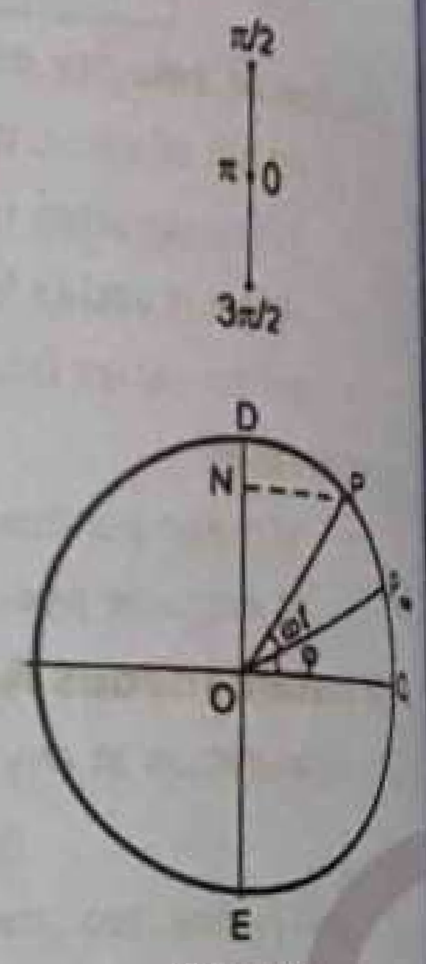


Fig 7.8(a)

General case(concept of initial phase)

Let at t=0, the angle made by rotating radius OP with the reference line $OO_1 = \phi$

After a time t, the radius rotate through angle $=\omega t$. The angle made by rotating radius OP with the reference line OO_1 at time t is $(\omega t+\phi)$

So, the displacement at time t is given by

$$x = x_o \sin(\omega t + \phi)$$

> **For Your Information**
> The wave from SHM is sine or cosine curve.

Initial Phase

Now the phase angle is

$$\theta = \omega t + \phi$$

When t = 0, $\theta = \phi$, So ϕ is called initial phase.

Now taking initial phase as 90° or $\left(\frac{\pi}{2}\right)$, then displacement is

$$x = x_o \sin(\omega t + 90^\circ)$$

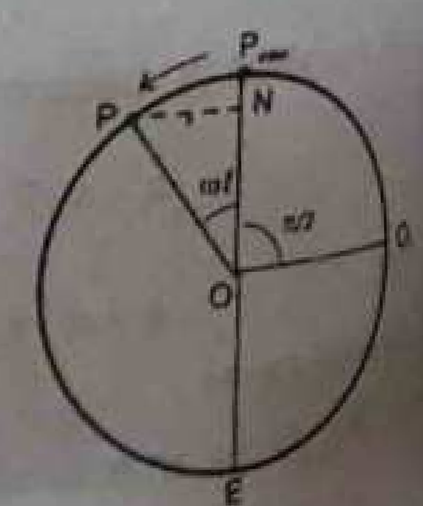


Fig 7.8(b)

OR $$\boxed{x = x_o \cos\omega t}$$

This equation gives the displacement of SHM, but in this case the point N is starting its motion form extreme position instead of the mean position, as shown in figure.

Q.7 Discuss the motion of a horizontal mass spring system and find the values of acceleration, angular velocity, time period, instantaneous displacement and instantaneous velocity.

Ans.

A Horizontal Mass Spring System

Consider a mass m is attached with a spring, while the other end of the spring is fixed with a rigid support and it is capable to oscillate on a friction less horizontal table.

When the mass m is displaced from mean position and released then its motion about mean position is SHM.

Instantaneous Acceleration

Let at displacement x the restoring force F produces the acceleration a. Then

$$\vec{F} = m\vec{a} \quad \text{———— (1)}$$

According to Hook,s law,

$$\vec{F} = -k\vec{x} \quad \text{———— (2)}$$

Comparing above two equations. We get,

$$\vec{F} = m\vec{a}$$

$$m\vec{a} = -k\vec{x}$$

$$\vec{a} = -\frac{k}{m}\vec{x} \quad \text{———— (3)}$$

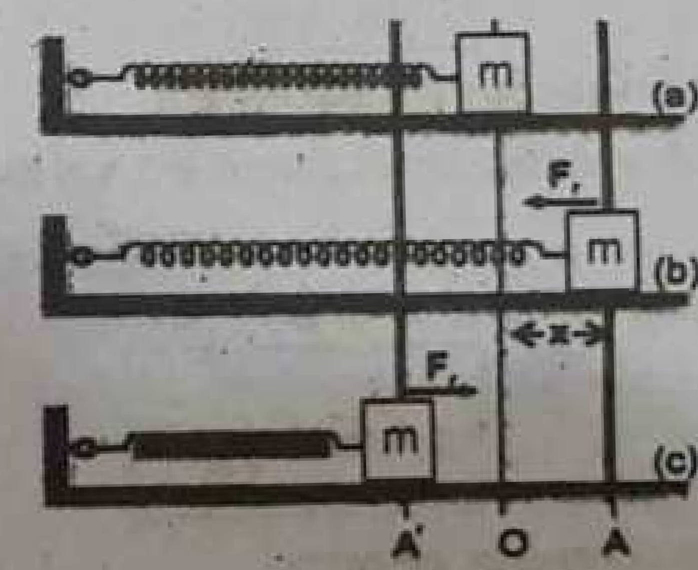


Fig. 7.2

Angular Frequency

As we know that

$$\vec{a} = -\omega^2 \vec{x} \quad \text{———— (4)}$$

Comparing equations (3) and (4), we get

$$-\omega^2 \vec{x} = -\left(\frac{k}{m}\right)\vec{x}$$

OR $$\omega^2 = \frac{k}{m}$$

OR $$\boxed{\omega = \sqrt{\frac{k}{m}}} \quad \text{———— (5)}$$

Which is the angular frequency.

Time period

As the time period of mass m having SHM can be expressed as,

$$T = \frac{2\pi}{\omega}$$

Putting values of ω, we get

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

OR $$\boxed{T = 2\pi\sqrt{\frac{m}{k}}} \quad \text{———} (6)$$

Frequency

As the reciprocal of the time period is called frequency. So,

$$f = \frac{1}{T}$$

Putting value of T, we get

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

Instantaneous Displacement

The instantaneous displacement x of mass m is

$$x = x_o \sin \omega t,$$

Putting values of ω we get

$$\boxed{x = x_o \sin\left(\sqrt{\frac{k}{m}}\right)t}$$

Instantaneous Velocity

The instantaneous velocity v of a point performing SHM is given by

$$v = \omega\sqrt{x_o^2 - x^2}$$

Putting value of ω, we get

OR $$v = \sqrt{\frac{k}{m}}\sqrt{x_o^2 - x^2}$$

$$v = \sqrt{\frac{k}{m}x_o^2\left(1 - \frac{x^2}{x_o^2}\right)}$$

$$\boxed{v = x_o\sqrt{\frac{k}{m}\left(1 - \frac{x^2}{x_o^2}\right)}} \quad \text{———} (9)$$

Maximum Velocity

Velocity of the mass is maximum at the mean position O where x = 0, so above equation becomes.

$$v_o = x_o\sqrt{\frac{k}{m}\left(1 - \frac{0}{x^2}\right)}$$

$$\boxed{v_o = x_o\sqrt{\frac{k}{m}}} \quad \text{———} (10)$$

Minimum Velocity

Velocity of vibrating mass is minimum at extreme position (i.e. at $x = x_o$)

So,
$$v = \omega\sqrt{x_o^2 - x_o^2}$$
$$v = \omega(0)$$
$$v_{min} = 0$$

Relation between maximum velocity (v_o) and instantaneous velocity (v)

Using equation (10) in equation (9), we get

$$\boxed{v = v_o\sqrt{1 - \frac{x^2}{x_o^2}}} \quad \text{———} (11)$$

This is the relation between maximum velocity and instantaneous velocity.

Q.8 What is simple pendulum? Show that the motion of pendulum is SHM. Also find relations for its time period and frequency.

Ans.

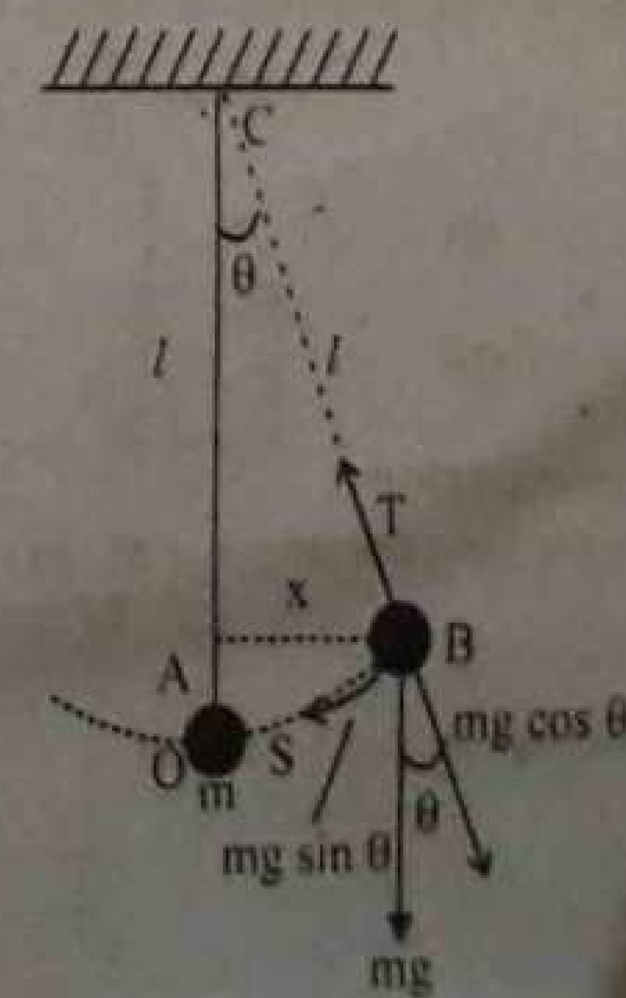
Simple Pendulum

An ideal simple pendulum consists of a small heavy mass suspended by a weightless, flexible and inextensible string fixed with a frictionless support and medium.

Practically, the above mentioned conditions are incompatible and we use the light weight and less extensible string.

Motion of simple Pendulum is SHM

Consider an object of mass m attached with the end of a light weight string.



Length of the pendulum

The length of the pendulum ℓ is the distance between the point of suspension and the center of the bob.

Working

When the pendulum is displaced from its mean position through a small angle θ and released then it starts to oscillate to and fro about mean position.

Components of weight

Resolve the weight mg into two components mg cosθ and mg sinθ.

The other force in this case is the tension T in the string.

mg cosθ and T are equal and opposite to each other. So, they cancel the effect of each other.i.e.,

$$mg \cos\theta = T$$

Restoring force

The only force responsible for motion of the pendulum is mgsin θ which brings the bob back towards its mean position. So, the restoring force for the bob is,

$$F = -mg\sin\theta \text{ ______ } (1)$$

Negative sign shows that force is directed towards mean position.

Also we know that

$$F = ma \text{ ______ } (2)$$

Comparing above two equations, we get

$$ma = -mg\sin\theta$$

OR

$$a = -g\sin\theta$$

For small value of angle θ, $\sin\theta \approx \theta$

So, $a = -g\theta \text{ ______ } (3)$

From figure $\theta = \frac{AB}{\ell}$ $\quad [\because S = r\theta \Rightarrow \theta = \frac{S}{r}]$

$\theta = \frac{x}{\ell}$ $\quad [\because \theta$ is small so arc AB $\approx x]$

So equation (3) becomes,

$$a = -g\left(\frac{x}{\ell}\right)$$

$$\boxed{a = -\left(\frac{g}{\ell}\right)x} \text{ ______ } (4)$$

$a = -\text{constant} \times x$ $\quad [\because \frac{g}{\ell} = \text{constant}]$

OR $\vec{a} \propto -\vec{x}$

This proves that the motion of pendulum is SHM.

Angular Frequency

We know that for a body having SHM,

$$a = -\omega^2 x \text{ ______ } (5)$$

Comparing equation (4) and (5), we have

$$-\omega^2 x = -\left(\frac{g}{l}\right)$$

OR $\omega^2 = \frac{g}{l}$

OR $\boxed{\omega = \sqrt{\frac{g}{\ell}}} \text{ ______ } (6)$

Time period

As the time period for SHM can be expressed as,

$$T = \frac{2\pi}{\omega}$$

Putting value of ω, we get

$$T = \frac{2\pi}{\sqrt{g/\ell}}$$

$$\boxed{T = 2\pi\sqrt{\frac{\ell}{g}}} \text{ ______ } (7)$$

Dependence of Time period

This equation shows that time period of the pendulum depends on Length of pendulum and Acceleration due to gravity

Note

Time period of the pendulum is independent of mass.

Frequency

As the reciprocal of the time period is called frequency. So,

$$f = \frac{1}{T}$$

Putting value of T, we get

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}} \text{ ______ } (8)$$

Q.9 What is second pendulum? Calculate its frequency and length.

Second Pendulum

The pendulum whose time period is 2 seconds is called second pendulum.

So $T = 2$ sec

Frequency of second Pendulum

$$f = \frac{1}{T}$$

$$f = \frac{1}{2}$$

$$f = 0.5\text{Hz}$$

Length of second Pendulum

As $T = 2\pi\sqrt{\frac{\ell}{g}}$

Or $T^2 = 4\pi^2\left(\frac{\ell}{g}\right)$

$$\ell = \frac{gT^2}{4\pi^2}$$

As T = 2 sec

$$\ell = \frac{9.8\times(2)^2}{4\times(3.14)^2} = 0.992\text{m}$$

Or $\ell = 99.2$ cm

Q.9 Prove that the (mechanical) energy is conserved in SHM.

Ans.

Energy Conversion in SHM

According to Law of Conservation of energy

"Energy can not be destroyed. It can be transformed from one kind into another kind but the total amount of energy remain constant."

Consider the case of vibrating mass-spring. When the mass m is pulled slowly, the spring is stretched by an amount x_o along a horizontal frictionless table. As it is assumed that stretching is done slowly, so that acceleration is zero, because change in velocity will be very small.

Instantaneous P.E.

Let for any instant t the mass m is at a distance x from mean position,

So according to Hook's law

$$F = kx$$

When displacement = 0

Then $F = 0$

When displacement = x

Then $F = Kx$

So average force is

$$F = \frac{0+kx}{2}$$

$$F = \frac{1}{2}(kx)$$

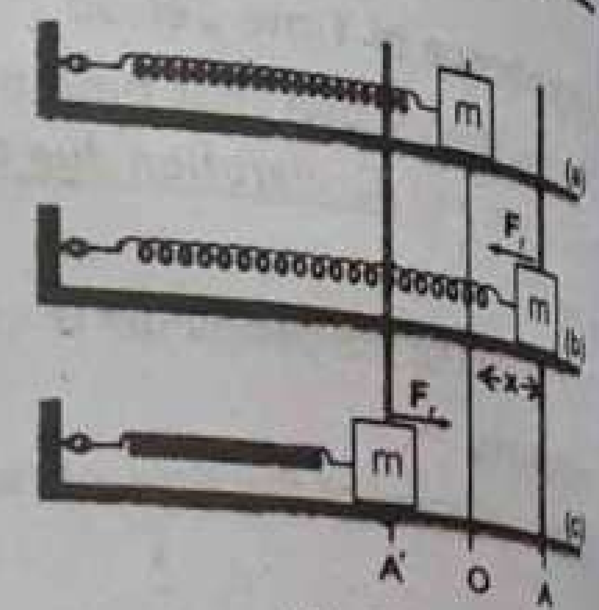


Fig. 7.2

Hence the work done in displacing the mass through displacement x is

$$W = Fd$$

$$= \frac{1}{2}(kx)(x)$$

$$= \frac{1}{2}kx^2$$

Work done appears as elastic P.E. So,

$$\boxed{(P.E)_{ins} = \frac{1}{2}kx^2}$$

Maximum P.E.

At extreme position, the P.E is maximum, as the displacement at extreme position $x = x_o$ So,

$$\boxed{(P.E)_{max} = \frac{1}{2}kx_o^2}$$

Minimum P.E.

P.E. is zero if the displacement x = 0, i.e., the mass is at mean position, thus

$$P.E. = \frac{1}{2}k(0)^2$$

$$\boxed{(P.E.)_{min} = 0}$$

Instantaneous K.E.

We know that

$$(K.E)_{ins} = \frac{1}{2}mv^2$$

Since $$v = x_o\sqrt{\frac{k}{m}\left(1-\frac{x^2}{x_o^2}\right)}$$

So, $$(K.E)_{ins} = \frac{1}{2}m\left(x_o\sqrt{\frac{k}{m}\left(1-\frac{x^2}{x_o^2}\right)}\right)^2$$

$$= \frac{1}{2}mx_o^2\left[\left(\frac{k}{m}\right)\left(1-\frac{x^2}{x_o^2}\right)\right]$$

So, $$\boxed{(K.E)_{ins} = \frac{1}{2}kx_o^2\left(1-\frac{x^2}{x_o^2}\right)}$$

Maximum K.E.

The K.E is maximum at mean position where x = 0

$$K.E = \frac{1}{2}kx_o^2\left(1-\frac{0^2}{x_o^2}\right)$$

$$\boxed{(K.E)_{max} = \frac{1}{2}kx_o^2}$$

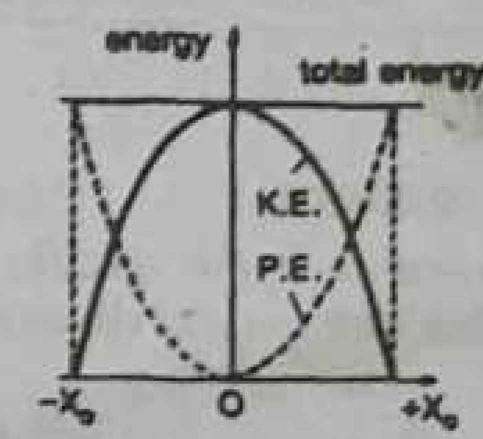
Minimum K.E.

The K.E is minimum at extreme position where $x = x_o$

$$K.E = \frac{1}{2}kx_o^2\left(1-\frac{x_o^2}{x_o^2}\right)$$

$$= \frac{1}{2}kx_o^2(1-1)$$

$$\boxed{(K.E)_{min} = 0}$$



Total Energy

At any position, the total energy is sum of partly P.E. partly and K.E.

So,

$$E = P.E + K.E$$

$$E = \frac{1}{2}kx^2 + \frac{1}{2}kx_o^2\left(1-\frac{x^2}{x_o^2}\right)$$

$$E = \frac{1}{2}kx^2 + \frac{1}{2}kx_o^2 - \frac{1}{2}kx^2$$

$$\boxed{E = \frac{1}{2}kx_o^2}$$

Thus the total energy of vibrating mass spring system always remains constant.

Note

During the oscillatory motion where the K.E is maximum, and the P.E is zero and when the P.E is maximum is K.E is zero. The change of P.E and K.E with displacement is required for maintaining the oscillation.

Thus *periodic exchange of energy is the property of all oscillatory systems.*

Q.11 What are free and forced oscillations? Also define driven harmonic oscillator.

Ans.

Free Oscillations

A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force.

For example

A simple pendulum vibrates freely with its natural frequency that depends only upon the length of the pendulum.

Forced Oscillations

A body is said to be executing forced vibrations if it oscillates with the interference of an external force.

For example

- If the mass of vibrating pendulum is struck repeatedly, then forced vibrations are produced.
- The vibrations of factory floor caused by the running of heavy machinery is another example.

Driven harmonic oscillator

The physical system undergoing forced vibrations is known as driven harmonic oscillator.

Q.12 What is resonance phenomenon? Explain it with examples?

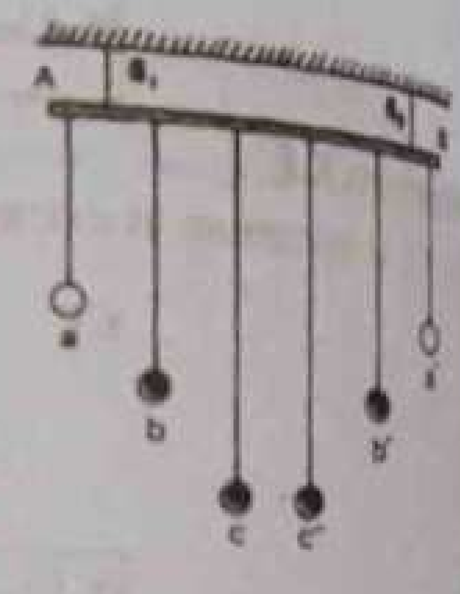


Fig 7.9

Ans.

Resonance

Resonance is the specific response of a vibrating system to a periodic force acting with natural vibrating period of system.

OR

The phenomenon in which the amplitude of a vibrating body increases when the frequency of an applied force is equal to the natural frequency of the harmonic oscillator.

Experiment to demonstrate resonance

Consider a horizontal rod AB is supported by strings S_1 and S_2. Three pairs of pendulum aa', bb' and cc' are suspended to rod 'AB'.

- The length of each pair is same but different for different pairs.
- Displace pendulum c is in a direction perpendicular to the plane of the paper
- A small force acts on all the pendulums through the rod AB.
- All the pendulums will oscillate with the pendulum c but with a slight periodic motion.
- The pendulum c', whose length and hence period is exactly the same as that of c oscillates with larger amplitude equal to c.
- The amplitude of other pendulums remains small because their natural periods are not same as that of the disturbing force due to rod AB.

Intrusting Information

The collapse of Tacoma Narrows Bridge (USA) is suspected to be due to violent resonance oscillations.

EXPLANATION
The Tacoma Narrows bridge (USA) showed gentle rolling oscillations due to resonance produced by the puffs of wind that shook the bridge at a frequency matched the one of its natural vibrational frequencies. Later the bridge develop violent torsional oscillations. Eventually the main span broke up sending the bridge road-way crashing into the water below.

Examples

Motion of swing (mechanical resonance)

- A swing is a good example of ***mechnical resonance.***
- We apply a ***periodic force*** on swing.
- When the frequency of periodic force ***becomes equal*** to the natural frequency of the swing, resonance is produced
- So energy absorption is ***maximum***.
- Hence, the amplitude of vibration is ***increased***

Collapse of suspended bridge

On a big span bridge the soldiers crossing the bridge are ordered to break their steps, if the frequency of steps coincides with natural frequency of the bridge. Then there is a chance to collapse the bridge due to resonance.

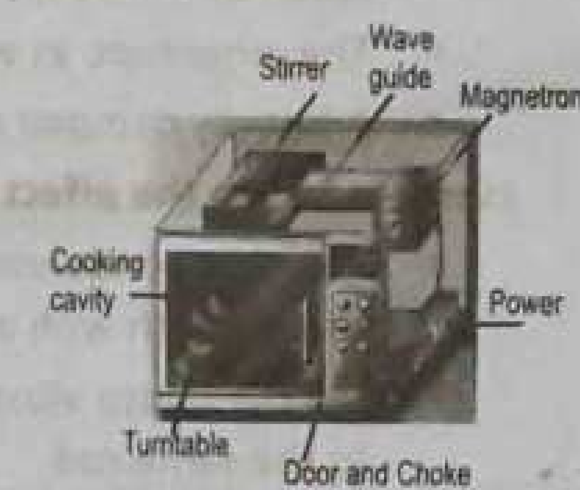
Tuning a radio (Electrical resonance)

Tuning of radio is a good example of ***electrical resonance.*** To tune a radio,

- We turn the knob of a radio.
- It ***changes*** the natural frequency of electrical circuit of receiver until it becomes equal to the frequency of transmitter.
- Now the resonance is produced and energy absorption is ***maximum***.
- Hence a station is tuned.

Cooking by microwave oven

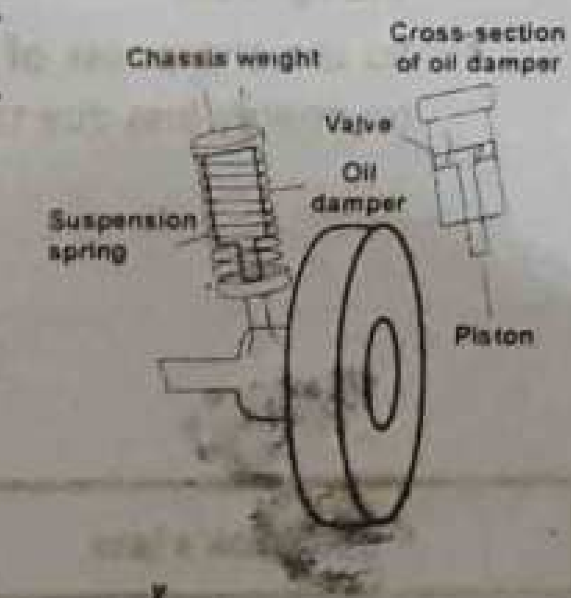
- Resonance plays an important role in heating and cooking food by microwave oven.
- The microwaves produced by oven are ***absorbed due to resonance*** by water and fats molecules in the food.
- This increases the internal energy of the molecules.
- They get ***heat up*** and so food is cooked



Note

The wave length of the microwaves produced in this type of oven is ***12 cm*** and frequency is ***2450 MHz***

Q.13 What are damped and undamped oscillations? What is damping?



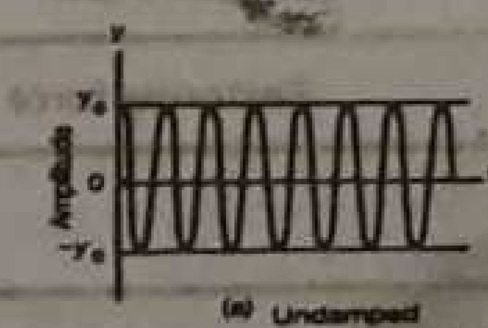
Ans.

Damped Oscillations

Oscillations in which amplitude decreases with time due to energy dissipation are called damped oscillations.

Explanation

The amplitude of the oscillating body gradually becomes smaller and smaller because of friction and air resistance. As the energy of the oscillator is used up in doing work against the resistive forces, that is why the amplitude decreases with time till it becomes zero.



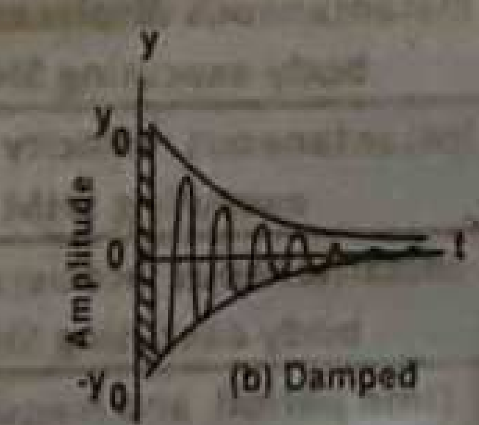
Graph between amplitude and time.

Application

An application of damped oscillation is the ***shock absorber*** of a car which provides a damping force to stop the excessive oscillations.

Damping

Damping is the process by which energy is lost by the oscillating system



Graph between amplitude and time

Undamped oscillations

Oscillations in which the amplitude remains same with time are called undamped oscillations.

In undamped oscillations energy is not dissipated from the oscillating system.

Example

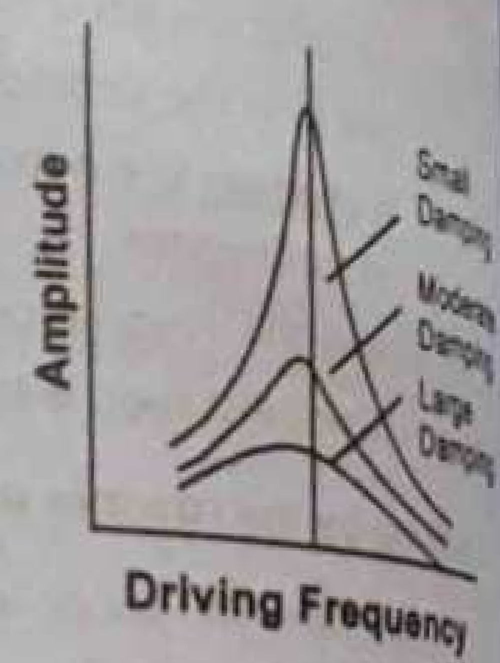
Oscillations of an ideal simple pendulum is the example of undamped oscillation.

Q.14 What is the effect of damping on the sharpness of resonance?

Ans.

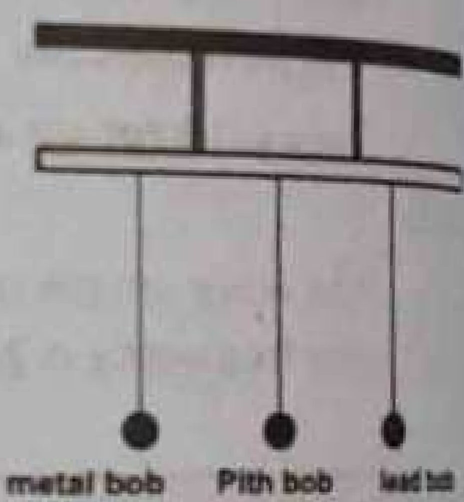
Sharpness of Resonance

- The amplitude of vibration of a body increases when the damping is small.
- Thus, the presence of damping prevents the amplitude from becoming sufficiently large.
- The amplitude decreases rapidly at a frequency slightly different from resonance frequency.
- The amplitude as well as sharpness depends upon damping.
- A heavily damped system has fairly flat resonance curve.



Example to see the effect of damping

- Attach a pendulum having very light mass such a pith ball and another of same length with a heavy mass of equal size such as lead ball.
- Set them into vibrations by third pendulum of equal length and attached to the same rod.
- It is observed that the amplitude of the heavy ball is much greater than the light ball.
- So the sharpness of the resonance curve of resonating system depends on energy loss due to friction.



FORMULAE

Hook's law	$\vec{F} = k\vec{x}$		
Restoring force	$\vec{F} = -k\vec{x}$		
	$\vec{a} = -\frac{k}{m}\vec{x}$		
Angular frequency	$\omega = \frac{2\pi}{T}$	$\omega = 2\pi f$	
Instantaneous displacement of body executing SHM	$x = x_o \sin\theta$	$x = x_o \sin\omega t$	$x = x_o \sin(\omega t+\phi)$
Instantaneous velocity of body executing SHM	$v = \omega\sqrt{x_o^2 - x^2}$		
Instantaneous acceleration of body executing SHM	$\vec{a} = -\omega^2\vec{x}$		
Time period and frequency of body executing SHM	$T = \frac{2\pi}{\omega}$	$f = \frac{\omega}{2\pi}$	

Value of ω for mass spring system	$\omega = \sqrt{\frac{k}{m}}$	
Time period of mass spring system executing SHM	$T = 2\pi\sqrt{\frac{m}{k}}$	
Frequency of mass spring system executing SHM	$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$	
Instantaneous displacement of mass spring system executing SHM	$x = x_o \sin\sqrt{\frac{k}{m}}t$	
Instantaneous velocity of mass spring system executing SHM	$v = \sqrt{\frac{k}{m}}\sqrt{x_o^2 - x^2}$	$v = x_o\sqrt{\frac{k}{m}}\sqrt{1-\frac{x^2}{x_o^2}}$
Maximum velocity of mass spring system executing SHM	$v_o = x_o\sqrt{\frac{k}{m}}$	
Instantaneous velocity of mass spring system in terms of maximum velocity executing SHM	$v = v_o\sqrt{1-\frac{x^2}{x_o^2}}$	
Restoring force for simple pendulum	$F = -mg\sin\theta$	
Acceleration of simple pendulum	$a = -\left(\frac{g}{\ell}\right)x$	
Value of ω for simple pendulum	$\omega = \sqrt{\frac{g}{\ell}}$	
Time period of simple pendulum	$T = 2\pi\sqrt{\frac{\ell}{g}}$	
Frequency of simple pendulum	$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$	
Instantaneous P.E. of mass spring system	$(P.E)_{ins} = \frac{1}{2}kx^2$	
Maximum P.E. of mass spring system	$(P.E)_{max} = \frac{1}{2}kx_o^2$	
Instantaneous K.E. of mass spring system	$(K.E)_{ins} = \frac{1}{2}m\left(x_o\sqrt{\frac{k}{m}(1-\frac{x^2}{x_o^2})}\right)^2$	
Maximum K.E. of mass spring system	$(K.E)_{max} = \frac{1}{2}kx_o^2$	
Total energy of mass spring system	$E = \frac{1}{2}kx_o^2$	

Multiple Choice Questions

- *Four possible answers to each statement are given below. Tick (✓) the correct answer:*

1. **The waveform of S.H.M is:**
   (a) Standing wave (b) Sine wave
   (c) Square wave (d) None of these
2. **Force need to produce and extantion of one metre in a spring is called:**
   (a) Stress (b) Strain
   (c) Spring constant (d) None of these
3. **When the amplitude of oscillation is doubled then energy of mass spring system becomes:**
   (a) Double (b) Four times
   (c) One half (d) Six times
4. **A simple pendulum suspended from the ceiling of a lift has time period T when the lift is at rest. When lift falls freely, the time period is:**
   (a) Infinite (b) $\frac{T}{g}$
   (c) Zero (d) $\frac{g}{T}$
5. **In SHM, at extreme position _______ is maximun:**
   (a) Velocity (b) Accerelation
   (c) Kinetic energy (d) All of these
6. **The working of ______ is not based upon the principle of resonance:**
   (a) T.V. (b) Radio
   (c) Microwave oven (d) Bulb
7. **Natural frequency of simple pendulum varies inversely:**
   (a) Its mass (b) Length
   (c) Square of length (d) Square root of length
8. **Total distance travelled by bob of simple pendulum in one vibration is:**
   (a) Amplitude (b) Square of amplitude
   (c) 2 times of amplitude (d) 4 times of amplitude
9. **When K.E. of SHM is maximum, its:**
   (a) P.E. is zero (b) Acceleration is zero
   (c) Restoring force is zero (d) All are zero
10. **In damped harmonic oscillation, which one of them decreases?**
    (a) Amplitude of vibration (b) Energy of vibration
    (c) Both amplitude and energy (d) Neither amplitude nor energy

11. **The distance covered by a body in one complete vibration is 20 cm. What is the amplitude of the body:**
    (a) 10 cm (b) 5 cm
    (c) 15 cm (d) 7.5 cm
12. **What will be total energy of a mass attached to a spring with spring constant 2 N/m when displaced through 2 m will be:**
    (a) 2 J (b) 8 J
    (c) 1 J (d) 4 J
13. **If the length of second pendulum is L, then length of pendulum having a period 1 sec will be:**
    (a) L / 2 (b) 2L
    (c) 4L (d) L / 4
14. **A spring of spring constant K is cut into two halves. Then the spring constant of each part will be:**
    (a) K (b) K / 2
    (c) 4K (d) 2K
15. **The time period of the hour hand of a watch is:**
    (a) 24 hr (b) 12 hr
    (c) 1 hr (d) 1 min
16. **Which of the following quantities is doubled on doubling the amplitude of a harmonic oscillator?**
    (a) Total energy (b) Kinetic energy
    (c) Maximum velocity (d) P.E.
17. **Which of the following characteristics must remain constant for undamped oscillations of the particle?**
    (a) Acceleration (b) Phase
    (c) Amplitude (d) Velocity
18. **Time period of the simple pendulum at Karachi and at Murree are related as:**
    (a) $T_K > T_M$ (b) $T_K < T_M$
    (c) $T_K = T_M$ (d) $2T_K = 3T_M$
19. **A simple pendulum is oscillating in a lift. If the lift starts moving upwards with uniform acceleration, the period will:**
    (a) Remain same (b) Be shorter
    (c) Be longer (d) Can't say anything
20. **In order to double the period of a simple pendulum:**
    (a) Its length should be doubled (b) Its length should be quadrupled
    (c) The mass of its bob should be doubled (d) The mass of its bob should be quadrupled

ANSWERS

1. b	2. c	3. b	4. a	5. b	6. d	7. d	8. d	9. d	10. c
11. b	12. d	13. d	14. d	15. b	16. c	17. c	18. b	19. b	20. b

Short Questions of Exercise

Q.7.1 Name two characteristics of simple harmonic motion

(D.G.Khan 2005, Fsd 2005, Mtn 2006, Bwp 2007-2008, Lhr 2010-2011, Grw 2011)

Ans. Characteristics of SHM

- *Restoring force* is directly proportional to displacement from mean position.
- *Acceleration* is directly proportional to displacement from mean position and is directed towards the mean position.
- *Total energy* of system is conserved in SHM.

Q.7.2 Does frequency depends on amplitude for harmonic oscillators?

(Mtn 2005-2009, D.G.Khan 2005, Lhr 2009)

Ans. No, it does not depend upon amplitude of harmonic oscillator.

Reason

In case of simple pendulum is

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

In case of mass spring system

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

These equations show that frequency of simple harmonic oscillator is ***independent*** of amplitude.

Q.7.3 Can we realize an ideal simple pendulum?

(Rwp 2005, Mir Pur 2006, Bwp 2006, Lhr 2008, Grw 2009-2010-2011)

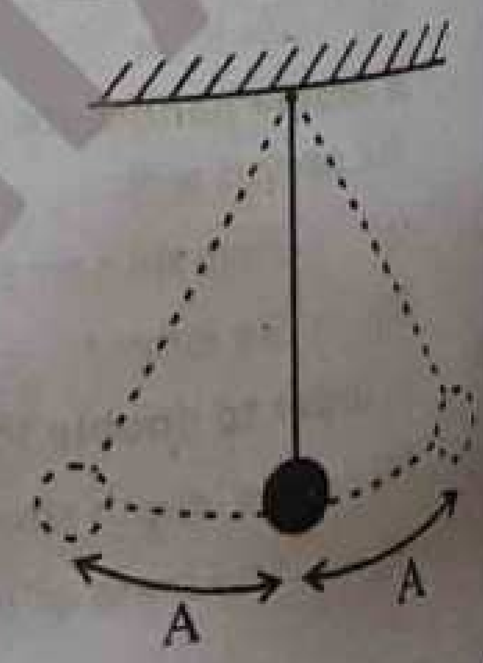
Ans. No, we can not realize an ideal simple pendulum.

Reason

An ideal simple pendulum consists of ***point mass*** suspended by ***massless*** and ***inextensible*** string. In practice, it is not possible.

Q.7.4 What is the total distance traveled by an object moving with SHM in a time equal to its period, if its amplitude is A?

(Fsd 2006, Bwp 2007, Rwp 2008, Mir Pur 2009)



Ans. The total distance covered by the body is ***4A***.

Explanation

Time period is the time during which the vibrating body completes one round trip. In one round trip

Total distance covered = A+A+A+A = 4A

Q.7.5 What happens to the time period of the simple pendulum if its length is doubled? What happens if the suspended mass is doubled?

(Lhr2008,Bwp2007, Rwp 2008, Mir Pur 2009, Lhr 2010,Grw 2010)

Ans. The time period of a simple pendulum is,

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

(i) Effect of doubling the length

When length becomes double, the time period increases $\sqrt{2}$ ***times.***

Explanation

If $\ell' = 2\ell$

$$T' = 2\pi\sqrt{\frac{2\ell}{g}}$$

$$T' = \sqrt{2}\left(2\pi\sqrt{\frac{\ell}{g}}\right)$$

$$T' = \sqrt{2}\,T$$

(ii) Effect of doubling the mass

When mass become doubled the time period remains ***same***

Explanation

As time period of simple pendulum is ***independent of mass***. So it does not change with mass.

Q.7.6 Does the acceleration of a simple harmonic oscillator remain constant during its motion? Is the acceleration ever zero? Explain?

(Federal 2004, Grw 2005, Lhr 2010-2011)

Ans. No, it does not remain constant.

Reason

The acceleration of the body executing SHM is

$$\vec{a} = -\omega^2\vec{x} \quad \text{\_\_\_\_\_\_ (1)}$$

$$\Rightarrow \quad \vec{a} \propto -\vec{x}$$

This shows that acceleration varies directly with ***displacement***

Zero Acceleration

Above equation shows that acceleration is zero at ***mean position*** where the value of displacement is zero (i.e. $x = 0$).

Q.7.7 What is meant by phase angle? Does it define the angle between maximum displacement and the driving force?

(Sgd 2005, Bwp 2008, Grw 2008-2009, D.G.Khan 2006)

Ans. (a) Phase Angle

It is the angle which gives the ***displacement*** as well as ***direction*** of motion of point executing SHM. It determines the ***state of motion*** of vibrating body.

(b)

No, it is actually the angle which the rotating vector makes with reference line.

Q.7.8 Under what conditions does the addition of two simple harmonic motions produce a resultant which is also simple harmonic? (Lhr 2005)

Ans. The addition of two simple harmonic motions produce a resultant, which is also simple harmonic when,

Conditions

The two simple harmonic motions have;

1) *same* frequency
2) *Same nature* (i.e., mechanical waves cannot be super posed with electromagnetic waves)
3) *constant* phase difference

Q.7.9 Show that in SHM the acceleration is zero when the velocity is greatest and the velocity is zero when the acceleration is greatest? (Lhr 2008, Mir Pur 2009)

Ans. **(a) Acceleration is zero when the velocity is greatest**

In simple harmonic motion, the instantaneous velocity and acceleration can be expressed as

$V = \omega\sqrt{x_o^2 - x^2}$	$\alpha = \omega^2 x$
At mean position (i.e. at x = 0)	
$V = \omega\sqrt{x_o^2 - x^2}$ $V = \omega x_o$ (max)	$\alpha = \omega^2(o)$ $\alpha = 0$ (min)
Thus at mean position acceleration is zero when velocity in is greatest.	
At extreme position (i.e. at $x = x_o$)	
$V = \omega\sqrt{x_o^2 - x_o^2}$ $V = \omega(o)$ $V = 0$ (min)	$\alpha = \omega^2 x_o^2$ (max)
Thus at extreme position, velocity is zero when acceleration is greatest.	

So in SHM when $\alpha = 0$, v is maximum

And when $v = 0$, α is maximum

Q.7.10 In relation to SHM, explain the equations;

(i) $y = A \sin(\omega t + \phi)$ (ii) $a = -\omega^2 x$ (Federal 2005)

Ans.

(i) $y = A \sin(\omega t + \phi)$

- Wave form of SHM is sinusoidal
- y = instantaneous displacement
- A = amplitude
- ϕ = initial phase angle
- $\omega t + \phi$ = phase angle

(ii) $a = -\omega^2 x$

- Acceleration of SHM depends upon directly proportional displacement and directed towards mean position
- a = instantaneous acceleration
- x = instantaneous displacement
- ω^2 = angular frequency

Q.7.11 Explain the relation between total energy, potential energy and kinetic energy for a body oscillating with SHM. (Federal 2005)

Ans: The total energy for a body oscillating with SHM always remains *constant*.

Explanation

- At extreme position, the whole energy is in form of P.E.
- At mean position, the whole energy is in form of K.E.
- At any point between mean and extreme position, total energy of simple harmonic oscillator in sum of P.E. and K.E.

Q.7.12 Describe some common phenomena in which the resonance plays an important role? (Grw 2005-2009)

Ans.

Tuning of radio (Electrical resonance)

Tuning of radio is a good example of *electrical resonance.* We turn the knob of a radio. It *changes* the natural frequency of electrical circuit of receiver until it becomes equal to the frequency of transmitter. So resonance is produced and energy absorption is ***maximum***. Hence a station is tuned.

Motion of the swing (Mechanical resonance)

A swing is a good example of *mechanical resonance.* We apply a ***periodic force*** on swing. When the frequency of periodic force ***becomes equal*** to the natural frequency of the swing, resonance is produced. So energy absorption is ***maximum***. Hence, the amplitude of vibration is ***increased***

Q.7.13 If a mass spring system is hung vertically and set into oscillation, why does motion eventually stop? (Mir Pur 2004-2005, Lhr 2006, Mtn 2009, Lhr 2009)

Ans. It eventually stops due to *damping.*

Reason

When a mass spring system vibrates, it gradually loses its energy in doing *work against frictional forces.* So amplitude of vibration becomes smaller and smaller and hence the motion eventually stops.

Solved Examples

Example 7.1

A block weighing 4.0 kg extends a spring by 0.16m from its unstretched position. The block is removed and a 0.50 kg body is hung from the same spring. If the spring is now stretched and then released, what is its period of vibration?

Given Data:

Mass of the block = m_1 = 4kg

Length of the stretched spring = x = 0.16 m

Mass of the body = m_2 = 0.5 kg

To Find:

Period of vibration = T = ?

Calculation:

The formula for time period is

$$T = 2\pi\sqrt{\frac{m_2}{k}} \quad ......(1)$$

To find value of k, we use the Hook's law

$$F = kx$$

or $k = \dfrac{F}{x}$

or $k = \dfrac{m_1 g}{x}$ As $F = m_1 g$

$$k = \frac{4 \times 9.8}{0.16}$$

$$k = 245\ Nm^{-1}$$

Putting values in equ. (1)

$$T = 2 \times 3.14 \sqrt{\frac{0.5}{245}}$$

$$T = 6.28 \times 0.045$$

$$\boxed{T = 0.28\ s}$$

Example 7.2

What should be the length of a simple pendulum whose period is 1.0 second at a place where $g = 9.8\ ms^{-2}$? What is the frequency of such a pendulum?

Given Data:

Time period = T = 1.0 second

Acceleration due to gravity = $g = 9.8\ ms^{-2}$

To Find:

Length of simple pendulum = ℓ = ?

Frequency of the pendulum = f = ?

Calculation:

Using the formula

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Squaring both sides

$$T^2 = 4\pi^2\left(\frac{\ell}{g}\right)$$

or $\ell = \dfrac{gT^2}{4\pi^2}$

Putting values, we get

$$\ell = \frac{9.8 \times (1.0)^2}{4 \times (3.14)^2}$$

$$\ell = \frac{9.8 \times 1}{4 \times 9.85}$$

$$\ell = \frac{9.8}{39.4}$$

$$\ell = 0.25\text{m}$$

Now frequency

$$f = \frac{1}{T}$$

$$f = \frac{1}{1.0}$$

$$\boxed{f = 1.0\ \text{Hz}}$$

Example 7.3

A spring, whose spring constant is $80.0\ Nm^{-1}$ vertically supports a mass of 1.0 kg in the rest position. Find the distance by which the mass must be pulled down, so that on being released, it may pass the mean position with a velocity of $1.0\ ms^{-1}$.

Given Data:

Spring constant = k = $80.0\ Nm^{-1}$

Mass = m = 1.0 kg

Velocity of mass = v = $1.0\ ms^{-1}$

To Find:

Distance by which mass is pulled = amplitude = x_o

Calculation:

As $v = x_o \omega$

Thus $v = x_o\sqrt{\dfrac{k}{m}}$ As $\omega = \sqrt{\dfrac{k}{m}}$

or $x_o = v\sqrt{\dfrac{m}{k}}$

Putting values, we get

$$x_o = 1.0\sqrt{\frac{1.0}{80}}$$

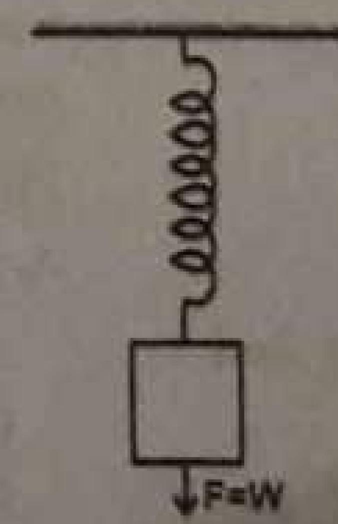
$$x_o = 1.0 \times \sqrt{0.0125}$$

$$x_o = 1.0 \times 0.11$$

$$\boxed{x_o = 0.11\ \text{m}}$$

Exercise Problems

7.1 **A 100.0g body hung on spring elongates the spring by 4.0cm.when a certain object is hung on the spring and set vibrating, its period is 0.568s. what is the mass of the object pulling the spring?**



Given data:

Mass of the body = $m = 100g = \dfrac{100}{1000} = 0.1kg$

Extension produced in the spring $= x = 4.0cm = \dfrac{4}{100} = 0.04m$

Time period = T = 0.568sec (when mass m' is suspended)

To find:

Mass of the object = m' = ?

Calculation:

According to Hook's law

$F = kx$, also $F = W = mg$

Thus $kx = mg$

or $k = \frac{mg}{x}$

putting values, we get

$$k = \frac{0.1 \times 9.8}{0.04}$$

$$k = 24.5\ Nm^{-1}$$

As the time period of mass attached spring is

$$T = 2\pi\sqrt{\frac{m'}{k}},$$

Squaring on both sides

$$T^2 = 4\pi^2\left(\frac{m'}{k}\right)$$

Or $m' = \frac{kT^2}{4\pi^2}$

Putting values, we get

$$m' = \frac{24.5 \times (0.568)^2}{4 \times (3.14)^2}$$

$$m' = 0.200\ kg$$

Or $\boxed{m' = 200\ gm}$

7.2 A load of 15.0g elongates a spring by 2.00cm. If body of mass 294 g is attached to the spring and is set into vibration with an amplitude of 10.0 cm, what will be its (i)period (ii) spring constant (iii)maximum speed of its vibration.

Given data:

Load = m = 15.0gm = 0.015kg

Extension produced = x = 2.0cm = 0.02m

Mass attached to the spring = m' = 294gm = 0.294kg

Amplitude = x_0 = 10.0cm = 0.10m

To find:

(i) time period = T = ?

(ii) spring constant = k = ?

(iii) maximum speed = v_o = ?

Calculation:

(ii) According to Hook.s law

$F = kx$ also $F = w = mg$

Thus $kx = mg$

Or $k = \frac{mg}{x}$

Putting values, we get

$$k = \frac{0.015 \times 9.8}{0.02}$$

Or $\boxed{k = 7.35\ Nm^{-1}}$

(i) when m' is attached to the spring ,then

$$T = 2\pi\sqrt{\frac{m'}{k}}$$

Putting values, we get

$$T = 2 \times 3.14\sqrt{\frac{0.294}{7.35}}$$

$$T = 6.28 \times 0.2$$

$$\boxed{T = 1.26 sec}$$

(ii) Maximum speed = $v_o = x_o\sqrt{\frac{k}{m'}}$

Putting values, we get

$$v_o = 0.1 \times \sqrt{\frac{7.35}{0.294}}$$

$$v_o = 0.5 m/s$$

$$\boxed{v_o = 50\ cm/s}$$

7.3 An 8.0kg body executes SHM with amplitude 30cm. The restoring force is 60N, When the displacement is 30cm. Find

(i) period

(ii) Acceleration , speed , kinetic energy and potential energy when the displacement is 12 cm.

Given data:

Mass of body = m = 8.0kg

Amplitude = x_o = 30cm = 0.30m

Restoring force = F = 60N

Displacement = x = 30cm = 0.30m

To find:

(i) Period = T = ?

(ii) Acceleration = a = ?

(iii) Speed = v = ?

(iv) K.E = ?

(v) P.E = ?

When the displacement, x = 12cm = 0.12 m

Calculation:

(i) Time period

According to Hooks law, $F = kx_o$

Or $k = \frac{F}{x_o}$

Or $k = \frac{60}{0.30}$

$$k = 200 Nm^{-1}$$

Now using the formula for time period of mass spring system

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Putting values, we get

$$T = 2\times3.14\sqrt{\frac{8}{200}}$$

$$T = 6.28\times\sqrt{0.04}$$

$$T = 6.28\times0.2$$

$$T = 1.256\text{sec}$$

Or $\boxed{T = 1.3\text{second}}$

(ii) Acceleration:

$$a = -\omega^2 x \qquad \text{As } \omega^2 = \frac{k}{m}$$

Or $$a = -\left(\frac{k}{m}\right)x$$

Putting values, we get

$$a = -\frac{200}{8}\times0.12$$

$$\boxed{a = -3.0m/\sec^2}$$

Negative sign shows that acceleration in directed towards the mean position.

(iii) Speed:

The speed of the body executing SHM is given by

$$v = \omega\sqrt{x_o^2 - x^2}$$

Thus, $$v = \sqrt{\frac{k}{m}}\sqrt{x_o^2 - x^2} \qquad \text{As} \quad \left(\omega = \sqrt{\frac{k}{m}}\right)$$

Putting values, we get

$$v = \sqrt{\frac{200}{8}}\sqrt{(0.3)^2 - (0.12)^2}$$

$$v = 5\sqrt{0.09 - 0.0144}$$

$$v = 5\sqrt{0.0756}$$

$$v = 1.37\text{m/sec}$$

$$\boxed{v = 1.4\text{m/sec (approx)}}$$

(iv) K.E:

Using the formula, for calculating K.E.

$$K.E. = \frac{1}{2}mv^2$$

Putting values, we get

$$K.E = \frac{1}{2}\times8\times(1.37)^2$$

$$K.E = 4\times1.89$$

$$K.E = 7.56\text{ J}$$

Or $\boxed{K.E = 7.6\text{ J}}$

(v) P.E:

The formula for P.E. is

$$P.E = \frac{1}{2}kx^2$$

$$P.E = \frac{1}{2}\times200\times(0.12)^2$$

$$P.E = 100\times0.0144$$

$$\boxed{P.E = 1.44\text{ J}}$$

7.4 A block of mass 4.0kg is dropped from a height of 0.80 m on to a spring of spring constant $k = 1960\ Nm^{-1}$. Find the maximum distance through which the spring will be compressed.

Given data:

Mass of the block = m = 4kg

Height = h = 0.80m

Spring constant = k = 1960 Nm^{-1}

To find:

Maximum distance = x_o = ?

Calculation:

As $P.E = mgh$

$P.E = 4.0\times9.8\times0.80$

$P.E = 31.36\text{joules}$

When the block is dropped on spring, the spring will be compressed through Maximum distance x_o, so that gravitational P.E will be converted into elastic P.E.

So gravitational P.E = elastic P.E

$$mgh = \frac{1}{2}kx_o^2$$

Putting values, we get

$$31.36 = \frac{1}{2}\times1960\times x_o^2$$

Or $$31.36 = 980\times x_o^2$$

Or $$x_o^2 = \frac{31.36}{980}$$

Or $$x_o^2 = 0.32$$

Or $$\boxed{x_o = 0.18m}$$

7.5 A simple pendulum is 50.0 cm long. What will be its frequency of vibration at a place where $g = 9.8\ ms^{-2}$?

Given data:

Length of simple pendulum = ℓ = 50.0cm = 0.50m

Acceleration due to gravity = g = $9.8 m/s^2$

To find:

Frequency of simple pendulum = f = ?

Calculation:

For simple pendulum, formula for frequency is

$$f=\frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

putting values, we get

$$f=\frac{1}{2\times3.14}\sqrt{\frac{9.8}{0.50}}$$

$$f=\frac{1}{6.28}\sqrt{19.6}$$

$$f=\frac{1}{6.28}\times4.427$$

$$\boxed{f=0.70\text{Hz}}$$

7.6 **A block of mass 1.6 kg is attached to a spring with spring constant 1000 Nm^{-1}, as shown in Fig. 7.14 The spring is compressed through a distance of 2.0cm and the block is released from rest. Calculate the velocity of the block as it passes through the equilibrium position, x=0 if the surface is frictionless.**

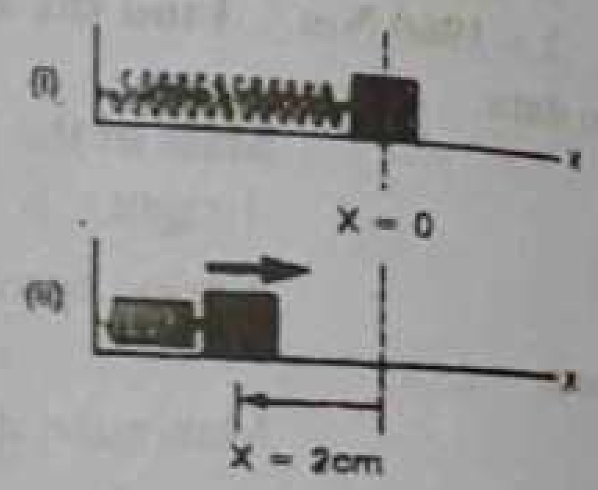


Fig. 7.14

Given data:

Mass of block = m= 1.6 kg

Spring constant=k= 1000 Nm^{-1}

Maximum displacement = x_o = 2cm =0.02m

To find:

Velocity at mean position = v_o =?

Calculation:

As the velocity is maximum at mean position, so

$$v_o=x_o\sqrt{\frac{k}{m}}$$

Putting the values, we get

$$v_o=0.02\sqrt{\frac{1000}{1.6}}$$

$$v_o=0.02\times\sqrt{625}$$

$$v_o=0.02\times25$$

$$\boxed{v_o=0.50\text{m/s}}$$

7.7 **A car of mass 1300kg is constructed using a frame supported by four springs. Each spring has a spring constant 20,000 Nm^{-1}. If two people riding in the car have a combined mass of 160kg, find the frequency of vibration of the car, when it is driven over a pot hole in the road. Assume the weight is evenly distributed.**

Given data:

Mass of the car = m_1 = 1300 kg

Mass of two people $= m_2 = 160\,kg$

Total mass = m = $m_1 + m_2$

$m = 1300 + 160 = 1460kg$

Spring constant of each spring = k'=20,000Nm^{-1}

Combined spring constant =k=4k'=4×20,000=80000Nm^{-1} (because they are connected in parallel)

To find:

Frequency of vibration = f = ?

Calculation:

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Putting values, we get

$$f=\frac{1}{2\times3.14}\sqrt{\frac{80000}{1460}}$$

$$f=\frac{1}{6.28}\sqrt{54.794}$$

$$f=\frac{1}{6.28}\times7.40$$

$$\boxed{f=1.18\text{Hz}}$$

7.8 **Find the amplitude frequency and period of an object vibration at the end of a spring, if the equation for its position, as a function of time, is**

$$x=0.25\cos\left(\frac{\pi}{8}\right)t$$

what is the displacement of the object after 2.0 s?

Given data:

$$x=0.25\cos\left(\frac{\pi}{8}\right)t$$

Time = t =2sec

Amplitude = x_o = ?

To Find:

Frequency = f = ?

Period = T = ?

Displacement = x = ?

Calculation:

Amplitude:

As given displacement is

$$x=0.25\cos\left(\frac{\pi}{8}\right)t \quad ....(1)$$

And general equation for displacement is

$x = x_o \cos \omega t$ ....(2)

Comparing equation (1)and(2),we get

$$\boxed{x_o=0.25\text{m}}$$

From above equation, angular frequency is

requency:

$$\omega = \frac{\pi}{8}$$

Or $2\pi f = \frac{\pi}{8}$ (as $\omega = 2\pi f$)

$$\boxed{f = \frac{1}{16}\ Hz}$$

ime period:

Now $T = \frac{1}{f}$

$$T = \frac{1}{1/16}$$

$$\boxed{T = 16\ sec}$$

ow calculation of displacement when t =2sec
utting value in equation (1)

$$x = 0.25\cos(\frac{\pi}{8}) \times 2$$

Or $x = 0.25\cos(\frac{\pi}{4})$

Or $x = 0.25\cos 45^{\circ}$

Or $x = 0.25 \times 0.707$

Or $x = 0.1768$

Hence $\boxed{x = 0.18m}$

....

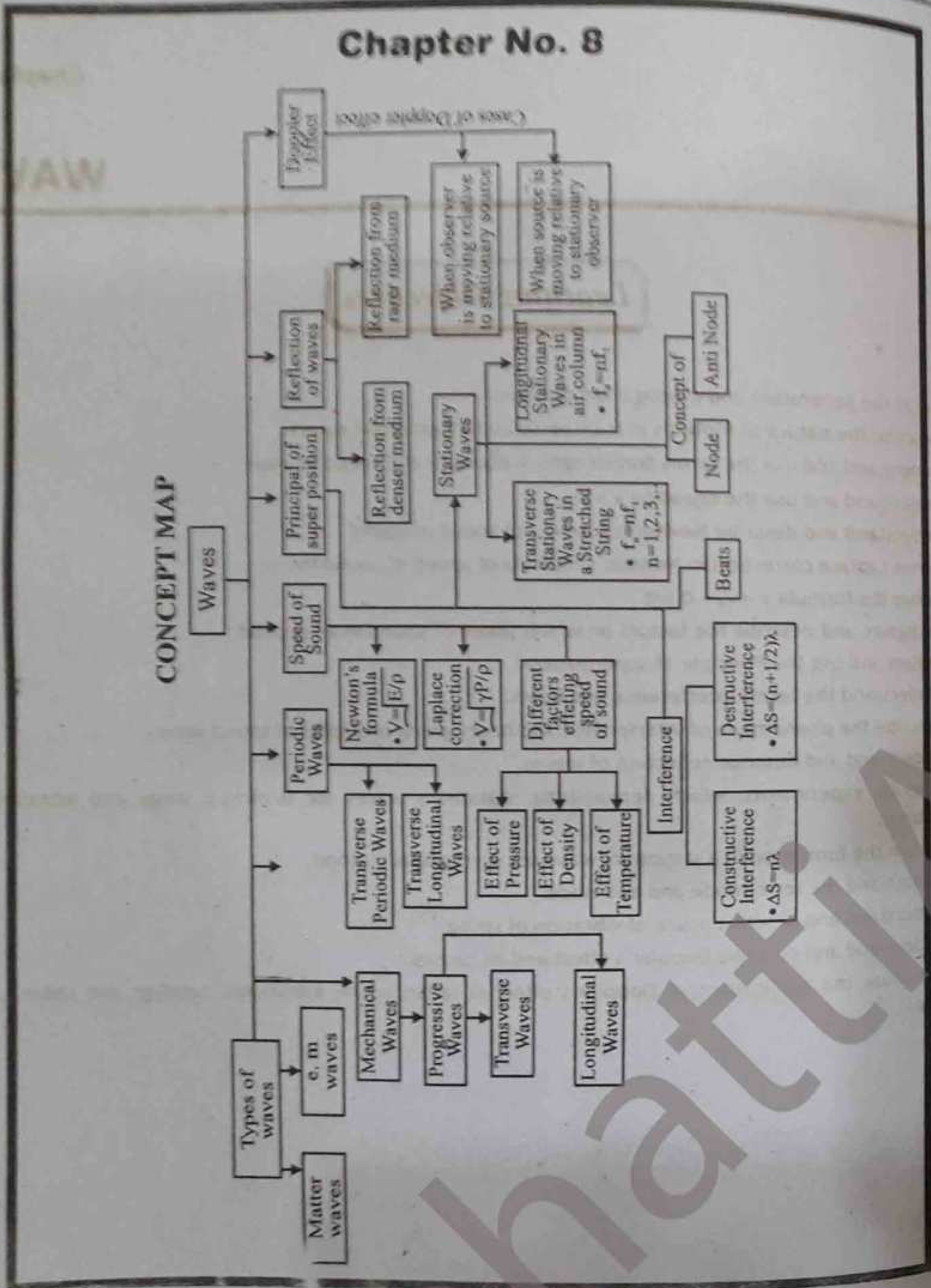
Chapter 8

WAVES

Learning Objectives

1. Recall the generation and propagation of waves.
2. Describe the nature of motions in transverse and longitudinal waves.
3. Understand and use the terms wavelength, frequency and speed of wave.
4. Understand and use the equation $v = f\lambda$.
5. Understand and describe Newton's formula of speed of sound.
6. Derive Laplace correction in Newton's formula of speed of sound for air.
7. Derive the formula $v = v_o + 0.61t$.
8. Recognize and describe the factors on which speed of sound in air depends.
9. Explain and use the Principle of superposition.
10. Understand the terms interference and beats.
11. Describe the phenomena of interference and beats giving examples of sound waves.
12. Understand and describe reflection of waves.
13. Describe experiments, which demonstrate stationary waves for stretched stings and vibrating air columns.
14. Explain the formation of a stationary wave using graphical method.
15. Understand the terms node and anti-node.
16. Understand and describe mode of vibration of string.
17. Understand and describe Doppler's effect and its causes.
18. Recognize the applications of Doppler's effect in radar, sonar, astronomy, satellite and radar speed traps.

Chapter No. 8



Waves

A wave is the mechanism by which energy is transferred from one place to another. In mechanical waves transport energy without transporting matter.

The nature of wave may be different, but the mechanism by which it transports energy is the same.

Types of Waves

Waves are of three types.

1) Mechanical waves

*The waves which need a **material medium** for their propagation are called mechanical waves.*

OR

*The waves which propagate by the **oscillation of material particles** are called mechanical waves*

For example

Water waves, sound waves, string waves etc.

2) Electromagnetic waves

*The waves which are produced due to **oscillating electric and magnetic fields** and they require **no medium** for their propagation are called electromagnetic waves.*

These waves propagate due to oscillations of electric and magnetic field.

For example

Radio waves, light waves, micro waves, x-rays etc.

3) Matter waves

The waves, in which the energy transfers in the form of kinetic energy of very fast moving particles, are called mater waves or de Broglie waves. These waves are associated with particles in motion.

For example

Wave associated with the motion of electron.

> **Do You Know?**
>
> Ultrasonic waves are particularly useful for undersea communication and detection systems. High frequency radio waves, used in radars travel just a few centimeters in water, whereas highly directional beams of ultrasonic waves can be made to travel many kilometers.
>
> **EXPLANATION:**
>
> Radio waves are electromagnetic waves and ultrasonic waves are mechanical waves. The mechanical waves (ultrasonic waves) can produce disturbance in matter and therefore, can travel for long distance through matter (water) without any loss in energy. The electromagnetic waves travel through matter for very short distance and lose their energy because they are absorbed.

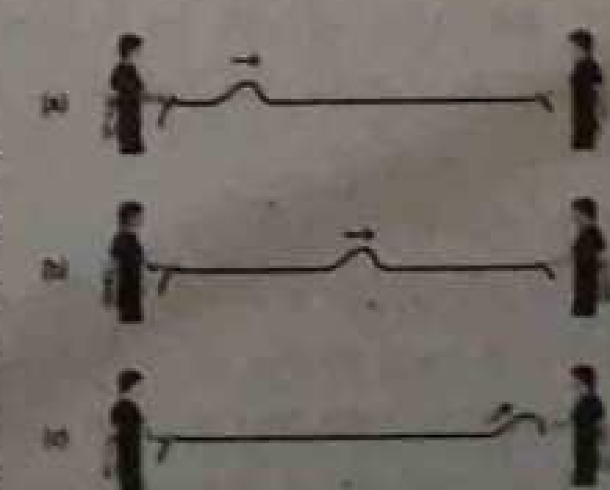
Q.1 What are progressive waves? Give its types.

Ans.

Progressive Waves

The waves which transfer energy by moving away from the source of disturbance are called progressive or traveling waves.

Example



Consider two persons holding the opposite ends of the rope. Suddenly one person gives a jerk to the rope. The disturbance in the rope produces a pulse which moves toward other person. When this reaches the other person it pushes his hand upward. So the ***energy and momentum*** transferred from one person towards the other person. This is an example of progressive wave.

The jerking of hand is its source and rope is the medium for the propagation of the waves.

Kinds of waves

... re two kinds of progressive waves

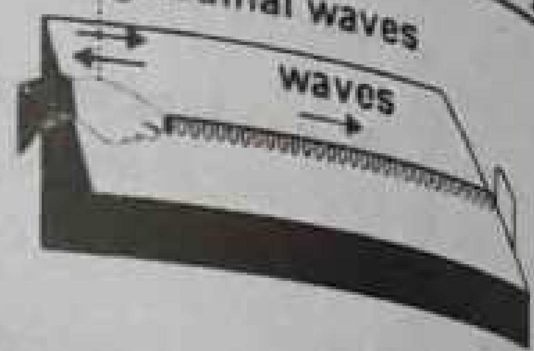
(i) Transverse waves (ii) Longitudinal waves

Transverse Waves

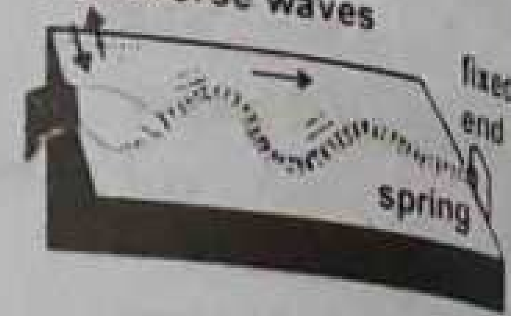
The waves, in which particles of the medium are displaced perpendicular to the direction of propagation of waves are called as transverse waves.

Take a loose spring coil (slinky spring) for illustration of motion of source in generating waves in a medium. *Slinky is the soft spring which has small initial length but relatively large extended figure.* Consider a horizontal spring system with its one end fixed. When the free end is moved from side to side, a pulse of wave having a displacement pattern as shown in figure, which will move along the spring. This shows that displacement of particles is perpendicular to the direction of propagation of wave, hence transverse waves are produced.

Longitudinal waves



Transverse waves

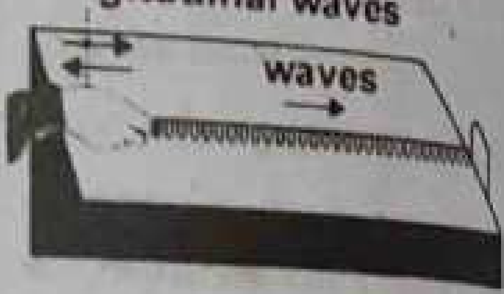


Longitudinal Waves (Compressional waves)

The waves, in which particles of the medium are displaced along the direction of propagation of the waves, are called as longitudinal waves.

If one end of the spring is moved back and forth, along the direction of the spring. Then the waves are produced in which displacement of the spring is along to the direction of propagation of wave and such waves are called longitudinal waves, as shown in figure.

Longitudinal waves



Note

(Why, sound waves in air are longitudinal in nature?)

Both types of waves can be set up in solids. In fluids, however, transverse waves die out very quickly and usually cannot be produced at all. That is why; sound waves in air are longitudinal in nature.

Q.2 What are periodic waves? Also discuss its different types?

Ans

Periodic Waves

The waves which are produced by continuous and rhythmic disturbances in a medium are called periodic waves.

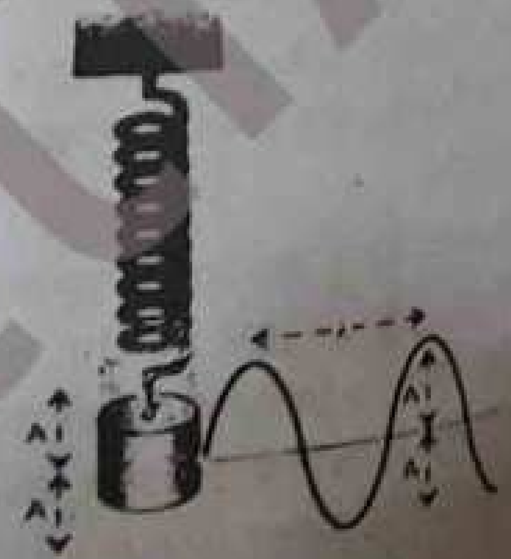
These may be transverse or longitudinal in nature. A good example of periodic waves is an oscillating mass-spring system which executes SHM.

Transverse Periodic Waves

The periodic waves in which the displacement of particles of medium is perpendicular to the direction of motion of waves are called transverse periodic waves.

Experiment

Let us consider a mass-spring system which can vibrate vertically as shown in figure. A long string of uniform thickness is stretched horizontally and its one end is attached with the oscillating mass m. Due to oscillation of mass-spring system a transverse wave is produced in the string.



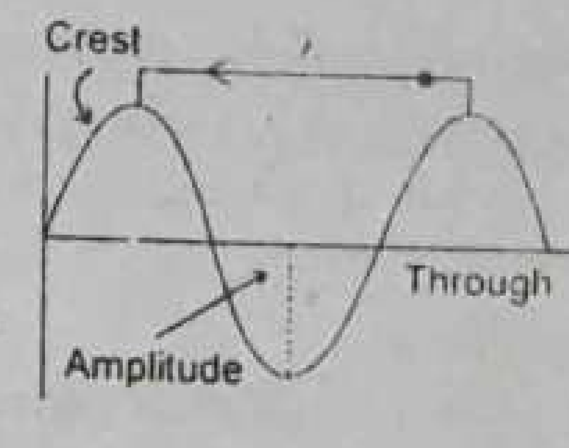
The wave appears to be traveling on the spring, from its one end to the other. In this case each part of string vibrates at right angle to the length of stretched string. The crest and troughs are being replaced by one another periodically and waves appear to be traveling.

Crest

The portion of the wave above the mean level is called as crest.

Trough

The portion of the wave below its mean level is called as trough.



Amplitude

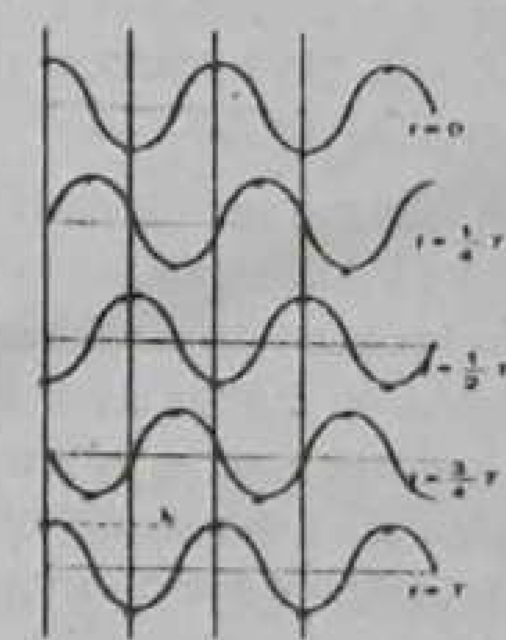
The amplitude is the maximum displacement of point in a crest or a trough of the string.

Wave Length

The distance between two consecutive crests or two consecutive troughs is known as wave length. It is denoted by λ.

Time Period

The time for which the wave travels a distance of wave length is called time period. The time period of wave is equal to be the time period of the oscillator which produces it.



Speed of Wave

When a wave progresses, each particle in the medium performs SHM. The time that the crest requires to moves a distance, of one wave length is equal to the time required for a point in the medium to go trough one complete oscillation. If 'v' be the speed of wave, then

$$v = \frac{\text{Distance covered}}{\text{Interval of time}}$$

$$v = \lambda / T$$

$$v = \lambda \left(\frac{1}{T}\right)$$

$$\text{Or} \quad v = f\lambda \qquad \left[\frac{1}{T} = f\right]$$

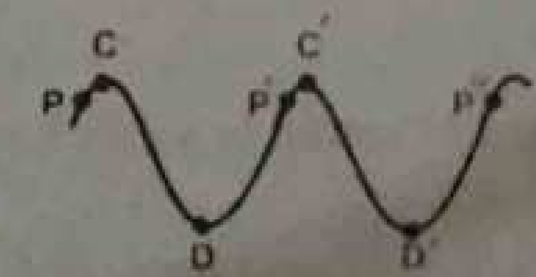
Where f is the frequency of the wave which is same as the frequency of oscillator (crest or trough) which produce it.

Wave Profile

Relation between path difference and phase difference

Consider the snapshot of the periodic waves moving through the medium. As any distance x from the reference point then phase difference can be described as

$$\boxed{\phi = \frac{2\pi}{\lambda} x}$$



The points C and C′, as they move up and down are always in the same state of vibrations. (i.e. they have identical displacement and velocities). There are many points or particles along the medium which are vibrating in phase. *The points separated from one another through distance λ, 2λ, 3λ ... are all in phase with each other.*

Some points are exactly out of step, for example, when point C reaches its maximum up wall displacement, at the same time D reaches its down wall displacement.

The point separated from one another through a distance of $\lambda/2$, $3\lambda/2$, $5\lambda/2$, ... are opposite in phase.

Longitudinal Periodic Waves

The periodic waves in which particles of the medium vibrate along the direction of motion of waves are called longitudinal periodic waves or compressional periodic waves.

Experiment

To explain longitudinal periodic waves, we take an example of a spring which is suspended by the help of threads. Longitudinal wave is produced in this spring by applying horizontally varying force at one end of the spring. This force produces compressions and rarefaction in the spring as shown in the figure.

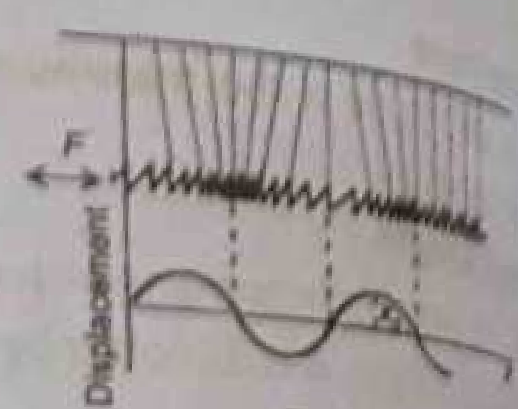


Fig. 8.6

In this case the various parts of the spring vibrate along the length of the spring (or along the direction of motion of wave). When spring was undisturbed then all the suspension threads were vertical. But when the longitudinal wave is produced in the spring then these suspension threads are displaced. Their displacement is same as the displacement of corresponding parts of the spring. The graph of displacement of various parts of spring and corresponding values of the distances of these various parts of spring, from its one end is shown in the figure.

Q.3 What are the factors on which the speed of sound depends upon? What was Newton's formula for the speed of sound? What was drawback in it, how it was corrected by Laplace?

Ans.

Speed of Sound in Air

Sound waves are longitudinal waves and their speed depends upon

- compressibility (i.e. elasticity) of the medium
- inertia (i.e. density) of the medium.

If E be the modules of elasticity (reciprocal of compressibility) and ρ be the density of the medium, then the speed v can be expressed as,

$$v = \sqrt{\frac{E}{\rho}}$$

Speed of sound in solids is much greater than in gases

Reason

Since molecules are closer in solids than in the gases, so they respond more **quickly** to a disturbance.

In other words, so the speed of sound in gases is smaller than in solids because the gases are **more compressible** and thus have smaller modulus of elasticity.

Newton's formula for the speed of sound in air

If E be the modules of elasticity and ρ be the density of the medium, then the speed v is

$$v = \sqrt{\frac{E}{\rho}} \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

Calculation of modulus of elasticity

Newton's Assumption

In order to calculate the elastic modulus for air, Newton assumed that the *temperature of the air during a compression remains constant* (i.e. an isothermal change)

So $\quad PV = \text{constant}$

When the pressure increases from P to P+ΔP then the volume decreases from V to V$-\Delta V$. According to Boyle's Law,

$$PV = (P + \Delta P)(V - \Delta V)$$

$$PV = PV - P\Delta V + \Delta P\, V - \Delta P \Delta V \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

Since changes ΔP and ΔV represent the small. So their product $\Delta P \Delta V$ can be neglected. Hence above equation becomes,

$$PV = PV - P\Delta V + V\Delta P$$

OR
$$0 = -P\Delta V + V\Delta P$$
$$P\Delta V = V\Delta P$$

OR
$$P = \frac{V\Delta P}{\Delta V}$$

OR
$$P = \frac{\Delta P}{\Delta V/V} \qquad \left[\text{where } \frac{\Delta P}{\Delta V/V} = \frac{\text{volumetric stress}}{\text{volumetric strain}} = E\right]$$

OR
$$P = E \quad \text{\_\_\_\_\_\_\_\_ (3)}$$

So equation (1) becomes

$$v = \sqrt{\frac{P}{\rho}}$$

At S.T.P, for air P = 0.76 mHg $=1.01\times10^5$ N/m^2

and $\quad \rho = 1.29$ kg/m^3

$$v = \sqrt{\frac{1.01\times15^5}{1.29}}$$

$$v = 280\text{m/sec}$$

The experimental value of speed of sound is **332m/sec.** The theoretical value is about **16%** less than the experiment value.

Drawback in Newton's Formula

During a compression the temperature of air does not remain constant but *increases* .i.e. it is an adiabatic change.

Laplace Correction

Laplace assumed that compressions and rarefactions in air take place so rapidly that heat of compression does not able to transfer to the neighboring cooler regions. Therefore the temperature of the medium does not remain constant. i.e. it is an adiabatic change.

In this case, Boyle's law can be takes the form

$$PV^{\gamma} = \text{constant} \quad \text{\_\_\_\_\_\_\_\_ (4)}$$

When the pressure increases from P to P+ΔP then the volume decreases from V to (V$-\Delta$V), so

$$PV^{\gamma} = (P+\Delta P)(V-\Delta V)^{\gamma}$$

Table 8.1

Speed of sound in different media

Medium	Speed ms^{-1}
Solids at 20°C	
Lead	1320
Copper	3690
Aluminium	5100
Iron	5130
Glass	5500
Liquids at 20°C	
Methanol	1120
Water	1482
Gases at S.T.P	
Carbon dioxide	256
Oxygen	315
Air	332
Helium	972
Hydrogen	1286

For Your Information

Values of constant	
Types of Gas	γ
Monoatomic	1.67
Diamatic	1.40
Polyatomic	1.29

Where $\gamma = \frac{C_p}{C_v} = \frac{\text{Molar heat capacity at constant pressure}}{\text{Molar heat capacity at constant volume}}$

$$\cancel{P V^{\gamma}} = (P+\Delta P)\cancel{V^{\gamma}}\left(1-\frac{\Delta V}{V}\right)^{\gamma}$$

Or $$P = (P+\Delta P)\left(1-\frac{\Delta V}{V}\right)^{\gamma}$$

By Binomial expansion

$$(1+x)^n = 1+nx+n\frac{(n-1)x^2}{2!}+\ldots\ldots\ldots\ldots$$

So $$P = (P+\Delta P)\left(1-\gamma\frac{\Delta V}{V}\right) + \text{neglecting square and higher powers of } \frac{\Delta V}{V}$$

$$\cancel{P} = \cancel{P} - \gamma P\frac{\Delta V}{V} + \Delta P - \gamma\frac{\Delta P \Delta V}{V}$$

Since ΔP and ΔV both are small so neglecting the term $\gamma\frac{\Delta P\Delta V}{V}$

So, $$P = P - \gamma P\frac{\Delta V}{V} + \Delta P$$

OR $$\Delta P = \gamma P\frac{\Delta V}{V}$$

$$\gamma P = \frac{\Delta P}{\Delta V/V} \qquad \left[\text{where } \frac{\Delta P}{\Delta V/V} = \frac{\text{stress}}{\text{strain}} = E\right]$$

$$\gamma P = E \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_} (4)$$

So equation (4) becomes

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

At S.T.P.

$$P = 0.76\ \text{mHg} = 1.01\times10^5\ \text{N/m}^2 \quad \rho = 1.29\ \text{kg/m}^3,$$

Value of γ is different for different gases. For air, $\gamma = 1.41$(for diatomic gas)

$$v = \sqrt{\gamma}\sqrt{\frac{P}{\rho}}$$

$$v = \sqrt{1.41}\times280$$

$$v = 333\ \text{m/sec}$$

which is close to the experimental value of 332m/sec.

Q.4 How the variation of pressure, density and temperature effect the speed of sound in a gas?

Ans

Effect on Speed of Sound in Air

(1) Effect of Pressure

As $$v = \sqrt{\frac{\gamma P}{\rho}}$$

For Your Information
Range of Hearing

Organisms	Frequency (Hz)
Dolphin	[illegible]
Bat	[illegible]
Cat	[illegible]
Dog	[illegible]
Human	[illegible]

...nce density is directly proportional to the pressure. When pressure of gas is ...creased, density of gas also increases, so the speed of sound remains **same.**

(2) Effect of Density

...t constant temperature and pressure of gases having same value of γ, the ...elocity is inversely proportional to the square root of their densities, which ...hows, smaller the density, greater the speed. i.e.

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$$v = \sqrt{\gamma P}\ \frac{1}{\sqrt{\rho}}$$

$$v = \text{constant}\ \frac{1}{\sqrt{\rho}}$$

$$v \propto \frac{1}{\sqrt{\rho}}$$

Note

The speed of sound in hydrogen is four times to its speed in oxygen because ...ensity of oxygen is sixteen times as that of oxygen

(3) Effect of temperature

When a gas is heated at constant pressure then its volume is increased and ...ensity is decreased

As

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

So the speed of sound is increased with the rise in temperature.

Let

v_o = speed of sound at $0^\circ C$

v_t = speed of sound at $t^\circ C$

ρ_o = density of gas at $0^\circ C$

ρ_t = density of gas at $t^\circ C$

So
$$v_o = \sqrt{\frac{\gamma P}{\rho_o}} \quad \text{———— (1)}$$

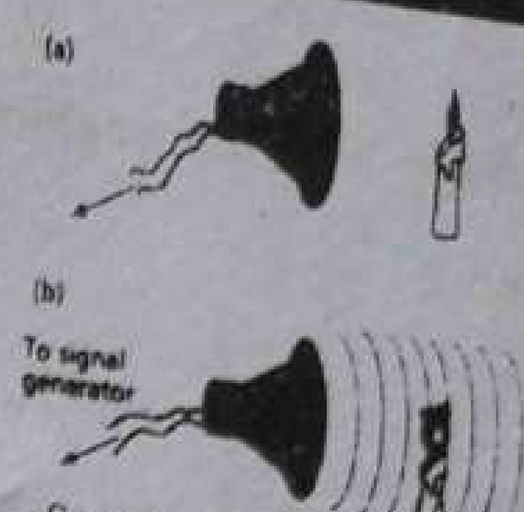
And
$$v_t = \sqrt{\frac{\gamma P}{\rho_t}} \quad \text{———— (2)}$$

Dividing the equation (2) by (1)

$$\frac{v_t}{v_o} = \sqrt{\frac{\gamma P/\rho}{\gamma P/\rho_o}}$$

$$\frac{v_t}{v_o} = \sqrt{\frac{\rho_o}{\rho_t}} \quad \text{———— (3)}$$

Tidbits



Sound waves cause the candle flame to flicker

EXPLANATION:
As the sound waves are mechanical waves. They can transfer energy and therefore cause flickering of candle flame.

If V_o is the volume of gas at 0° C and V_t is the volume at t°C. Then

$$V_t = V_o [1 + \beta t]$$

Where β is the coefficient of volume expansion. For all gases, its value is about $\frac{1}{273}$.

$$V_t = V_o\left[1+\frac{t}{273}\right]$$

As $\quad V_o = \frac{m}{\rho_o} \quad$ and $\quad V_t = \frac{m}{\rho_t} \quad [\because \rho = \frac{m}{V}]$

So $\quad \frac{m}{\rho_t} = \frac{m}{\rho_o}\left[1+\frac{t}{273}\right]$

OR $\quad \frac{\rho_o}{\rho_t} = 1+\frac{t}{273}$ ______ (4)

Using equation (4) in equation (3), we have

$$\frac{v_t}{v_o} = \sqrt{1+\frac{t}{273}} \text{ ______ (5)} \quad \text{Or} \quad \frac{v_t}{v_o} = \sqrt{\frac{t^\circ C + 273}{0^\circ C + 273}}$$

Or $\quad \frac{v_t}{v_o} = \sqrt{\frac{T}{T_o}}$ ______ (6)

Where $273 + t^\circ C = T$ = Absolute temperature corresponding to t°C

And $\quad 273 + 0^\circ C = T_o$ = Absolute temperature corresponding to 0°C

Thus, the speed of sound varies directly as the square root of absolute temperature.

Speed of sound in air at t°C

As $\quad \frac{v_t}{v_o} = \sqrt{1+\frac{t}{273}}$

Or $\quad \frac{v_t}{v_o} = \left(1+\frac{t}{273}\right)^{1/2}$

By using the Binomial expansion and neglecting square and higher powers, we have

$$\frac{v_t}{v_o} = \left[1+\left(\frac{1}{2}\right)\left(\frac{t}{273}\right)+\ldots\ldots\right]$$

$[\because (1+x)^n = 1 + nx + \ldots]$

So $\quad v_t = v_o + \frac{v_o}{546}t$

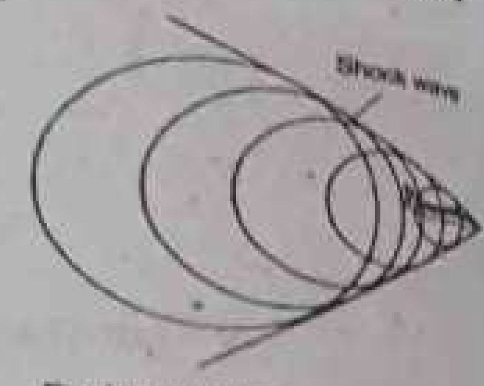
$\quad v_t = v_o + \frac{332}{546}t \qquad$ [as v_o = 332 m/sec]

Or $\quad v_t = v_o + 0.61t$

This equation shows that ***with one degree celsius rise in temperature, the speed of sound is increased by approximately 0.61m/sec. (or 61 cm/sec).***

Do You Know?

Slower than the speed of sound



Faster than the speed of sound

What happens when a jet plane like Concorde flies faster than the speed of sound?

A conical surface of concentrated sound energy sweeps over the ground as a supersonic plane passes overhead. It is known as sonic boom.

EXPLANATION:

There are many instances in which the source moves through a medium at a speed greater than the speed of the wave in that medium.

In such cases the wavefront takes the shape of a cone with the moving body at its apex. Jet airplanes, missiles and rockets moving with speed greater than the speed of sound, propagate Shockwaves that trail behind them, producing a so called sonic boom. Shock waves are longitudinal waves like sound but with larger amplitudes that possess much pressure. Both explosions and supersonic objects produce this type of shock waves. The sound waves produced by airplanes travel out in all directions but may become crowded together at the front as the airplane increases its speed. Fig (8) The supersonic airplanes leave a cone shaped trail of sound waves behind it. The cone-shaped wavefront produces a shock effect on reaching the ground. They cause an increase of pressure of about 2 pound per square foot for about one-half a second/followed by a decrease in pressure below atmospheric pressure. When the edge of that cone intercepts the ground below, we hear a "sonic boom" which is not associated with an. aircraft "breaking the sound barrier". The sonic boom is merely the total effect Of the concentration on one surface of the aircraft's radiated sound energy, which would radiate in all directions at subsonic speeds. It might be possible to hear two sonic booms from the same aircraft, one from the leading edge and another from the trailing edge. The cone never intercepts the projectile itself thus the aircraft passengers do not hear the sonic boom.

Q.5 State and explain on principle of super position.

Ans.

Superposition Principle

If a particle of medium is simultaneously acted upon by a number of waves then the resultant displacement of the particle is the algebraic sum of their individual displacements. This is called superposition principle.

Let the displacement of the individual waves be $y_1, y_2, y_3, \ldots y_n$,

Then by super position principle the resultant displacement be.

$$y = y_1 + y_2 + y_3 + \ldots\ldots + y_n$$

Consider the two waves coming from opposite direction through a coil of spring, as shown in figure. When theses waves combine with each other then during the time of overlapping, the displacements of waves are added up as shown in figure (c). After having crossed each other they again adopt their original shapes and continue their motion along the spring in their respective directions as shown in figure (d). We can study three important cases of super-position principle.

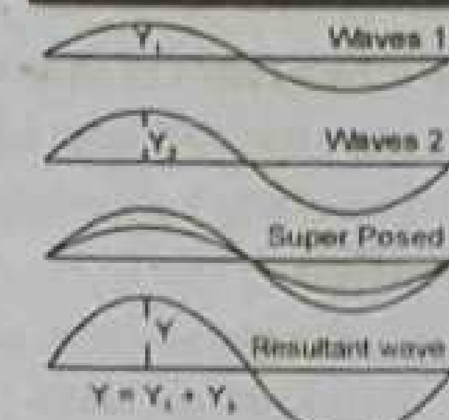
Cases of Superposition principle

(i) When two waves having same frequency and traveling in the same direction produce the phenomenon of **interference**.

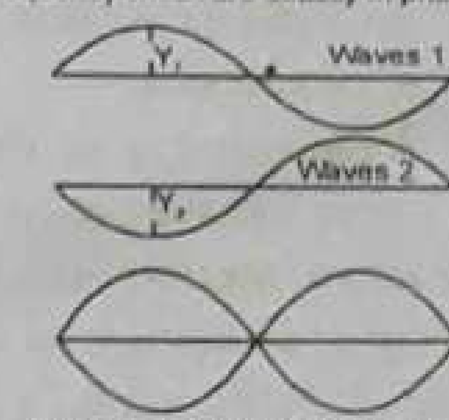
(ii) When two waves of slightly different frequencies and traveling in the same direction produce **beats**.

(iii) When two waves of same frequency traveling in opposite direction produce **stationary waves**.

For Your Information



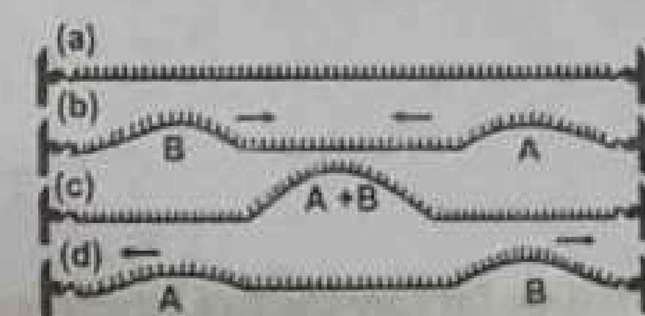
Superposition of two waves of the same frequency which are exactly in phase



Wave 1 and wave 2 super posed Resultant wave

Y = 0

Superposition of two waves of the same frequency which are exactly out of phase

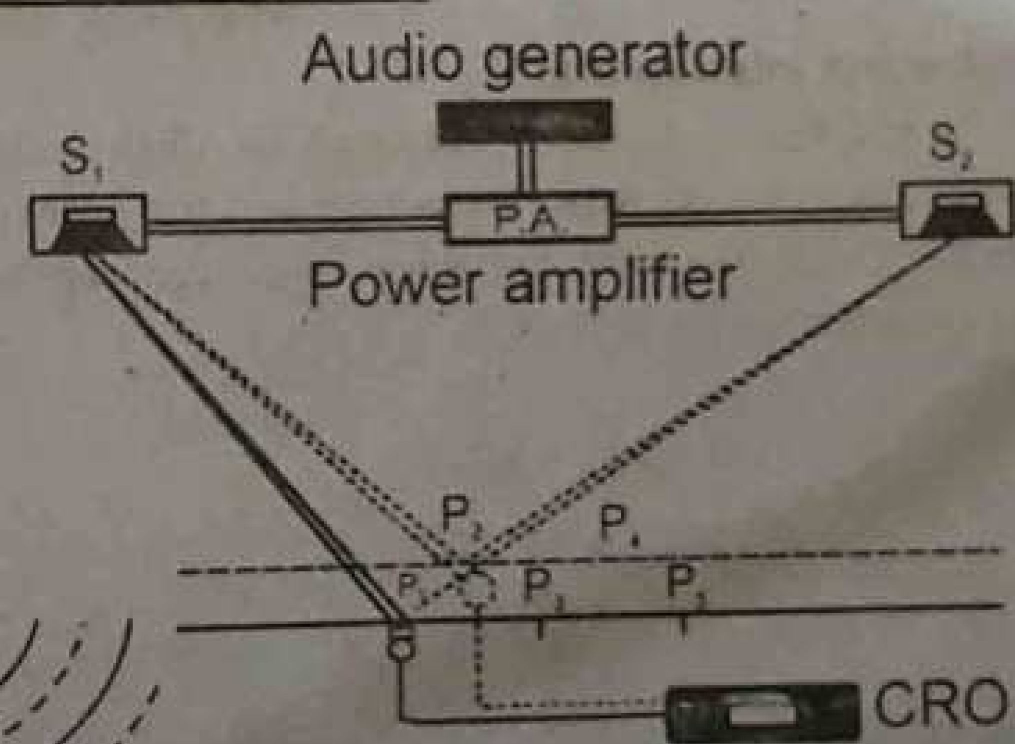


Q.6 What is interference? Describe its types. Also write down the conditions for constructive and destructive interference.

Ans.

Interference

When two identical waves meet each other in a medium then at some points they reinforce the effect of each other and at some points they cancel the effect of each other. This phenomenon is called interference.



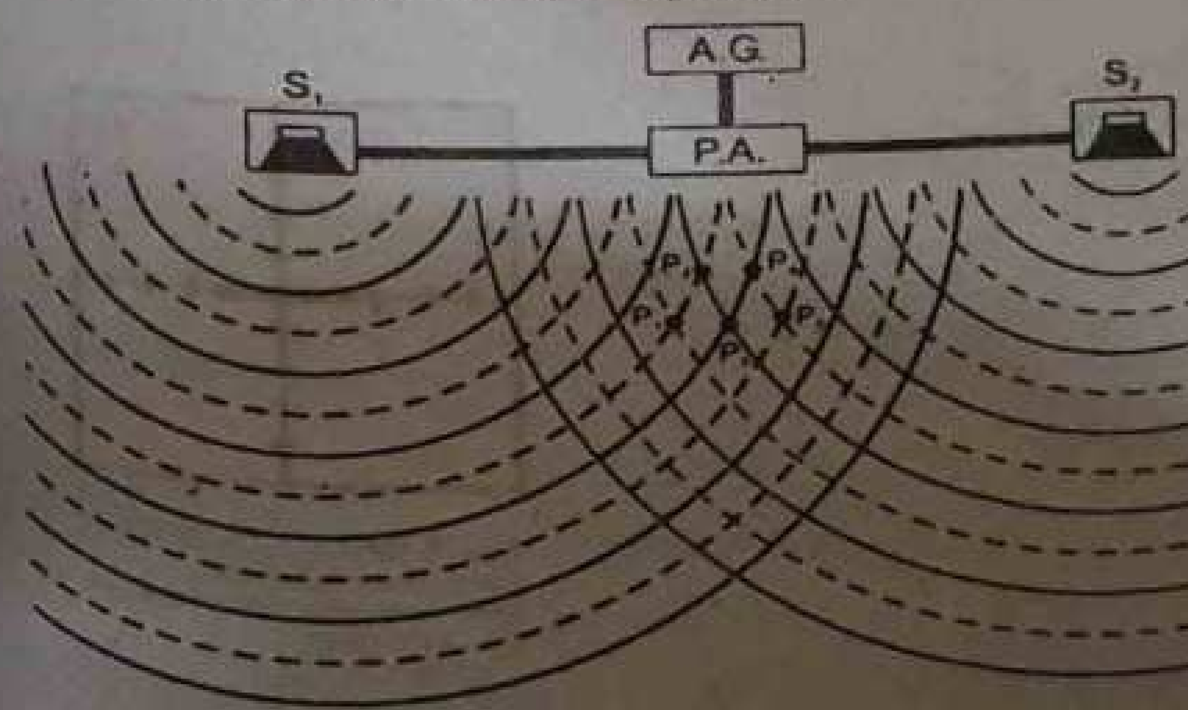


Fig. 8.8 (b)

Interference of sound waves
Points P_1, P_2, P_3 are points of constructive interference.
Points P_2 and P_4 are points of destructive interference.

Types of interference

They are two types of interference

- Constructive interference
- Destructive interference

Explanation

Consider an experimental arrangement shown in figure. It consists of:

Consider an experimental arrangement shown in figure. It consists of:

- Two loud speakers S_1 and S_2 for the production of harmonic sound waves of fixed frequency.
- An audio-generator.
- A microphone
- A cathode ray oscilloscope (CRO) is attached to microphone to see the input signal wave-form.

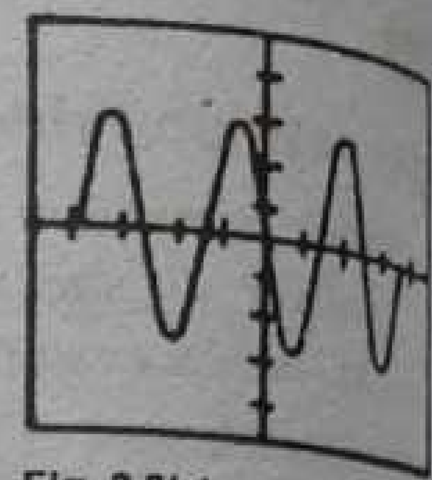


Fig. 8.8(c)
Constructive Interference
Large displacement is displayed on the CRO screen

Constructive Interference

The micro-phone is placed at various points in front of loud speakers, as shown in figure. At P_1, P_3 and P_5, a large signal is seen on CRO, as shown in fig. At these points compression meets with compression and rarefaction meets with rarefaction. So the displacements of two waves are added up at these points and a large resultant displacement is produced. We can find the path difference at point P_1 between two waves is,

$$\Delta S = S_2P_1 - S_1P_1$$
$$= 4\frac{1}{2}\lambda - 3\frac{1}{2}\lambda$$
$$= \lambda$$

Condition for constructive interference

Whenever path difference is an integral multiple of wave length, displacements of two waves are added up. This effect is called constructive interference.

$$\Delta S = n\lambda \qquad \text{where } n=0, \pm1, \pm2, \pm3, \ldots$$

Destructive Interference:

At P_2 and P_4, no signal is obtained on CRO, as shown in fig. At these points compression meets with rarefaction and they cancel the effect of each other, so resultant displacement becomes zero. We can calculate the path difference between the waves

At P_2,

$$\Delta S = S_2P_2 - S_1P_2$$
$$= 4\lambda - 3\frac{1}{2}\lambda$$
$$= \frac{1}{2}\lambda$$

Similarly at P_4,

$$\Delta S = S_2P_4 - S_1P_4$$
$$= 3\frac{1}{2}\lambda - 4\lambda$$
$$= \frac{1}{2}\lambda$$

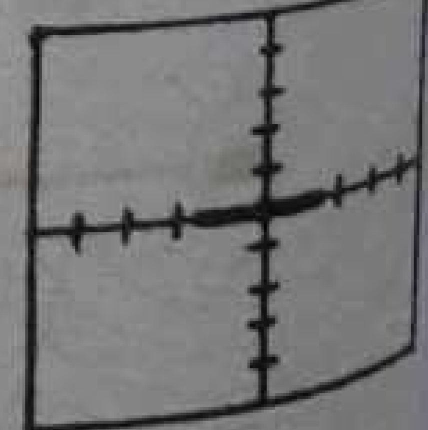


Fig. 8.8(d)
Destructive Interference
Zero displacement is displayed on the CRO screen

Condition for destructive interference

Whenever the path difference is odd integral multiple of half of wavelength, the displacements of two waves cancel the effect of each other. This effect is called destructive interference

$$\Delta S = (2n+1)\frac{\lambda}{2} \qquad \text{where } n=0, \pm1 \pm2, \pm3\ldots$$

Or

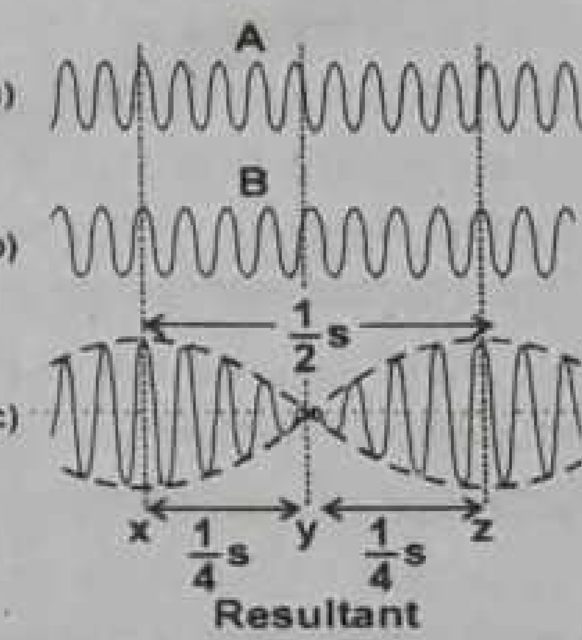
$$\Delta S = (n + \frac{1}{2})\lambda$$

Q.7 What are beats? Explain it with the help of example as well as graphically? Also mention some uses of beats?

Ans.

Beats

When two waves of slightly different frequencies, traveling in the same direction overlap each other then there is a periodic variation of sound between maximum and minimum loudness called beats.



Formation of Beats

Consider two tuning forks A and B of same frequency say 32Hz are sounded separately then they will produce pure notes. But when they are sounded simultaneously then it is difficult to differentiate the notes. The sound waves of two will be superposed on each other and will be heard by the human ear as single pure notes. If the frequency of tuning fork B is lowered slightly by loading it with some wax, say it becomes 30Hz.

Now if A and B sounded together, a sound of alternately increasing and decreasing intensity will be heard. Such a not is called beat, which is due to interference between the sound waves from A and B as shown in figure below.

At some instant X is the displacement of the two waves is in the same direction. The resultant displacement is large and a loud sound is heard.

After time ¼ sec, the displacement of wave due to one tuning fork is opposite to the displacement of waves due to the other tuning fork. As a result, a minimum displacement is produced at Y. So a low or no sound is heard. After next ¼ sec, the displacements are again in the same direction and a loud sound is heard again at Z. It represents a loud sound is heard two times in one second because the frequency difference is 2Hz.

Conclusion

Number of beats per second is equal to the difference between frequencies of tuning forks.

Mathematically,

$$f_A - f_B = n = \text{number of beats per second}$$

If the frequency difference is greater than 10Hz, then it is difficult to recognize them.

Uses of beats

Beats are used to;

(1) tune a string instrument such as piano or violin, by beating a note against a note of known frequency. The string can then be adjusted to the desired frequency by tightening or loosening it until no beats are heard.

(2) find unknown frequency of vibrating body.

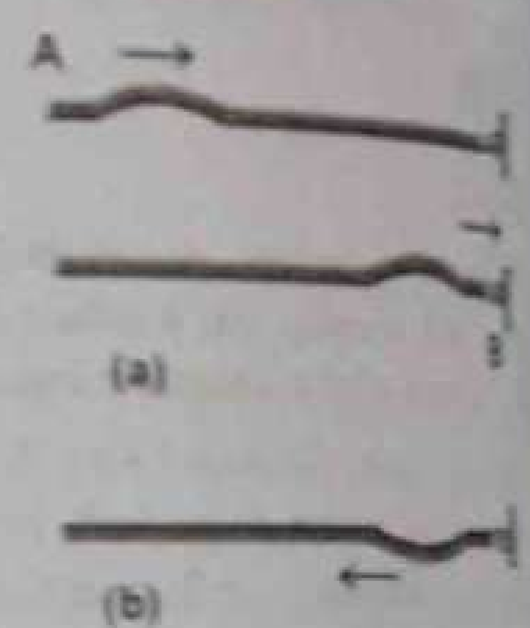
(3) produce variety in music.

Q.8 Explain the reflection of waves from rare and denser media.

Ans.

Reflection of Waves

The bouncing back of wave from the boundary of a medium is called reflection of wave.



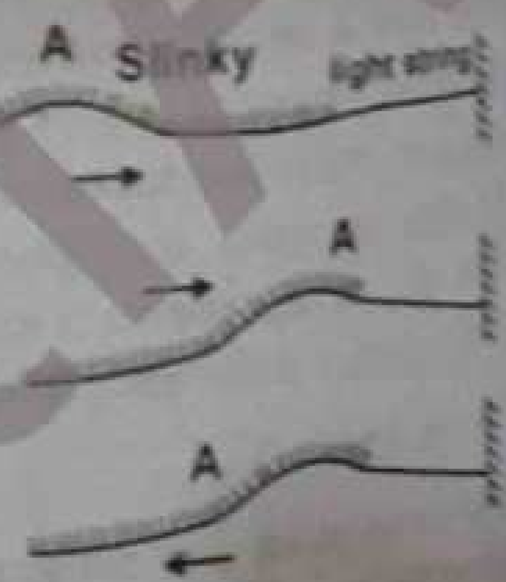
When a wave produced in one medium travel to the boundary to enter into another medium, then a part of incident wave is reflected from the boundary. This reflected part has same frequency and wavelength as the incident wave has.

But there may be the change of phase which depends upon the nature of boundary of medium.

Reflection of waves from the boundary of denser medium:

Let us take a long slinky spring whose one end is fixed to a rigid support on a horizontal surface of a table. The other end of this spring is free to oscillate. A sharp jerk is give to which a crest or through is produced on it. It travels on the spring from its end A towards the end B. On reaching at end B spring exerts a force on the rigid support to produce similar motion in it. But the rigid body has large density so it exerts equal and opposite reaction on the spring. Due to which crest is converted into trough and it travel back from end B to A.

Reflection of waves from the boundary of a rare medium



If we attach end B of long slinky spring with a light string and keep its end A free, like before. Then giving a sudden jerk to end A, a crest is produced which travels on the spring from its end A to end B. When the crest produced reaches the boundary of string. Then string being rare medium, do not give reaction to the spring. So that a crest is reflected back as a crest on the spring from end B to end A.

Results:

(i) If a transverse wave traveling in a rarer medium is incident on a denser medium, it is reflected such that it undergoes a phase change of 180°. (path difference of $\lambda/2$)

(ii) If a transverse wave traveling in a denser medium is incident on a rarer medium, it is reflected with out any change in phase. (no path difference)

Q.9 Define stationary waves and how they are produced? Give properties of stationary waves. Also define node and anti node.

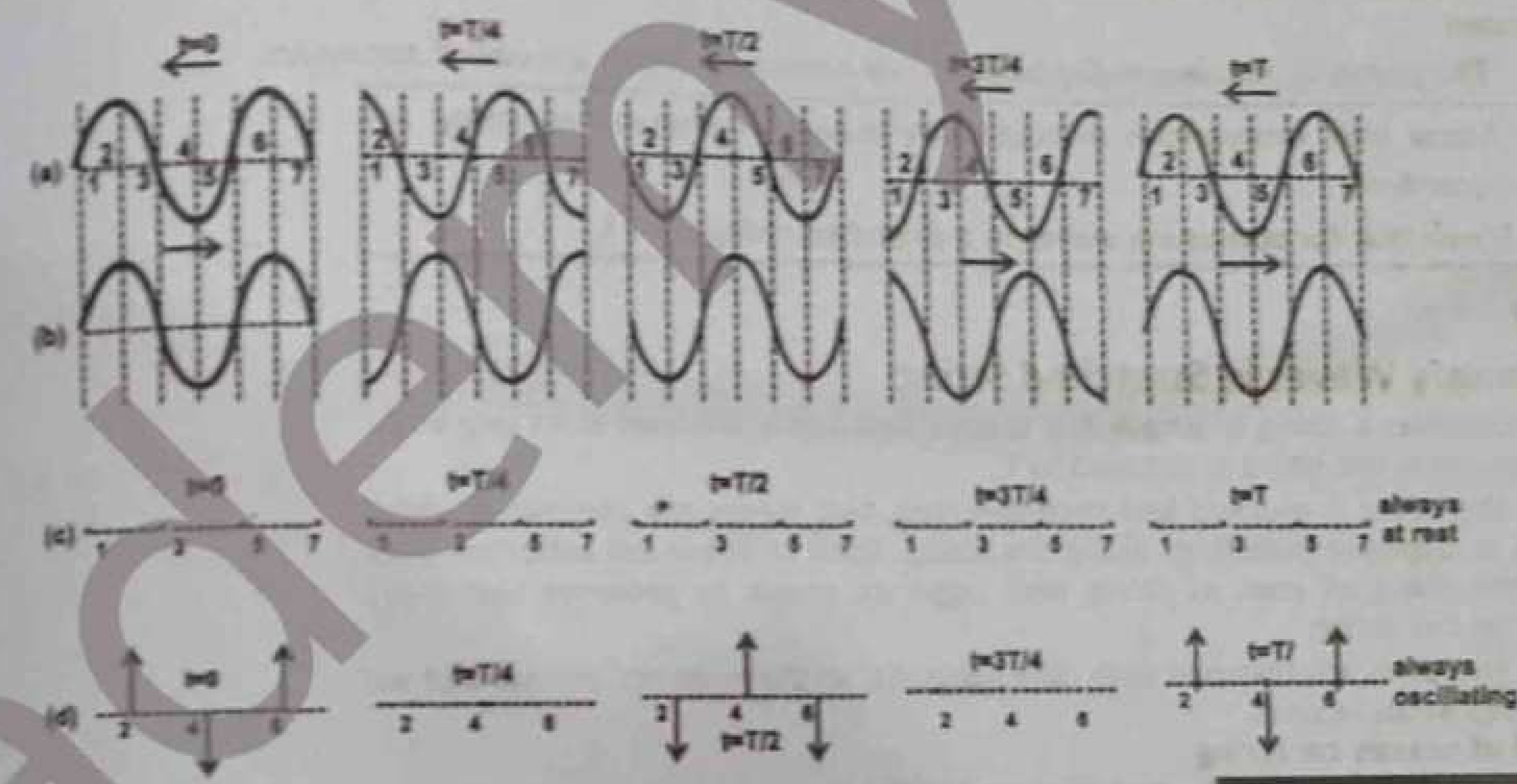
Ans.

Stationary Wave

The resultant wave produced by the superposition of two identical waves traveling in opposite direction is called stationary wave.

Production of Stationary Waves

Consider the superposition of two waves moving along a string in opposite directions. The picture of such two waves at instants t=0, T/2, T/4, 3T/4 and T, as shown in figure (a) and (b).



When two waves superpose to each other, we want to find out the displacements of the points 1,2,3,4,5,6 and 7 shown in fig. It clear that points 1,2,3 etc are at a distance $\lambda/4$ apart from each other. The resultant displacements at these are calculated by applying the super position principle. Fig (c) shows that the resultant displacement of the point 1,3,5,7 at the instants t=0, T/4, T/2, 3T/4 and T. It can be seen that the resultant displacement is always zero at all the instants.

Fig (d) shows the resultant displacement of points 2, 4 and 6 at instant t = 0, T/4, T/2, 3T/4 and T. These points move with maximum displacement from mean positions.

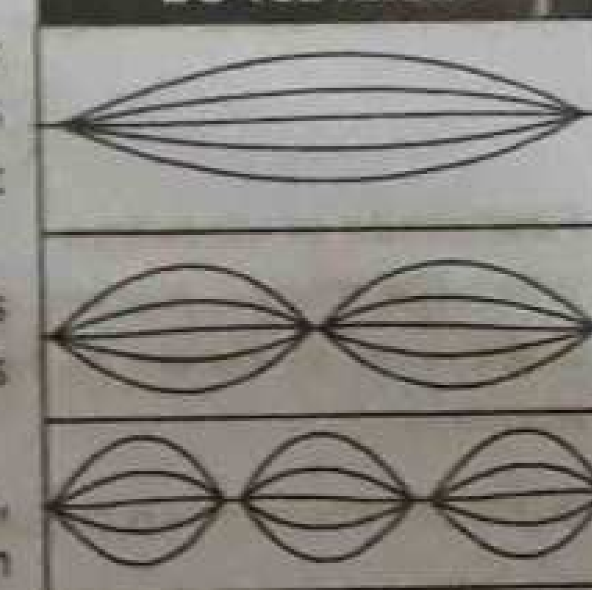
Properties of stationary waves

1. There are points of medium in stationary waves which permanently show zero displacement are called *nodes*.
2. The points between two successive nodes are *in phase* with each other.
3. Each point along the stationary waves vibrates with *different amplitudes*.
4. There are points of medium in stationary waves which have maximum amplitude are called *antinodes*.
5. The distance between two consecutive nodes is $\lambda/2$.
6. The distance between one node and next antinode is $\lambda/4$.
7. The energy remains standing in the medium between nodes because the nodes remain at rest, so energy cannot flow through these points. That is why stationary waves are also called standing waves.
8. Energy oscillates between P.E. and K.E. Between nodes.

Note

* When antinodes are at their extreme positions the whole energy is P.E while at passing through equilibrium position, the whole energy is K.E.

DO You Know?



A standing-wave pattern is formed when the length of the string is an integral multiple of half wave-length; otherwise no standing wave is formed.

EXPLANATION:

If L is the length of the string and λ_1, λ_2 and λ_3 are the wavelengths in the figures a, b and c respectively then

from Fig a, $L = \lambda_1/2 = 1 \times \lambda_1/2$

from Fig b, $L = \lambda_2/2 + \lambda_2/2 = \lambda_2 = 2 \times \lambda_2/2$

from Fig c, $L = \lambda_3/2 + \lambda_3/2 + \lambda_3/2 = 3 \times \lambda_3/2$

Thus the length of the string L is an integral (1, 2 and 3) multiple of half the wavelength.

- Commonly the standing waves are produced due to superposition of a wave traveling down a string with its reflection traveling in opposite direction.

Nodes

The points of zero displacement on stationary waves are called node.

Antinodes

The points of maximum displacement on stationary waves are called antinodes.

Q.10 Show that frequencies of stationary waves in a stretched string are quantized. OR

Prove that for stationary waves in a stretched string $f_n = n\,f_1$

Ans.

Stationary Waves in Stretched String

Let us consider a string of length ℓ. It is stretched and is clamped at its two ends. The tension in the string is denoted by F.

When the string is plucked and then released, two waves are generated which moves in opposite directions along the string. Both of these are reflected back from the clamped ends of string with opposite phase to generate stationary waves on the string.

As the two ends are clamped with rigid supports, so these do not vibrate and we get nodes at these ends.

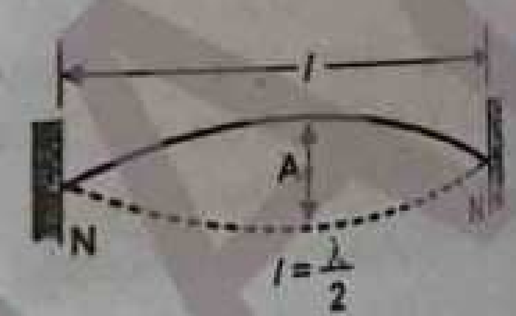
Speed of waves on string

The speed of wave depends upon tension F in the string and mass per unit length m (i.e. thickness and nature of wire).

$$v = \sqrt{\frac{F}{m}} \quad \text{———} (1)$$

First mode of vibration

When the string is plucked at the middle of its length then the string vibrates in a single loop as shown in figure. Such a mode is called fundamental mode of vibration.



Distance between two consecutive nodes $= \frac{\lambda}{2}$

If λ_1 be the wave length and f_1 be the frequency of vibration in this mode, then

$$\ell = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2\ell \quad \text{———} (2)$$

Thus, speed of wave v is

$$v = f_1 \lambda_1$$

OR

$$f_1 = \frac{v}{\lambda_1}$$

putting value of λ_1, we get

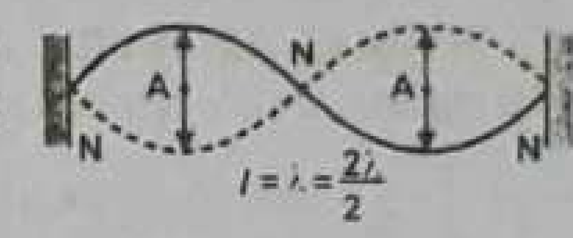
$$f_1 = \frac{v}{2\ell} \quad \text{———} (3)$$

Putting value of v from equation (1) in equation (3), we get

$$\boxed{f_1 = \frac{1}{2\ell}\sqrt{\frac{F}{m}}} \quad \text{———} (4)$$

Second Mode of Vibration

When the string is plucked from one quarter of its length then the string vibrates into two loops as shown in figure. If λ_2 be the wave length and f_2 be the frequency of vibration in this mode, then



$$\ell = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$$

OR

$$\ell = \lambda_2$$

OR

$$\boxed{\lambda_2 = \ell}$$

Thus, speed of wave v is

$$v = f_2 \lambda_2$$

OR

$$f_2 = \frac{v}{\lambda_2}$$

Putting value of λ_2, we get

$$f_2 = \frac{v}{\ell}$$

or

$$f_2 = \frac{2v}{2\ell}$$

or

$$f_2 = 2\left(\frac{v}{2\ell}\right)$$

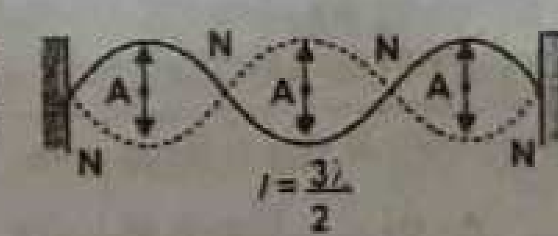
So

$$\boxed{f_2 = 2f_1} \qquad \left[\text{since } \frac{v}{2\ell} = f_1\right]$$

Thus when the string vibrates in two loops, its frequency is double than when it vibrates in one loop. f_2 is called second harmonic.

Third mode of vibration

When the string is plucked from one sixth (1/6) of its length then the string vibrates into three loops as shown in figure. If λ_3 be the wave length and f_3 be the frequency of vibration in this mode, then



$$\ell = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$$

$$\ell = \frac{3\lambda_3}{2}$$

or

$$\lambda_3 = \frac{2\ell}{3}$$

So the speed becomes,

$$v = f_3 \lambda_3$$

$$f_3 = \frac{v}{\lambda_3}$$

$$f_3 = \frac{3v}{2\ell}$$

$$f_3 = 3\left(\frac{v}{2\ell}\right)$$

$$\boxed{f_3 = 3f_1} \qquad \left[\text{since } \frac{v}{2\ell} = f_1\right]$$

The frequency f_3 is called third harmonic.

nth mode of vibration

If string vibrates in n loops then,

$$f_n = n\left(\frac{v}{2\ell}\right) = n\,f_1$$

And wavelength is

$$\lambda_n = \frac{2\ell}{n} \qquad \text{where } n = 1,2,3,4,5 \ldots\ldots\ldots$$

So the stationary wave have a discrete set of frequencies f_1, $2f_1$, $3f_1$, ... nf_1, which is known as harmonic series. The frequency f_1 is known as *fundamental frequency*, and the other are called *over tones*.

Note

The stationary waves can be set up in the string only with the frequencies of harmonic series determined by the tension, length and mass per unit length of the string. Waves not in harmonic series are quickly damped out.

Q.11 How can we change the frequency of string on a musical instrument?

Ans.

The frequency of a string on a musical instrument can be changed either by varying the tension in string and length of string.

For example

The tension in guitar and violin strings is varied by tightening the pegs on the neck of the instrument. Once the instrument is tuned, the musicians vary the frequency by moving their fingers along the neck. By doing so the change the length of the vibrating portion of the string.

Q.12 Find the frequencies produced in organ pipe when it is

(i) Open at both ends

(ii) Closed at one end

Ans.

Stationary Waves in Air Column

Stationary waves can be set up in air column inside a pipe or tube. A common example of vibrating air column is an organ pipe.

Organ pipe

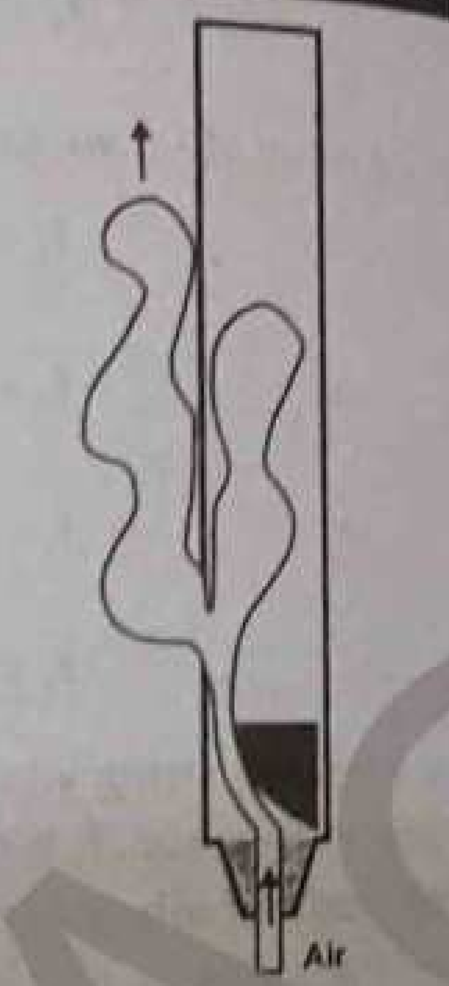
An organ pipe is a wind instrument in which sound is produced, due to setting up of stationary waves in air column.

It consists of a hollow long tube with both ends open or with one end open and the other closed. There are two types of organ pipes

(i) **Open Organ pipe:** It is that organ pipe whose both ends are open.

(ii) **Closed organ pipe:** It is that organ pipe whose one end is closed.

For Your Information



In an organ pipe, the primary driving mechanism is wavering sheet like jet of air from flute-slit, which interacts with the upper lip and the air column in the pipe to maintain a steady oscillation

EXPLANATION:

All wind instruments are provided with a column of air, called a resonator, which may be in the form of a pipe. The periodic movement is caused by mouth piece. The mouth-piece acts as a generator and supplies the energy necessary to maintain the vibrations in air column. The air from the mouth-piece strikes the upper lip and sets up vibrations in the resonating pipe.

Modes of vibrations in organ pipe open at both ends

Let us consider an organ pipe of length ℓ which is open at both ends. As at the open ends air molecules have complete freedom of motion so it acts as antinode.

Longitudinal waves set up inside, the pipe have been represented by transverse curves which represent the displacement and amplitude variations of air at various points.

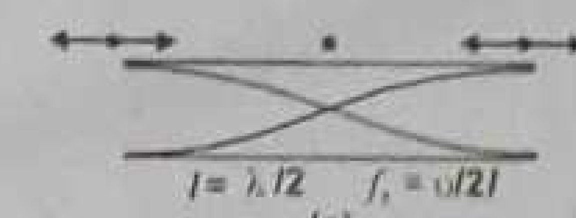
Fundamental Mode of Vibration

In this case there is only one node at the middle of the pipe. As both ends of pipe are open, so there are two antinodes at both the ends. If λ is the wavelength of sound,

$$\ell = \frac{\lambda_1}{4} + \frac{\lambda_1}{4}$$

$$\ell = \frac{\lambda_1}{2}$$

Or $$\boxed{\lambda_1 = 2\ell}$$



(a)

If f_1 is the frequency of sound, then the velocity of sound is

$$v = f_1 \lambda_1$$

OR $$f_1 = \frac{v}{\lambda_1}$$

putting value of λ_1, we get

$$f_1 = \frac{v}{2\ell}$$

This frequency is called fundamental frequency or first harmonic

Second mode of vibration:

In this case, there are three antinodes and two nodes.

If λ_2 is the wavelength of sound then

(b)

$$\ell = \frac{\lambda_2}{4} + \frac{\lambda_2}{2} + \frac{\lambda_2}{4}$$

$$\ell = \left(\frac{1+2+1}{4}\right)\lambda_2$$

OR $$\lambda_2 = \ell$$

If f_2 is the frequency of sound, then speed becomes,

$$v = f_2 \lambda_2$$

OR $$f_2 = \frac{v}{\lambda_2}$$

Putting value of λ_2, we get

$$f_2 = \frac{v}{\ell}$$

or $$f_2 = \frac{2v}{2\ell}$$

or $$f_2 = 2\left(\frac{v}{2\ell}\right)$$

So $$\boxed{f_2 = 2f_1} \qquad \left[\text{since } \frac{v}{2\ell} = f_1\right]$$

Third mode of vibration

For three loops, there are four antinodes and three nodes. If λ_3 is the wavelength of sound, then length of the pipe is

$$\ell = \frac{\lambda_3}{4} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{4}$$

$$\ell = \left(\frac{1+2+2+1}{4}\right)\lambda_3$$

$$\ell = \frac{3\lambda_3}{2}$$

$$\boxed{\lambda_3 = \frac{2\ell}{3}}$$

So the speed becomes,

$$v = f_3\lambda_3$$

$$f_3 = \frac{v}{\lambda_3}$$

$$f_3 = \frac{3v}{2\ell}$$

$$f_3 = 3\left(\frac{v}{2\ell}\right)$$

$$\boxed{f_3 = 3f_1} \qquad [\text{since } \frac{v}{2\ell} = f_1]$$

The frequency f_3 is called third harmonic.

nth mode of vibration

If air column vibrates in n loops then,

$$f_n = n\left(\frac{v}{2\ell}\right) = n f_1$$

And wavelength is

$$\lambda_n = \frac{2\ell}{n} \qquad \text{where } n = 1,2,3,4,5 \ldots\ldots$$

So the longitudinal stationary waves have a discrete set of frequencies f_1, $2f_1$, $3f_1$, ... nf_1, which is known as harmonic series. The frequency f_1 is known as fundamental frequency, and the other are called over tones.

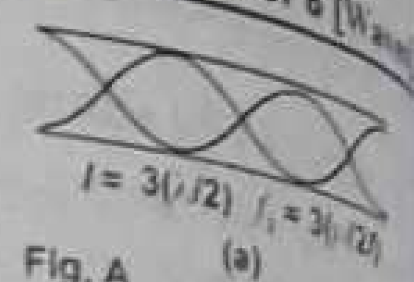


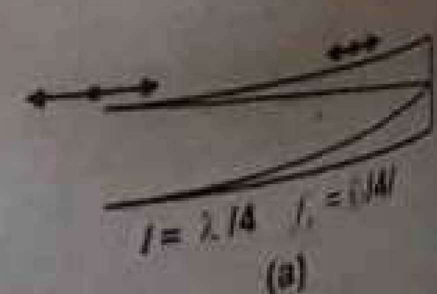
Fig. A
Stationary longitudinal waves in a pipe open at both ends.

Modes of vibration in organ pipe closed at one end

Let us consider an organ pipe of length ℓ which is closed at one end. Then at the closed end we get node while at the open end we get anti node.

Fundamental mode of vibration:

Fundamental mode of vibration has one node and one antinode. If λ_1 is the wavelength of fundamental mode, then length of the pipe is.



$$\ell = \frac{\lambda_1}{4}$$

Or $\boxed{\lambda_1 = 4\ell}$

So the speed becomes,

$$v = f_1\lambda_1$$

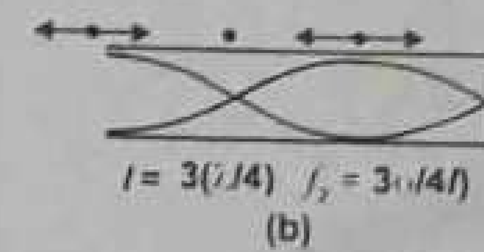
Or $f_1 = v/\lambda_1$ As $\lambda_1 = 4\ell$

$$\boxed{f_1 = v/4\ell}$$

The frequency f_1 is called fundamental frequency.

Second Mode of vibration

Second mode of vibration contains two nodes and two anti-nodes If λ_3 is the wavelength, then length of the pipe is



$$\ell = \frac{\lambda_3}{4} + \frac{\lambda_3}{2}$$

$$\ell = \left(\frac{1+2}{4}\right)\lambda_3$$

$$\ell = \frac{3\lambda_3}{4}$$

OR $$\lambda_3 = \frac{4\ell}{3}$$

If f_3 is the frequency of sound, then speed becomes,

$$v = f_3\lambda_3$$

OR $$f_3 = \frac{v}{\lambda_3}$$

Putting value of λ_3, we get

$$f_3 = \frac{v}{4\ell/3}$$

or $$f_3 = \frac{3v}{4\ell}$$

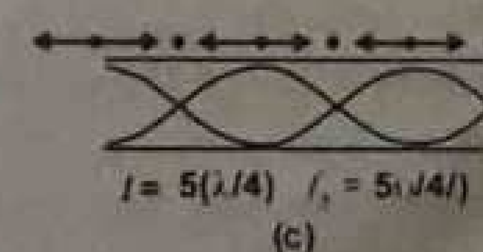
or $$f_3 = 3\left(\frac{v}{4\ell}\right)$$

So $$\boxed{f_3 = 3f_1} \qquad [\text{since } \frac{v}{4\ell} = f_1]$$

This is called second harmonic or first overtone.

Third Mode of vibration:

Third mode of vibration contains there nodes and three anti-nodes. If λ_5 is the wavelength, the length of the pipe is.



Stationary longitudinal waves in a pipe closed at one end. Only odd harmonics are present.

$$\ell = \frac{\lambda_5}{4} + \frac{\lambda_5}{2} + \frac{\lambda_5}{2}$$

$$\ell = \left(\frac{1+2+2}{4}\right)\lambda_5$$

$$\ell = \frac{5\lambda_5}{4}$$

OR $$\lambda_5 = \frac{4\ell}{5}$$

If f_5 is the frequency of sound, then speed becomes,

$$v = f_5 \lambda_5$$

OR $$f_5 = \frac{v}{\lambda_5}$$

Putting value of λ_5, we get

$$f_5 = \frac{v}{4\ell/5}$$

or $$f_5 = \frac{5v}{4\ell}$$

or $$f_5 = 5\left(\frac{v}{4\ell}\right)$$

So $$\boxed{f_5 = 5f_1} \quad \left[\text{since } \frac{v}{4\ell} = f_1\right]$$

Which is the frequency of third harmonic or second overtone.

nth mode of vibration

If air column vibrates in n loops then,

$$f_n = n\left(\frac{v}{4\ell}\right)$$

$$f_n = n f_1$$

And wavelength is

$$\lambda_n = \frac{4\ell}{n} \qquad \text{where } n = 1, 3, 5, 7, \ldots\ldots$$

Conclusion

By studying the both these cases, we conclude that ***the pipe which is open at both ends is richer in harmonics.***

for this long ques. see on azeem guides

Q.13 What is Doppler Effect? Discuss its different cases.

Ans.

Doppler Effect

The apparent change in the frequency (or pitch) of waves due to the relative motion between the source and observer is called Doppler's Effect.

Note

This effect was first observed by John Doppler while he was observing the frequency of light emitted from a star. In some cases the frequency of emitted light was found to be slightly different from that emitted from a similar source on the Earth. He found that the change of frequency of light depends upon motion of star relative to Earth.

Example

1) The pitch of whistle of an engine coming towards the platform appears to become higher to an observer standing on the platform
2) The pitch of whistle of an engine going away from the platform appears to become lower to an observer standing on the platform

Different cases

Consider a source of sound S at rest emits sound waves having wavelength λ .Let speed of the sound for a stationary observer is v. Then,

the number of waves received by observer in one second is

$$f = \frac{v}{\lambda} \quad \text{——— (1)}$$

Case I

[When observer moves towards stationary source]

Let observer A moves towards the source with velocity u_o. Then

The relative velocity of the waves and the observer = $v + u_o$.

Now, the number of waves received by observer in one second is

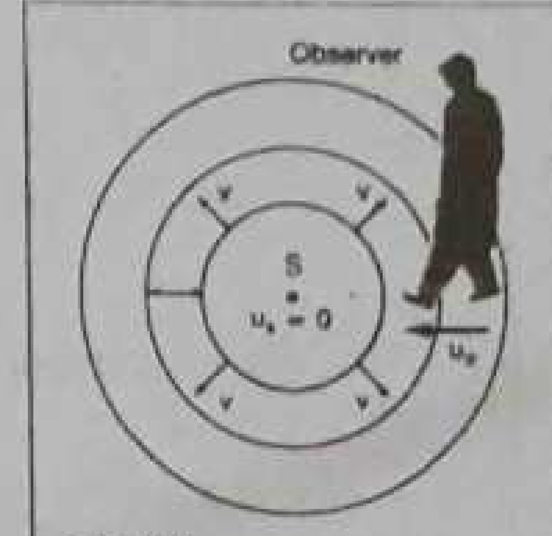


Fig. 8.18
An observer moving with velocity u_o towards a stationary source hears a frequency f_A that is greater than the source frequency.

$$f_A = \left[\frac{v + u_o}{\lambda}\right]$$

$$f_A = \left[\frac{v + u_o}{v/f}\right] \quad \text{(using equation 1)}$$

$$f_A = \left[\frac{v + u_o}{v}\right] f \quad \text{——— (2)}$$

$$\because \left[\frac{v + u_o}{v}\right] > 1 \text{ so}$$

$$\Rightarrow \quad f_A > f$$

Result

Thus the apparent frequency of sound heard by the observer will ***increase.***

Case II

[Observer moves away from the stationary source]

Let observer B movers away from the source with velocity u_o, then

The relative velocity of the sound and observer = $v - u_o$. Thus, the number of waves received by observer in one second is

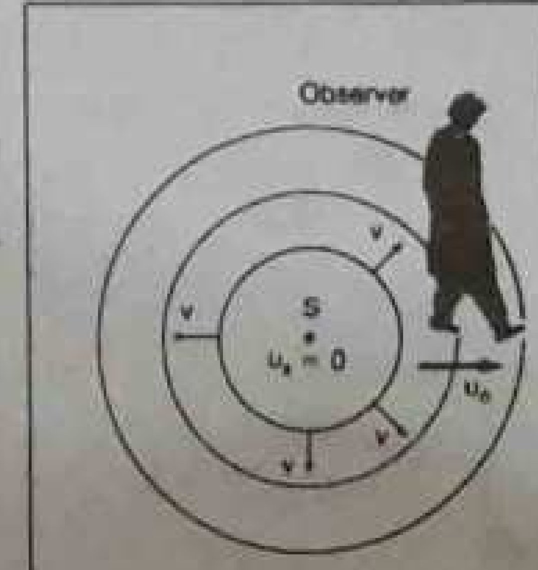


Fig. 8.19
An observer moving with velocity u_o away from stationary source hears a frequency f_B that is smaller than the source frequency.

$$f_B = \left[\frac{v - u_o}{\lambda}\right]$$

$$f_B = \left[\frac{v - u_o}{v/f}\right] \quad \text{(Using equation 1)}$$

$$f_B = \left[\frac{v - u_o}{v}\right] f$$

Since $$\frac{v - u_o}{v} < 1$$

so $$f_B < f$$

Result

Thus the apparent frequency of sound heard by the observer will ***decrease.***

Case III

[When source moves towards the stationary observer]

When source moves towards the stationary observer C with velocity u_s then waves are compressed and their wavelength is decreased. In this case the waves are compressed in a distance equal to $v - u_s$ in one second

Thus
the wavelength of sound waves for observer is

$$\lambda_c = \frac{v - u_s}{f} \quad \text{\_\_\_\_\_\_\_\_ (3)}$$

OR

$$\lambda_c = \frac{v}{f} - \frac{u_s}{f}$$

$$\lambda_c = \lambda - \Delta\lambda$$

Where

$\frac{u_s}{f} = \Delta\lambda$ = decrease in wavelength in one second called **Doppler shift** .So

$$\Delta\lambda = \frac{u_s}{f} \quad \text{\_\_\_\_\_\_\_\_ (4)}$$

Thus, the number of waves received by observer C in one second is

$$f_C = \frac{v}{\lambda_c}$$

putting the value of λ_c from equation (3),we get

$$f_c = \left(\frac{v}{\frac{(v - u_s)}{f}}\right)$$

$$f_c = \left(\frac{v}{v - u_s}\right) f \quad \text{\_\_\_\_\_\_\_\_ (5)}$$

Since $\frac{v}{v - u_s} > 1$

So $f_c > f$

Result

Thus the apparent frequency of sound heard by the observer will ***increase.***

Case IV

[Source moves away from the stationary observer]

When source moves away the stationary observer D with velocity u_s then waves are expanded and their wavelength is increased. In this case the waves expand in a distance equal to $v - u_s$ in one second

Thus

The wavelength of sound waves for observer D is

$$\lambda_D = \frac{v + u_s}{f} \quad \text{\_\_\_\_\_\_\_\_ (6)}$$

OR

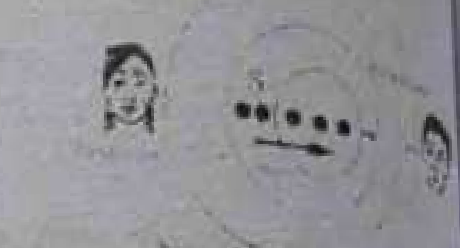
$$\lambda_D = \frac{v}{f} + \frac{u_s}{f}$$

$$\lambda_D = \lambda + \Delta\lambda$$

Where

$\frac{u_s}{f} = \Delta\lambda$ = increase in wavelength in one second called **Doppler shift**. So

$$\Delta\lambda = \frac{u_s}{f} \quad \text{\_\_\_\_\_\_\_\_ (7)}$$



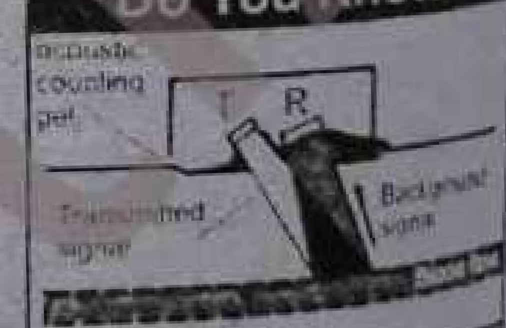
Do You Know?

Bats navigate and find food by echo location.

EXPLANATION

During flight, they emit ultrasound sound in a series of pulses and the reflected waves from the neighboring objects provide the bat the information and location of the food.

Do You Know?



The doppler effect can be used to monitor blood flow through major arteries. Ultrasound waves of frequencies 5MHz to 10MHz are directed towards the artery and a receiver detects the back scattered signal. The apparent frequency depends on the velocity of flow the blood.

EXPLANATION

Velocity of blood flow is detected by using Doppler effect by noting the Doppler shift (change in frequency)

Thus, the number of waves received by observer D in one second is

$$f_D = \frac{v}{\lambda_D}$$

Putting the value of λ_D from equation (6), we get

$$f_D = \left(\frac{v}{\frac{(v + u_s)}{f}}\right)$$

$$f_D = \left(\frac{v}{v + u_s}\right) f \quad \text{\_\_\_\_\_\_\_\_ (8)}$$

Since $\frac{v}{v + u_s} < 1.$

So $f_D < f$

Result

Thus the apparent frequency of sound heard by the observer will ***decrease.***

Q.14 Write down the uses of Doppler's Effect.

Ans.

Applications of Doppler Effect

Now we discuss some important applications of Doppler's effect.

(i) Radar System

RADAR is an acronym. It is derived from Rsdio Detection And Ranging.

Radar is a device which transmits and receives ***radio waves***. The radar system uses to determine the height and speed of aeroplane. This system emits radio waves which are reflected from aeroplane and received by the system.

If reflected waves have shorter wavelengths, then the aeroplane is moving towards a radar system. If reflected waves have longer wavelengths, then the object is moving away from the radar system.

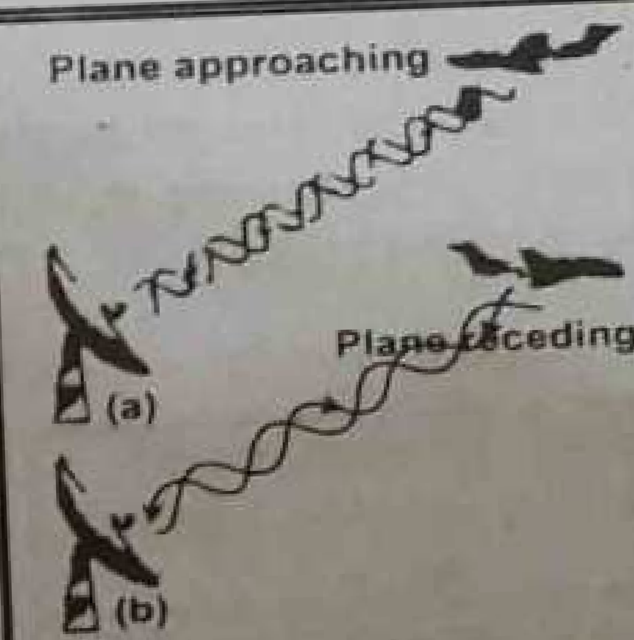


Fig. A
A frequency shift is used in a radar to detect the motion of a aeroplane

(ii) Speed of Satellite

The speed of satellite can also be determined by sending electromagnetic radiations from Earth. When these are reflected back after colliding with the satellite, then these are received on the earth. The value of Doppler's shift in wave length of these radiations gives the estimation of speed of satellite.

(iii) Sonar (Sound Navigation and Ranging)

Sonar is a technique for detecting the presence of objects under water by an *acoustic echo*. This system uses the *ultrasonic* waves because they can travel longer distances in water. Doppler detection depends upon the relative speed of the target and the detector. The apparent change in frequency is observed, which is Doppler's shift. In this way we can locate and detect submarines, antisubmarine weapons and mines etc. Also, the depth of sea can be measured.

For Your Information

When source and observer ***move towards*** each other with velocities u_s and u_o respectively, then waves are compressed in a distance equal to $v - u_s$ in one second and the relative velocity of the sound and observer becomes $v + u_o$. In this case both the relative velocity and the wavelength of wave changes. So, the apparent frequency is

$$f' = \left[\frac{v + u_o}{\lambda'}\right] = \left[\frac{v + u_o}{\frac{v - u_s}{f}}\right] = \left[\frac{v + u_o}{v - u_s}\right] f$$

When source and observer move away each other with velocities u_s and u_o respectively, then waves expand in a distance equal to $v + u_s$ in one second and the relative velocity of the sound and observer becomes $v - u_o$. In this case both the relative velocity and the wavelength of wave changes. So, the apparent frequency is

$$f' = \left[\frac{v - u_o}{\lambda'}\right] = \left[\frac{v - u_o}{\frac{v + u_s}{f}}\right] = \left[\frac{v - u_o}{v + u_s}\right] f$$

(iv) Speed of Star

By comparing the line spectrum of light coming from a distant star and the light emitted from laboratory source, Doppler's shift can be measured to calculate the speed of star with respect to Earth.

Stars moving towards the earth show blue shift and away from source show red shift.

Blue shift

The frequency of light emitted by the star increases (i.e. wavelength decreases) if it is moving towards the earth, as compared to the light emitted by stationary star. Thus spectrum is shifted towards *shorter wavelength* i.e. to the Blue end of spectrum, which is called Blue shift.

Red Shift

The frequency of light emitted by the star decreases (i.e. wavelength increases) if it is moving away from earth. Thus spectrum is shifted towards the *longer wavelength* i.e. towards the Red end of the spectrum, which is called red shift.

For Your Information

Echolocation allows dolphins to detect small differences in the shape, size and thickness of objects.

EXPLANATION

The dolphins emit ultrasound waves in the form of echoes which spread in water. By reflecting ultrasound waves from the obstacles, dolphin is able to locate the location, shape and size of the obstacles. The ultrasounds are emitted at frequencies of about 50000 c/s.

Note

As astronomers have also discovered that all the distant galaxies are moving away from us and by measuring their red shifts, they have estimated their speed.

(v) Speed of Car

Microwaves are emitted from a source in form of short bursts. Each burst is reflected back by any moving car in their way. The reflected bursts are detected in the detector. If these are reflected by moving car, the Doppler's shift is observed. By which speed of car is calculated. Microwaves are used to calculate the speed of car by computer program.

FORMULAE

Relation between velocity, frequency and wavelength of waves	$v = f\lambda$	
Relation between frequency and time period	$f = \frac{1}{T}$	
Relation between phase difference and path difference	$\phi = \frac{2\pi}{\lambda}x$	
Speed of sound	$v = \sqrt{\frac{E}{\rho}}$	
Boyle's law for isothermal process	PV = constant	
Newton's formula for speed	$v = \sqrt{\frac{P}{\rho}}$	
Boyle's law for adiabatic process	PV^{γ} = constant	
Speed of sound in air	$v = \sqrt{\frac{\gamma P}{\rho}}$	

Relation between speed of sound and temperature	$\frac{v_t}{v_o} = \sqrt{\frac{T}{T_o}}$	$v_t = v_0 + 0.61t$
Condition for constructive interference in sound waves	$\Delta S = n\lambda$ where $n = 0, \pm 1, \pm 2, \pm 3, ...$	
Condition for destructive interference in sound waves	$\Delta S = (2n+1)\frac{\lambda}{2}$ where $n = 0, \pm 1 \pm 2, \pm 3...$	$\Delta S = (n + \frac{1}{2})\lambda$ where $n = 0, \pm 1 \pm 2, \pm 3...$
Beat frequency	$f_A - f_B = n$	
Velocity of transverse waves on string	$v = \sqrt{\frac{F}{m}}$	
Fundamental frequency of stationary waves on string	$f = \frac{1}{2\ell}\sqrt{\frac{F}{m}}$	$f = \frac{v}{\lambda}$
Frequency of stationary waves on string(nth mode)	$f_n = n\left(\frac{v}{2\ell}\right)$	$f_n = nf_1$ where n = 1, 2, 3, 4, ----
Wave length of stationary waves on string (nth mode)	$\lambda_n = \frac{2\ell}{n}$ where n = 1,2,3, ...	
Frequency of longitudinal stationary waves in organ pipe open at both ends (nth mode)	$f_n = n\left(\frac{v}{2\ell}\right)$	$f_n = nf_1$ where n = 1, 2, 3, 4, ----
Wave length of longitudinal stationary waves in organ pipe open at both ends (nth mode)	$\lambda_n = \frac{2\ell}{n}$ where n = 1,2,3, ...	
Frequency of longitudinal stationary waves in organ pipe closed at one end (nth mode)	$f_n = n\left(\frac{v}{4\ell}\right)$	$f_n = nf_1$ where n = 1, 3, 5, 7, ----
Wave length of longitudinal stationary waves in organ pipe closed at one ends (nth mode)	$\lambda_n = \frac{4\ell}{n}$ where n = 1,3,5 ...	
Apparent frequency when observer(A) moves towards stationary source	$f_A = \left[\frac{v+u_o}{v}\right]f$	
Apparent frequency when observer(B) moves away from stationary source	$f_B = \left[\frac{v-u_o}{v}\right]f$	
Doppler shift	$\Delta\lambda = \frac{u_s}{f}$	
Apparent frequency when source moves towards stationary observer(C)	$f_C = \left(\frac{v}{v-u_s}\right)f$	
Apparent frequency when source(s) moves away from stationary observer(D)	$f_D = \left(\frac{v}{v+u_s}\right)f$	

Multiple Choice Questions

◆ *Four possible answers to each statement are given below. Tick (✓) the correct answer:*

1. **Longitudinal waves can not pass through:**
   (a) Liquid (b) Solids
   (c) Vacuum (d) Gases
2. **When two identical waves superimposed, the velocity of the resultant wave:**
   (a) Decreases (b) Increases
   (c) Remains unchanged (d) is zero
3. **Waves produced in organ pipes are:**
   (a) Transverse waves (b) Longitudinal waves
   (c) E.M. wave (d) All of above
4. **Speed of sound in summer ________ than in winter.**
   (a) Decreases (b) Increases
   (c) Remains same (d) Can not tell
5. **Newton calculated speed of sound using ________ process.**
   (a) Adiabatic (b) Isothermal
   (c) Both (a) and (b) (d) None of these
6. **Speed of sound in hydrogen is ________ times its speed in oxygen.**
   (a) 4 (b) ¼
   (c) 2 (d) ½
7. **Error in calculation of Newton's formula for speed of sound is about:**
   (a) 10% (b) 35%
   (c) 16% (d) 27%
8. **The frequency range of human ear is:**
   (a) 20 Hz – 20 kHz (b) 20 Hz - 20 MHz
   (c) 20 Hz - 200 Hz (d) 200 Hz - 2 KHz
9. **Beats are the results of:**
   (a) diffraction of sound waves (b) constructive and destructive interference
   (c) destructive interference (d) None of these
10. **If the time required for the tuning fork to make one complete vibration, the wave in air will travel a distance equal to:**
    (a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$
    (c) λ (d) 2λ
11. **The term which can tell us the stage of vibration of the particles of the medium is called:**
    (a) Time period (b) Phase
    (c) Wavelength (d) Amplitude
12. **It is common characteristics of all types of wave motion without the transport of particles:**
    (a) Particles (b) Drown
    (c) Energy transferred (d) Mass decreased
13. **When a string, which is tied at both ends is plucked from the centre of waves, produces:**
    (a) Transverse waves (b) Longitudinal waves
    (c) Standing waves (d) Electromagnetic waves

14. In a stretched string, if speed of the wave is doubled, the tension will be:

(a) 2 (b) 4

(c) 8 (d) 6

15. Frequency of a stretched string is proportional to the:

(a) Tension (b) Linear density

(c) Reciprocal of length (d) Square of tension

16. A string fixed at two ends vibrates in one segment. The standing wave pattern is:

(a) first overtone (b) second over tone

(c) fundamental (d) second overtone

17. Two sounds of the same frequency in air must have same:

(a) amplitude (b) Intensity

(c) loudness (d) wavelength

18. Which of the following phenomenon can not take place with sound waves?

(a) reflection (b) interference

(c) diffraction (d) polarization

19. Presence of moisture in air:

(a) Increases speed of sound (b) Decreases speed of sound

(c) May increase/decrease the sound velocity (d) Does not have effect

20. Doppler's shift measure the change in ________ of the wave due to relative motion of source and observer.

(a) Intensity (b) Frequency

(c) Velocity (d) Wavelength

ANSWERS

1. c	2. c	3. b	4. b	5. b	6. a	7. c	8. a	9. b	10. c
11. b	12. c	13. c	14. b	15. c	16. c	17.	18.	19. a	20. d

Short Questions of Exercise

Q.1 What features do longitudinal waves have in common with transverse waves?
(Federal 2005, Mir Pur 2006-2009, Bwp 2008, Lhr 2010-2011, Grw2011)

Ans. Common Features

- Both types of waves ***transport*** energy.
- Both types of waves can produce ***interference, diffraction, refraction*** and ***reflection.***
- For determination of the speed of wave, $\underline{v = f\lambda}$ is applicable for both these waves

Q.2 The five possible waveforms obtained, when the output from a microphone is fed into the Y-input of cathode ray oscilloscope, with the time base on, are shown in fig. These waveforms are obtained under the same adjustment of the cathode ray oscilloscope controls. Indicate the wave form

a) Which trace represents the loudest note?

b) Which trace represents the highest frequency?

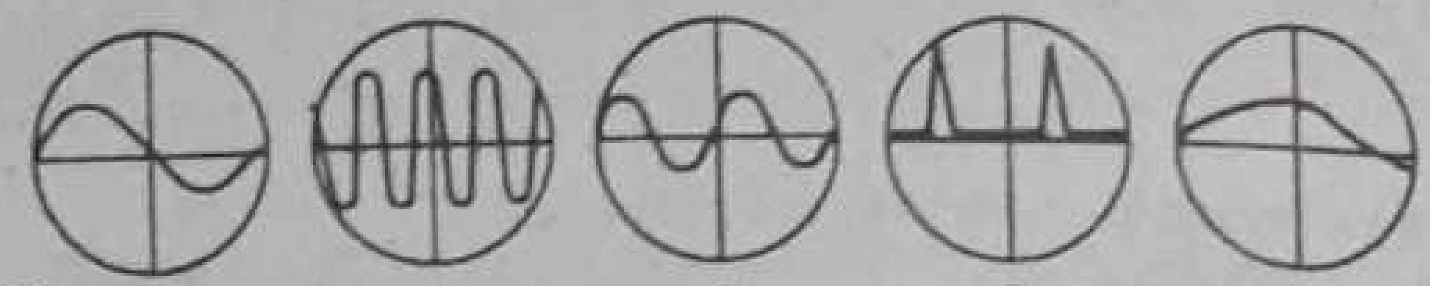


Fig. 8.23

(Lhr 2004, Bwp 2009, Fsd 2008, Grw 2008-2009)

Ans. **Loudest Note**

In fig. (d), the amplitude is maximum, so loudness is maximum for this case.

Maximum Frequency

In fig (b), the number of waves is maximum, so frequency is maximum in this case.

Q.8.3 Is it possible for two identical waves travelling in the same direction along a string to give rise to a stationary waves?

(Mtn 2003-2004, D.G.Khan 2005, Lhr 2006, Fsd 2005-2008, Rwp 2006, Grw 2008-2009-2010)

Ans. ***No***, it is not possible.

Reason

Stationary waves are produced only when two identical waves traveling in ***opposite*** direction along the same string superpose.

Q.8.4 A wave is produced along a stretched string but some of its particles permanently show zero displacement. What type of wave is it?

Ans. These are ***stationary*** waves.

Reason

Only in stationary waves some points of the medium permanently show zero displacement called ***nodes*** and some points show maximum displacement called ***antinodes***.

Q.8.5 Explain the terms crest and trough, node and antinode?

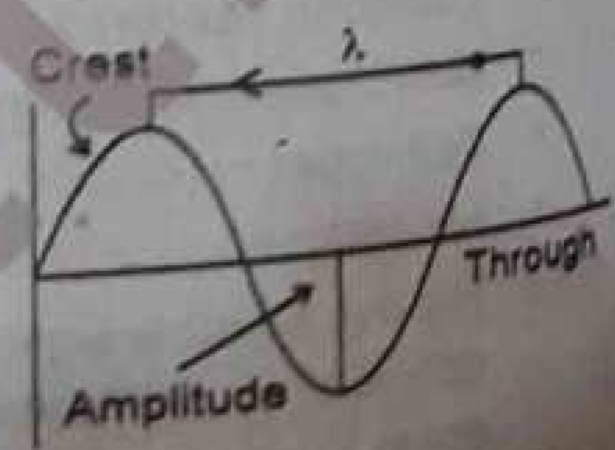
(Mtn 2003-2004-2009, Fsd 2005, Rwp 2006, Bwp 2009, Grw 2010, Lhr 2010-2011)

Ans. **Crest**

The portion of transverse wave ***above*** the equilibrium position is called trough

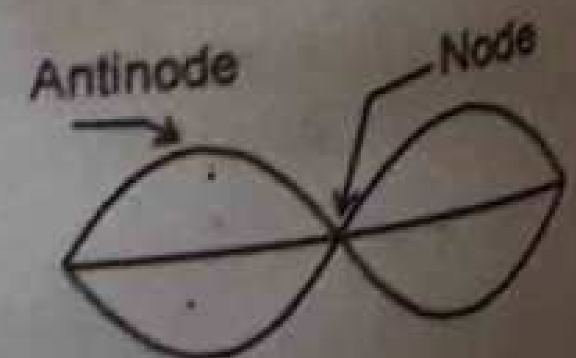
Trough

The portion of transverse wave ***below*** the equilibrium position is called trough.



Node

The points of ***zero*** displacement on stationary waves are called nodes.



Antinode

The points of ***maximum*** displacement on stationary waves are called antinodes.

Q.8.6 Why does sound travel faster in solids than in gases?

(Mtn 2006, Sgd 2003-2005, Fsd 2006-2008, Rwp 2006, Grw 2003-2004, D.G,Khan 2005, Mir Pur 2004, Bwp 2006)

Ans. Sound travels faster in solids than in gases

Reason:

Speed of sound is given by:

$$v=\sqrt{\frac{E}{\rho}}$$

where E is the modulus of elasticity and ρ is the density of medium. Although the density of solid is greater as compared to gases but modulus of elasticity for solids is ***much greater*** as compared to gases. So

$$\sqrt{\frac{E_{solid}}{\rho_{solid}}}>\sqrt{\frac{E_{gas}}{\rho_{gas}}}$$

Hence, sound travel faster in solids than in gases.

Q.8.7 How are beats useful in tuning musical instruments?

(D.G.Khan 2005-2006, Rwp 2005, Bwp 2006-2008, Lhr 2006- 2010-2011,Grw2011)

Ans. **Tuning of musical instruments**

In order to tune a musical instrument;

Sound the instrument against a note of known frequency. If the two frequencies do not match, beats will be produced. Adjust the frequency of the untuned instrument by ***tightening or loosening*** the string. When ***no beats*** are heard, the instrument is said to be tuned.

Q.8.8 When two notes of frequencies f_1 and f_2 are sounded together, beats are formed. If $f_1 > f_2$ what will be the frequency of beats?

(a) f_1+f_2 (b) $\frac{1}{2}(f_1+f_2)$

(c) f_1-f_2 (d) $\frac{1}{2}(f_1-f_2)$

(Federal 2005, Mir Pur 2005, Bwp 2008, Rwp 2008)

Ans. Since beat frequency is the equal to the difference of individual interfering frequencies

Beat frequency is $f_1 - f_2$.

Q.8.9 As a result of distant explosion, an observer senses a ground tremor and then hears the explosion. Explain the time difference? **(Lhr 2009)**

Ans. **Explanation**

The speed of sound is given by

$$v=\sqrt{\frac{E}{\rho}}$$

Since the speed of sound in solids (earth) is much greater than the speed of sound in gases (air) due to ***much greater*** value of elastic modulus. That is why the observer senses the ground tremor first and then hears the explosion.

Q.8.10 Explain why sound travels faster in warm air than in cold air?

(Fsd 2005, Sgd 2005, Bwp 2003, Federal 2004, Mtn 2009, Grw 2009)

Ans. **Reason**

The speed of sound is given by

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Since gases expand on heating. So the density of warm air ***decreases***. Hence, according to above equation, the speed of sound will be greater in warm air than in the cold air.

Q.8.11 How should a sound source move with respect to an observer so that the frequency of its sound does not change?

(Bwp 2007, Rwp 2008)

Ans. If the relative velocity between the source and the observer is ***zero***, there will be no change in frequency of sound.

Examples

- When the observer is at origin and sound source moves along the circumference of the circle then their distance remain the same and the frequency of sound does not change.
- Source and observer are moving in same direction with same velocity.

Solved Examples

Example 8.1

Find the temperature at which the velocity of sound in air is two times its velocity at 10°C.

Given Data:

Temperature = T_0 = 10°C = 10 + 273 = 283K
Velocity of sound at 283K = $v_{283} = v_0$
Velocity of sound at T= v_t
As velocity of sound is two times its velocity at 283 K
Therefore

$$v_t = 2v_o = 2v_{283}$$

To Find:

Temperature = T = ?

Calculation:

Using the formula

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_o}}$$

Or $$\frac{v_t}{v_{283}} = \sqrt{\frac{T}{283}}$$

Or $$\frac{2v_{283}}{v_{283}} = \sqrt{\frac{T}{283}},$$

$$2 = \sqrt{\frac{T}{283}}$$

Squaring on both sides

$$4 = \frac{T}{283}$$

$$T = 283 \times 4$$

$$T = 1132K$$

Or $$T = 1132 - 273 = 859C°$$

Example 8.2

A tuning fork A produces 4 beats per second with another tuning fork B. It is found that by loading B with some wax, the beat frequency increases to 6 beats per second. If the frequency of A is 320 Hz, determine the frequency of B when loaded.

Given Data:

Frequency of tuning fork A = f_A = 320 Hz
Number of beats per second before loading = n = 4
Number of beats per second after loading = n′= 6

To Find:

Frequency of tuning fork B after loading = f_B = ?

Calculation:

Case 1: Before loading:

As $f_A - f_B = \pm n$

or $f_B = f_A \pm n$

Putting values, we get

$f_B = 320 \pm 4$

f_B = 324 Hz or 316 Hz

By loading B, its frequency will decrease. If 324 Hz is the original frequency, the beat frequency will reduce. On the other hand, if it is 316 Hz, the beat frequency will increase which is the case.

So, the original frequency of B is 316 Hz.

Case 2: After Loading:

$$f_A - f_B' = n'$$

or $$f_B' = f_A - n'$$

$$f_B' = 320 - 6$$

$$f_B' = 314 \text{ Hz}$$

Thus f_B' when loaded is 314 Hz

Example 8.3

A steel wire hangs vertically from a fixed point, supporting a weight of 80N at its lower end. The diameter of the wire is 0.50 mm and its length from the fixed point to the weight is 1.5 m. Calculate the fundamental frequency emitted by the wire when it is plucked?

(Density of steel wire = 7.8×10^3 kgm^{-3})

Given Data:

Weight = W = F = 80 N
Diameter of steel wire = D = 0.50 mm = 0.50×10^{-3}m
Radius of steel wire = $r = \frac{D}{2} = \frac{0.5 \times 10^{-3}}{2} = 0.25 \times 10^{-3}$m
Length of the wire = ℓ = 1.5 m
Density of steel wire = $\rho = 7.8 \times 10^3$ kg m^{-3}

To Find:

Fundamental frequency = f_1 = ?

Calculation:

As formula for fundamental frequency is

$$f_1 = \frac{1}{2\ell}\sqrt{\frac{F}{m}} \quad ....(1)$$

Where m is the mass per unit length of the string.

So first we have to calculate the value of m.

As volume of the wire = Length × Area of cross section of wire.

volume of the wire = $\ell \times \pi r^2$

And

Mass of the wire = volume × density

$$M = \ell \times \pi r^2 \times \rho$$

Mass per unit length of the string = $m = \frac{M}{\ell} = \frac{\ell \times \pi r^2 \times \rho}{\ell}$

$$m = \pi r^2 \rho$$

Putting values, we get

$$m = 3.14 \times (0.25 \times 10^{-3})^2 \times 7.8 \times 10^3$$

$$m = 1.53 \times 10^{-3}\ \text{kg m}^{-1}$$

Putting values in equ. (1), we get

$$f_1 = \frac{1}{2\times1.5}\sqrt{\frac{80}{1.53\times10^{-3}}}$$

$$f_1 = \frac{1}{3.0}\sqrt{\frac{80}{0.00153}}$$

$$f_1 = \frac{1}{3}\sqrt{52287.5}$$

$$f_1 = \frac{1}{3}(228.6)$$

$$f_1 = 76.2\ \text{Hz}$$

Example 8.4

A pipe has a length of 1m. Determine the frequencies of the fundamental and the first two harmonics (a) if the pipe is open a both ends and (b) if the pipe is closed at one end.

(Speed of sound in air = 340 ms^{-1})

Given Data:

Length of Pipe = ℓ = 1m

Speed of sound = v = 340 m

To Find:

(a) If the Pipe is open at both ends.

Fundamental frequency = f_1 = ?

First harmonic = f_2 = ?

Second harmonic = f_3 = ?

(b) If the Pipe is open at one end.

Fundamental frequency = f_1 = ?

First harmonic = f_3 = ?

Second harmonic = f_5 = ?

Calculation:

(a) **When Pipe is open at both ends.**

Using the formula.

$$f_n = \frac{nv}{2\ell}$$

For fundamental frequency, n = 1

$$f_1 = \frac{1\times v}{2\ell}$$

$$f_1 = \frac{v}{2\ell}$$

Putting values

$$f_1 = \frac{340}{2\times1}$$

$$f_1 = 170\ \text{Hz}$$

So, for first harmonic n = 2

$$f_2 = 2f_1$$

$$f_2 = 2\times170$$

$$f_2 = 340\ \text{Hz}$$

For second harmonic n = 3

$$f_3 = 3f_1$$

$$f_3 = 3\times170$$

$$f_3 = 510\ \text{Hz}$$

(b) **When Pipe is open at one end**

Using the formula

$$f_n = \frac{nv}{4\ell}$$

For fundamental frequency n = 1

Thus $$f_1 = \frac{1\times v}{4\ell}$$

$$f_1 = \frac{v}{4\ell}$$

Putting values, we get

$$f_1 = \frac{340}{4\times1}$$

$$f_1 = 85\ \text{Hz}$$

In this case only odd harmonics are present.

So, for first harmonic n = 3

$$f_3 = 3f_1$$

$$f_3 = 3\times85$$

$$f_3 = 255\ \text{Hz}$$

For second harmonic = n = 5

$$f_5 = 5f_1$$

$$f_5 = 5\times85$$

$$f_5 = 425\ \text{Hz}$$

Example 8.5

A train is approaching a station at $90 kmh^{-1}$ sounding a whistle of frequency 1000 Hz. What will be the apparent frequency of the whistle as heard by a listener sitting on the platform? What will be the apparent frequency heard by the same listener if the train move away from the station with the same speed? (Speed of sound = $340\ ms^{-1}$)

Given Data.

Speed of train = $U_s = 90\ Kmh^{-1} = \frac{90 \times 1000}{60 \times 60} = 25\ ms^{-1}$

Speed of sound = $v = 340\ ms^{-1}$

Frequency of source = $f = 1000$ Hz

To Find:

(a) Apparent frequency (when train is reaching a stationery observer) = $f' = ?$

(b) Apparent frequency (when train is moving away from the observer) = $f'' = ?$

Calculation:

(a) When train is approaching toward listener, then using the formula

$$f' = \left(\frac{v}{v - Us}\right) f$$

Putting values, we get

$$f' = \left(\frac{340}{340 - 25}\right) \times 1000$$

$$f' = \frac{340}{315} \times 1000$$

$$f' = 1079.4\ Hz$$

(b) When the train is moving away from the listener, then using the formula

$$f'' = \left(\frac{v}{v + Us}\right) f$$

Putting values, we get

$$f'' = \left(\frac{340}{340 + 25}\right) \times 1000$$

$$f'' = \frac{340}{365} \times 1000$$

$$f'' = 931.5\ Hz$$

Exercise Problems

8.1 The wave length of the signals from a radio transmitter is 1500m and the frequency is 200 kHz. What is the wave length for a transmitter operating at 1000 kHz and with what speed the radio waves travel?

Given data:

Wavelength of the signal = $\lambda_1 = 1500m$

Frequency of the signal = $f_1 = 200kHz = 200 \times 10^3$ Hz

Frequency for transmitter = $f_2 = 1000kHz = 1000 \times 10^3$ Hz

To find:

Wavelength for the transmitter = $\lambda_2 = ?$

Speed of radio waves = $v = ?$

Calculation:

As $v = f\lambda$

Thus

or $v = f_1\lambda_1$

Putting values, we get

$$v = 200 \times 10^3 \times 1500$$

$$\boxed{v = 3.0 \times 10^8 m/sec}$$

As the transmitter emits radio waves, so

$$v = f_2\lambda_2$$

$$\lambda_2 = \frac{v}{f_2}$$

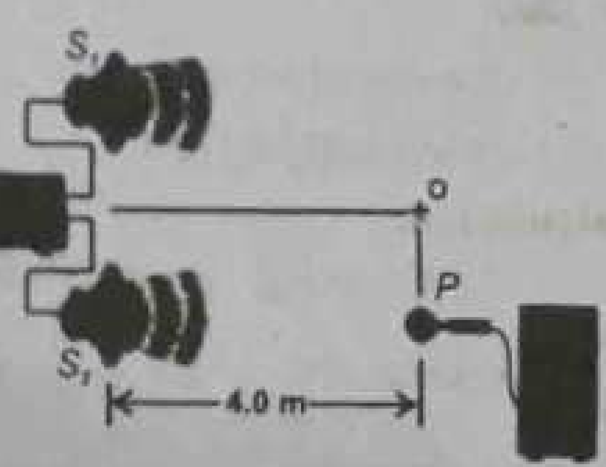
Putting values, we get

$$\lambda_2 = \frac{3 \times 10^8}{1000 \times 10^3}$$

$$\lambda_2 = 3 \times 10^2 m$$

$$\boxed{\lambda_2 = 300m}$$

8.2 Two speakers are arranged as shown in Fig. 8.24. The distance between them is 3 m and they emit a constant tone of 344 Hz. A microphone P is moved along a line parallel to and 4.00 m from the line connecting the two speakers. It is found that tone of maximum loudness is heard and displayed on the CRO when microphone is on the center of the line and directly opposite each speakers. Calculate the speed of sound.



Given data:

Distance between speakers = 3m

Tone frequency = f = 344 Hz

Distance between speakers and line of motion of P = 4m

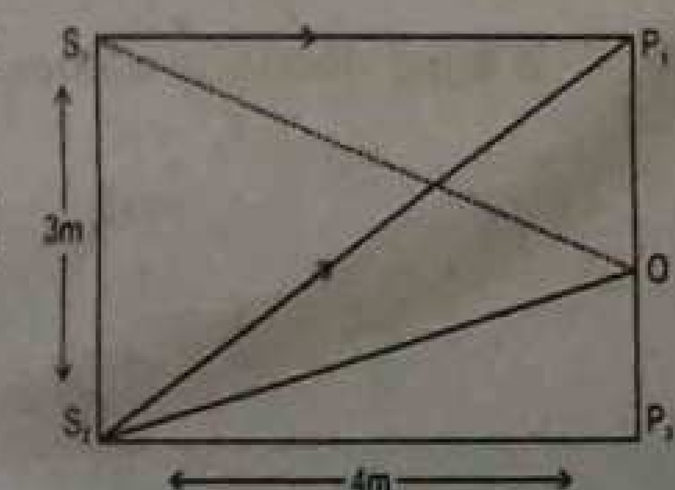
To find:

Speed of sound = v = ?

Calculation:

For tone of maximum loudness or the condition for constructive interference, the path difference must be $0, \pm 1\lambda, \pm 2\lambda, \pm 3\lambda, ...$

At middle point 'O' the path difference between two sound waves is zero (as $S_1O = S_2O$) thus at that point construction interference takes place.



For the next point Pof construction interference the path difference between waves should be λ.

So λ = path difference = $S_2p_1 - S_1p_1$

Now we calculate values of S_2p_1

From right angle triangle $S_1S_2p_1$

$$S_2p_1 = \sqrt{(S_1S_2)^2 + (S_1p_1)^2}$$

$$= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5m$$

Therefore path difference = $S_2S_1 - S_1P_1$

Or $\lambda = 5 - 4$

$\lambda = 1$ m

This is the path difference for constructive interference

Now $v = f\lambda$

putting the values, we get

$v = 344 \times 1$

$\boxed{v = 344 ms^{-1}}$

8.3 A stationary wave is established in a string which is 120 cm long and fixed at both ends. The string vibrates in four segments; at a frequency of 120 Hz. Determine its wavelength and the fundamental frequency?

Given data:

Length of string = ℓ = 120 cm = $\frac{120}{100} = 1.2m$

Number of loops = n = 4

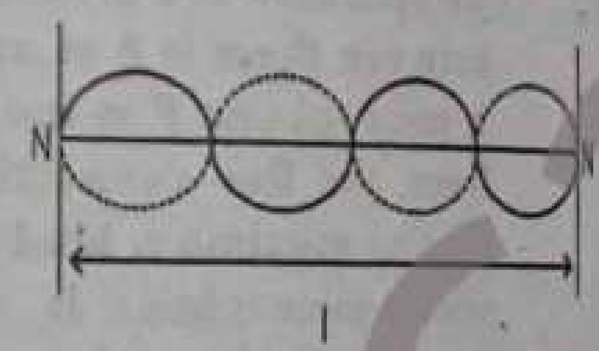
Frequency of vibration in four segments = f_4 = 120 Hz

To find:

Fundamental frequency = f_1 = ?

wavelenght = λ = ?

Calculation:



As the string vibrates in four segments and the distance between two consecutive nodes is $\frac{\lambda}{2}$, so the length of the string is ℓ

Or $\lambda = \frac{l}{2}$

Or $\lambda = \frac{1.2}{2}$

$\boxed{\lambda = 0.6m}$

If string vibrates in n loops, then frequency of stationery waves will be

$f_n = nf_1$

$f_4 = 4f_1$ As n=4

Or $f_1 = \frac{f_4}{4}$

$f_1 = \frac{120}{4}$

$\boxed{f_1 = 30Hz}$

8.4 The frequency of the note emitted by a stretched string is 300 Hz . What will be the frequency of this note when:

(a) the length of the wave is reduced by one – third without changing the tension.

(b) The tension is increased by one-third without changing the length of the wire.

Given data:

Frequency of the stretched string = f = 300Hz

To find:

(a) f' = ? (when length of the wave is reduced by one-third without changing the tension.)

(b) f'' = ? (when tension is increased by one-third without changing the length.)

Calculation:

(a) When length of the wave is reduced by one-third without changing the tension:-

When tension in the string is constant, then speed remains constant, hence

$v = f\lambda$

and $v = f'\lambda'$

Thus $f\lambda = f'\lambda'$ .... (1)

If λ is the original wavelength, then the reduced wavelength is,

$\lambda' = \lambda - \frac{1}{3}\lambda$

$\lambda' = \frac{2}{3}\lambda$

thus equation (1)becomes

$300\lambda = f' \times \frac{2}{3}\lambda$

or $300 = \frac{2}{3}f'$

or $\boxed{f' = 450Hz}$

(b) When tension is increased by one-third without changing the length:-

As the relation of fundamental frequency for a stretched string is

$f = \frac{1}{2l}\sqrt{\frac{F}{m}}$ ....(2)

when tension is increased by $\frac{1}{3}$F, then

$F' = F + \frac{1}{3}F$

$F' = \frac{4}{3}F$

$f'' = \frac{1}{2l}\sqrt{\frac{F'}{m}}$

$f'' = \frac{1}{2l}\sqrt{\frac{\frac{4}{3}F}{m}}$

Or $f'' = \sqrt{\frac{4}{3}}\left[\frac{1}{2l}\sqrt{\frac{F}{m}}\right]$

Or $f'' = \sqrt{\frac{4}{3}}\, f$ as f=300Hz

Thus $f'' = 1.15 \times 300$

$\boxed{f' = 346Hz}$

8.5 An organ pipe has a length of 50 cm. Find the frequency of its fundamental note and the next harmonic when it is

(a)- open at both ends **(b)- closed at one end**

(speed of sound =350mss^{-1})

Given data:

Length of organ pipe $= l = 50cm = 0.5m$

Speed of sound $= v = 350\ ms^{-1}$

To find:

(a)- (when pipe is open at both end)

fundamental frequency $= f_1 = ?$)

Next harmonic frequency $= f_2 = ?$)

(b)- (when pipe is closed at one end)

fundamental frequency $= f_1 = ?$)

Next harmonic frequency $= f_3 = ?$)

Calculation:

(a) When pipe is open at both end:-

The frequency for nth harmonic in open organ pipe is

$$f_n = \frac{nv}{2l} \quad \text{when } n=1,2,3,\ldots\ldots\ldots\ldots$$

So the fundamental frequency is

$$f_1 = \frac{1\times350}{2\times0.5} \qquad \text{put } n=1$$

$$\boxed{f_1 = 350Hz}$$

Next harmonic frequency i. e. n=2 is

$$f_2 = \frac{2v}{2l}$$

$$f_2 = \frac{v}{l} = \frac{350}{0.5}$$

$$\boxed{f_2 = 700Hz}$$

(b) When pipe is closed at one end:-

When the pipe is closed at one end , then frequency for nth harmonic is

$$f_n = \frac{nv}{4l} \quad \text{when } n=1,3,5,7,\ldots\ldots\ldots\ldots\ldots$$

So fundamental frequency is

$$f_1 = \frac{1\times350}{4\times0.5} \qquad \text{(Put } n=1)$$

$$f_1 = 175Hz$$

Next harmonic frequency i.e. n=3 is

$$f_3 = \frac{3v}{4l}$$

$$f_3 = \frac{3\times350}{4\times0.5}$$

$$\boxed{f_3 = 525Hz}$$

AZmK

8.6 A church organ consists of pipes ,each open at one end, of different lengths. The minimum length is 30 mm and the longest is 4m. Calculate the frequency range of the fundamental notes . (speed of sound =340 ms^{-1})

Given data:

Minimum length $= l_{min} = 30mm = 30\times10^{-3}m$

Maximum length $= l_{max} = 4m$

Speed of sound $= v = 340\ ms^{-1}$

To find:

Frequency range =? i.e. $f_1 = ?$ & $f_1' = ?$

Calculation:

For organ pipe open at one end only ,

$$f_n = \frac{nv}{4l}$$

Minimum length

For fundamental frequency, put n = 1

$$f_{1,max} = \frac{nv}{4l_{min}}$$

$$f_{1,max} = \frac{1\times340}{4\times30\times10^{-3}}$$

$$\boxed{f_{1,max} = 2833Hz}$$

Maximum length

For fundamental frequency, put n = 1

$$f_{1,min} = \frac{nv}{4l_{max}}$$

$$f_{1,min} = \frac{1\times340}{4\times4}$$

$$\boxed{f_{1,min} = 21Hz}$$

Result

So the fundamental frequency ranges from 21 Hz to 2833 Hz.

AZmK

8.7 Two tuning forks exhibit beats at a beat frequency of 3 Hz .The frequency of one fork is 256 Hz. Its frequency is then lowered slightly by adding a bit of wax to one of its prong. The two forks then exhibit a beat frequency of 1Hz. Determine the frequency of the second tuning fork.

Given data:

Frequency of first tuning fork $= f_1 = 256Hz$

Beat frequency before loading = 3 Hz

Beat frequency after loading = 1 Hz

To find:

Frequency of second tuning fork $= f_2 = ?$

Calculation:

As $\quad f_1 - f_2 = \pm n$

Or $\quad f_2 = f_1 \pm n$

Putting values, we get

$$f_2 = 256\pm3$$

$$f_2 = 259Hz \text{ or } 253Hz$$

To select the correct answer

If we consider 259 Hz as correct answer (i.e., frequency of second tuning fork). When first fork is loaded with wax, the frequency of first fork must fall below 256Hz i.e., 255, 254, …… and thus the number of beats produced per second will increase and will be greater then 3 beats.

Since the number of beats per second decreases on loading first fork is one, therefore 259 is not correct frequency of second tuning fork.
Thus

Correct frequency = f_2=253Hz As (254-253=1Hz)

8.8 Two cars P and Q travelling along a motorway in the same direction. The leading car P travels at a steady speed of 12 ms^{-1}; the other car Q, travelling at a steady speed of 20 ms^{-1}, sound its horn to emit a steady note which P's driver estimates, has a frequency of 830 Hz. What frequency does Q's own driver hear ?

(speed of sound = 340 ms^{-1})

Given data:

Speed of car P = U_p =12 ms^{-1}

Speed of car Q = U_Q =20 ms^{-1}

Speed of sound = v = 340 ms^{-1}

Frequency heard by P's driver = f_p=830Hz

To find:

Frequency heard by Q's driver = f_Q = ?

Calculation:

Speed of Q relative to P = $U_s=U_Q-U_p$ = 20-12 =8 ms^{-1}

Using Doppler's formula, when source is moving toward listener

$$f'=\left(\frac{v}{v-U_s}\right)f$$

Or $$f_p=\left(\frac{v}{v-U_s}\right)f_Q$$

Putting values, we get

$$830=\left(\frac{340}{340-8}\right)f_Q$$

$$830=\left(\frac{340}{332}\right)f_Q$$

Or $$f_Q=\frac{830\times332}{340}$$

$$\boxed{f_Q=810.47Hz}$$

8.9 A train sounds its horn before it sets off from the station and an observer waiting on the platform estimates its frequency at 1200Hz. The train then moves off and accelerates steadily. Fifty seconds after departure, the driver sounds the horn again and the platform observer estimates the frequency at 1140 Hz. Calculate the train speed 50 s after departure. How far from the station is the train after 50s ?

(speed of sound = 340 ms^{-1})

Given data:

Original frequency of horn = f = 1200Hz

Apparent frequency = f ' = 1140Hz

Speed of sound = v = 340 ms^{-1}

Time = t = 50 sec

To find:

Speed of source (i.e.train) = U_s=?

Distance covered by the train = S = ?

Calculation:

Using Doppler, s formula, when source is moving away from listener

$$f'=\left(\frac{v}{v+U_s}\right)f$$

Putting values, we get

$$1140=\left(\frac{340}{340+U_s}\right)1200$$

Or $$340+U_s=\frac{340\times1200}{1140}$$

Or $$340+U_s=357.89$$

Or $$U_s = 357.89-340$$

$$\boxed{U_s = 17.89\ ms^{-1}}$$

Distance covered by train after 50s

As the acceleration of the train is uniform,

So distance covered = S = $v_{av}\times t$

since $v_{av}=\frac{v_i+v_f}{2}=\frac{0+17.89}{2}$ = 8.95ms^{-1}

so $S = v_{av}\times t$

Putting values, we get

$$S = 8.95\times50$$

$$\boxed{S = 448m}$$

8.10 The absorption spectrum of faint galaxy is measured and the wavelength of one of the lines identified as the Calcium α line is found to be 478 nm. The same line has a wavelength of 397 nm when measured in a laboratory.

a) Is the galaxy moving towards or away from the Earth?

b) Calculated the speed of the galaxy relative to Earth?

(Speed of light = 3.0 × 10^8 ms^{-1})

Given data:

Original wavelength (laboratory measured) = λ= 397 nm = 397×10^{-9}m

Apparent wavelength = λ' =478nm=478×10^{-9}m

speed of light = v = c = 3.0×10^8 ms^{-1}

To Find:

a) Is the glaxy moving towards or away from earth = ?

b) Speed of galaxy relative to earth = U_s = ?

Calculation:

As v = fλ or c = fλ where c = speed of light

For original frequency,

$$f=\frac{c}{\lambda}$$

$$f=\frac{3\times10^8}{397\times10^{-9}}$$

$$f=7.56\times10^{14}Hz$$

For apparent frequency,

$$f'=\frac{c}{\lambda'}$$

$$f'=\frac{3\times10^8}{478\times10^{-9}}$$

$$f'=6.28\times10^{14}Hz$$

(a)- As $\lambda'>\lambda$ or $f'<f$ so galaxy is moving away from earth.

(b)- Using Doppler effect, When source is moving away from observer, then

$$f'=\left(\frac{v}{v+U_s}\right)f \qquad \text{As} \quad v=c$$

$$f'=\left(\frac{c}{c+U_s}\right)f$$

Putting values, we get

$$6.28\times10^{14}=\left(\frac{3\times10^8}{3\times10^8+U_s}\right)\times7.56\times10^{14}$$

$$6.28\left(3\times10^8+U_s\right)=3\times10^8\times7.56$$

$$6.28\times3\times10^8+6.28U_s=3\times10^8\times7.56$$

$$6.28U_s=22.68\times10^8-18.84\times10^8$$

$$6.28U_s=384\times10^6$$

$$U_s=\frac{384\times10^6}{6.28}$$

Or $\boxed{U_s=6.12\times10^7 ms^{-1}}$

****

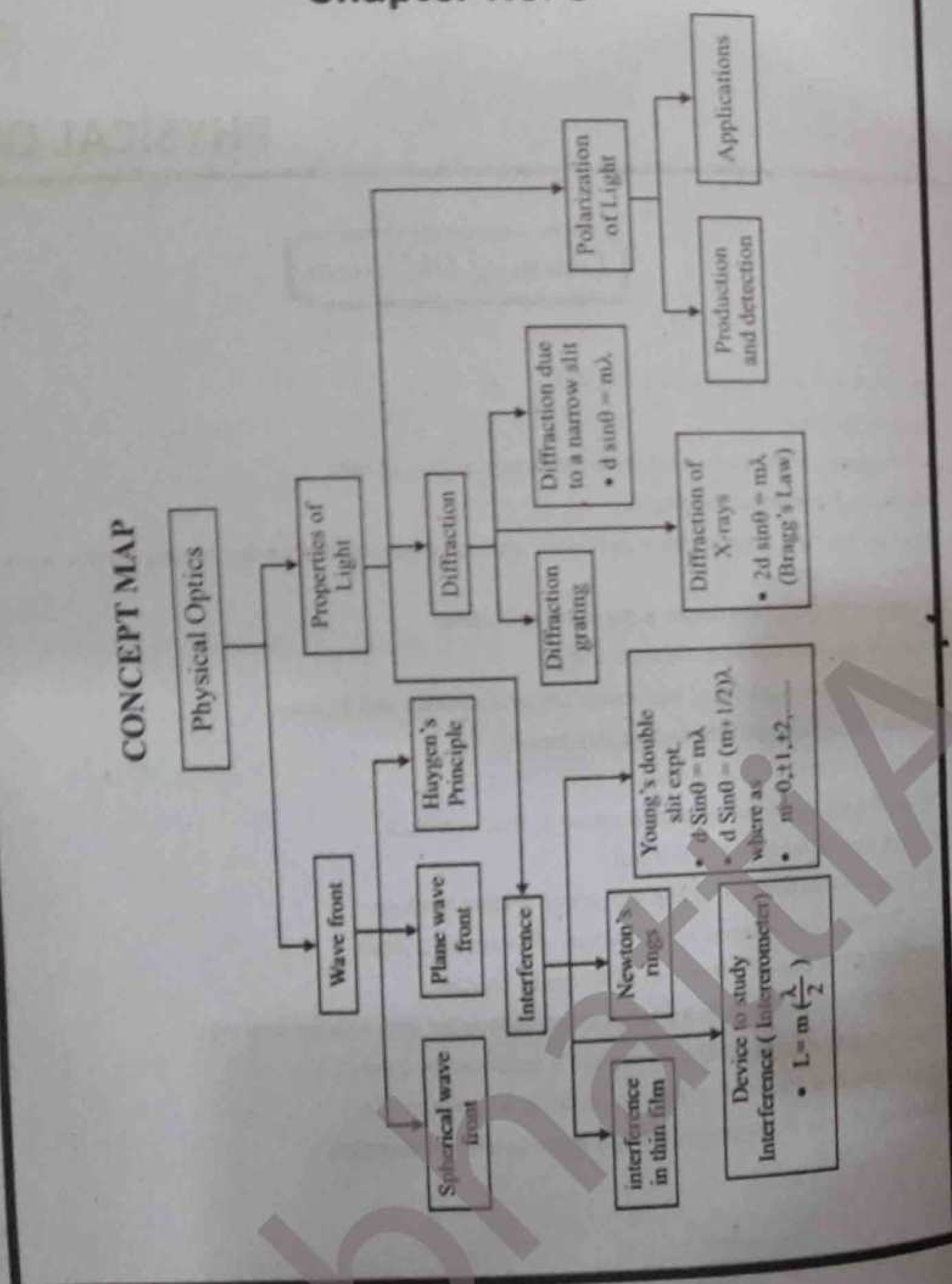
Chapter 9

PHYSICAL OPTICS

Learning Objectives

1. Understand the concept of wavefront.
2. State Huygen's principle.
3. Use Huygen's principle to explain linear superposition of light.
4. Understand interference of light.
5. Describe Young's double stilt experiment and the evidence it provided to support the wave theory of light.
6. Recognize and express colour patterns in thin films.
7. Describe the formation of Newton's rings.
8. Understand the working of Michelson's interferometer and its uses.
9. Explain the meaning of the term diffraction.
10. Describe diffraction at single slit.
11. Derive the equation for angular position of first minimum.
12. De4rive the equation d Sin $\theta = m\lambda$
13. Carry out calculations using the diffraction grating formula.
14. Describe the phenomenon of diffraction of X-rays by crystals.
15. Appreciate the use of diffraction of X-rays by crystals.
16. Understand polarization as a phenomenon associated with transverse waves.
17. Recognize and express that polarization is produced by a Polaroid.
18. Understand the effect of rotation of Polaroid on polarization.
19. Understand how plane polarized light is produced and detected.

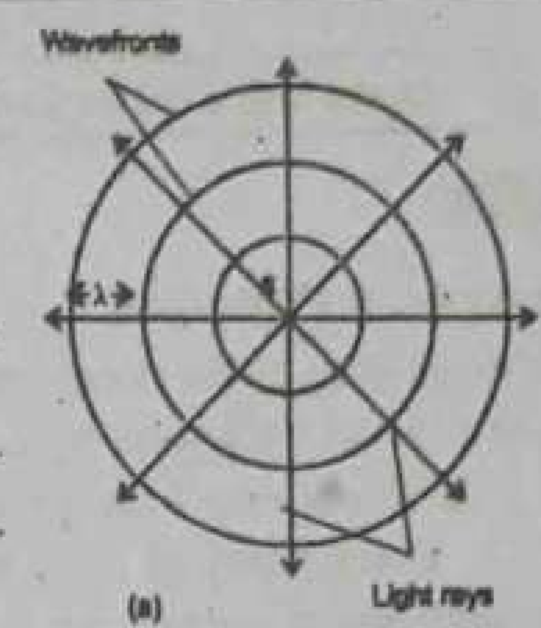
Chapter No. 9

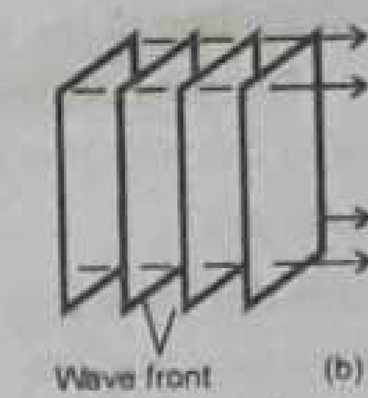


The branch of physics which deals with the nature of light and its different phenomenon is called physical optics.

Light is form of energy which produce the sensation of vision.

In 1678, *Huygens's*, an eminent *Dutch scientist*, proposed that light is the form of energy which travels in form of waves.





Spherical wave fronts (a) and plane wavefronts (b) spaced a wavelength apart. The arrows represent rays.

Q.1 What is a wave front?

Ans.

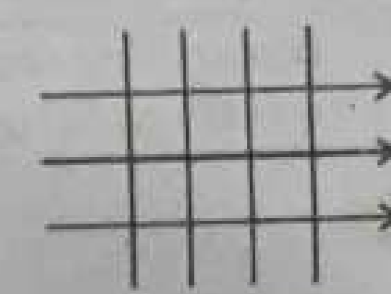
Wave Fronts

The surface on which all the points of waves have same phase of vibration is knows as wave front

Explanation

Suppose the light emitted from a point source propagates outward in all direction with speed c. After time t, the waves reaches the surface of an imaginary sphere with center as S and radius as ct.

As the distance of all these points from the source is same so all the points on the surface of the sphere have the same phase of vibration. Such as surface is known as wave front.

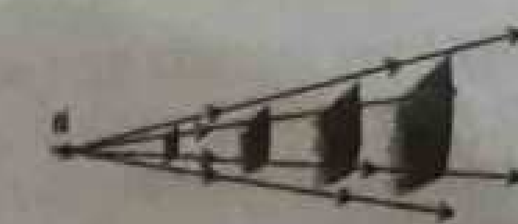


Note

The wave front from a point source are ***spherical.***

Thus wave propagates in space by the ***motion of wave fronts.***

The distance between two consecutive wave fronts is ***one wave length.***



Do You Know?

Small segments of large spherical wavefronts approximate a plane wavefront.

Ray of Light

The line normal to the wave front which shows the direction of propagation of light is called a ray of light.

(1) Spherical wave front

The wave front in which the light waves are propagated in spherical form with the source is called spherical wave front.

For a point source of light in a homogenous medium, the wave fronts are the concentric sphere of increasing radii.

EXPLANATION:
The wavefronts far away from the source are very large spheres. A small portion of the sphere will be the plane wavefront.

(2) Plane wave front

At very large distance (i.e. at infinity) from the source, a small portion of spherical wave front will become very nearly plane. Such a wave front is known plane wave front as shown in figure.

For example, *the sun light reaches the earth in plane wave fronts.*

On laboratory scale, in order to obtain plane wave from a point source, it is placed at the focus of convex lens.

Q.2 State and explain the Huygen's principle?

Ans.

Huygen's Principle

If the location of the wave front at any instant t is known then Huygen's principle enables us to determine shape and location of the new wave front at a later time $t + \Delta t$. This principle has two parts;

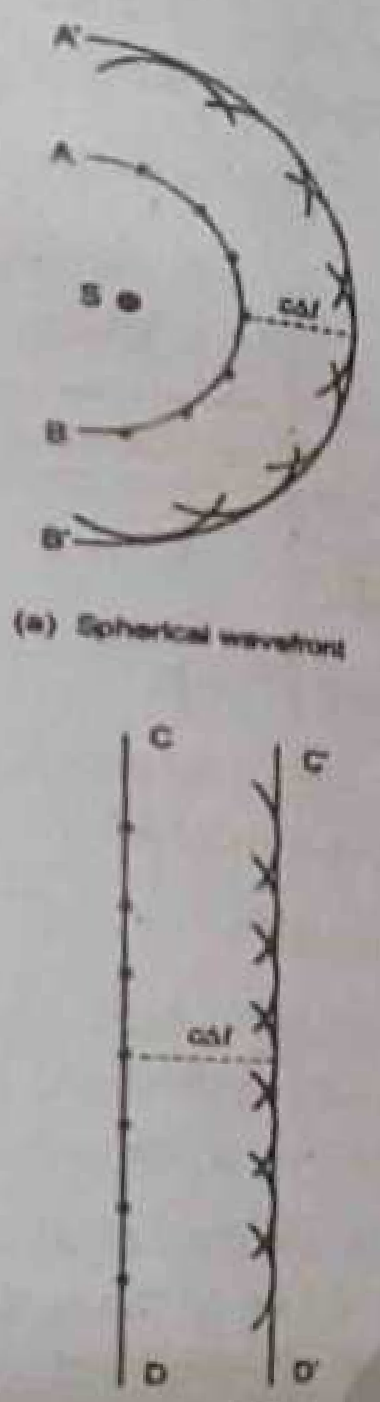
i. Every point of a wave front may be consider as a source of secondary wavelets which spread out in forward direction with a speed equal to the speed of propagation of the wave.

ii. The new position of the wave front after a certain interval of time can be found by constructing a surface that touches all the secondary wavelets.

Explanation:

Let AB is the wave front at time t.

To determine the wave front at time $t+\Delta t$, draw secondary wavelets with center at various points on the wave front $A'B'$ and radius as $c\Delta t$, where c is the speed of propagation of wave. The new wave front at time $t + \Delta t$ is $A'B'$ which is a tangent envelope to all the secondary wavelets.



(a) Spherical wavefront

(b) Plane wavefront

Fig.9.2

Huygens' construction for determining the position of the wavefronts AB and CD after a time interval Δt. A'B' and C'D' are the new positions of the wavefronts.

Q.3 Discuss the interference of light, discuss its different types and conditions for detectable interference.

Ans.

Interference of Light Waves

When two identical light wave travelling in the same direction are superimposed to each other in such a way that they reinforce each other at some points (constructive interference) While at some points they cancel the effect of each other (destructive interference). Such phenomenon is called interference of light."

Types of interference

There are two types of interference

(i) Constructive interference

If crest of one wave falls on the crest wave, then they support each other. Such a interference in known as constructive interference.

For constructive interference

$$\text{Path difference } = m\lambda \qquad \text{where } m = 0,1,2,3\ldots\ldots\ldots$$

(ii) Destructive interference

If crest of one wave falls on the trough of the other wave, then they cancel each other. Such a interference in known as destructive interference.

For destructive interference

$$\text{Path difference } = \left(m+\frac{1}{2}\right)\lambda, \qquad \text{where } m = 0,1,2,3,\ldots\ldots$$

$$=(2m+1)\frac{\lambda}{2}$$

For Your Information

Monochromatic Light

Sodium chloride in a flame gives out pure yellow light. This light is not a mixture of red and green.

Conditions for detectable interference pattern

The following condition must be met, in order to observe the interference phenomenon;

1. The interfering beams must be ***monochromatic.***
2. The interfering beams of light must be *coherent.*
3. The sources should be *narrow* and *very **close*** to each other.
4. The intensity of the two sources be ***comparable.***

Monochromatic Sources

The sources which should emit the light of single wave length are called monochromatic sources.

Coherent Sources

The monochromatic sources of light which emit waves, having a constant phase difference, are called coherent sources.

How to obtain coherent sources

A common method to obtain the coherent light beam is to use a monochromatic source to illuminate a screen containing two small closely spaced holes, usually in the shape of slits. The light emerging from the two slits is coherent because a single source produces the original beam and two slits serve only to split it into two parts. The points on a Huygen's wave front which sent out secondary wavelength are also coherent sources of light.

Q.4 Describe the Young's double slit experiment for demonstration of interference of light. Derive an expression for fringe spacing.

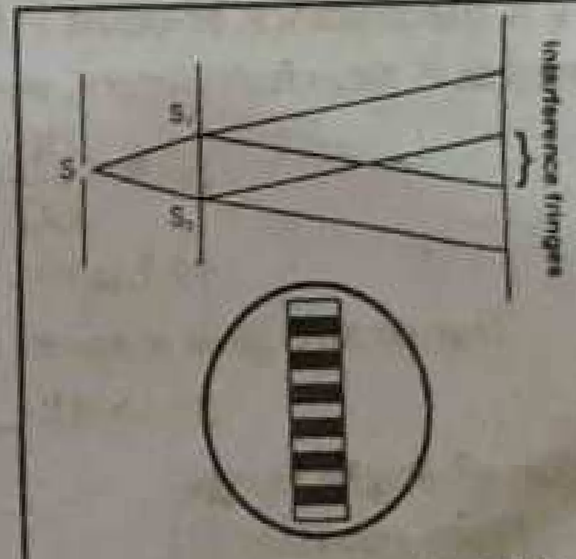
learn the diagrams, and mathematical derivation and read the theory only

Ans.

Young's Double Slit Experiment

In 1801, Thomas Young performed the interference experiment to prove the wave nature of light. A screen having two narrow slits is illuminated by a beam of monochromatic light.

The portion of wave front incident on the slit behaves like the source of secondary wavelets. The wavelets leaving the slits are coherent. Superposition of these wavelets results into the series of bright and dark bands which are observed on the second screen placed at some distance parallel to the first screen.



Ray geometry of Young's double slit experiment

The bright fringes are termed as maxima and dark as minima.

Conditions for Maxima and Minima

In order to derive the equations for maxima and minima, an arbitrary point P is taken on the screen on one side of the central point O as shown in figure. AP and BP are the parts of the rays reaching P. The line AD is drawn such that AP=DP. The separation between the centers of the two slits is AB=d.

The distance of the second screen from the slits is CO=L. The angle between CP and CO is θ, it can be proved that the angle BAD=θ by assuming that AD is nearly normal BP. The path difference between the wavelets, leaving slits and arriving at P, is BD. From right angled triangle ADB, we can write

$$\frac{BD}{AB} = \sin\theta$$

or $\quad BD = AB \sin\theta$

$(AB = d)$

Path difference = BD = d sin θ __________ (1)

Constructive Interference

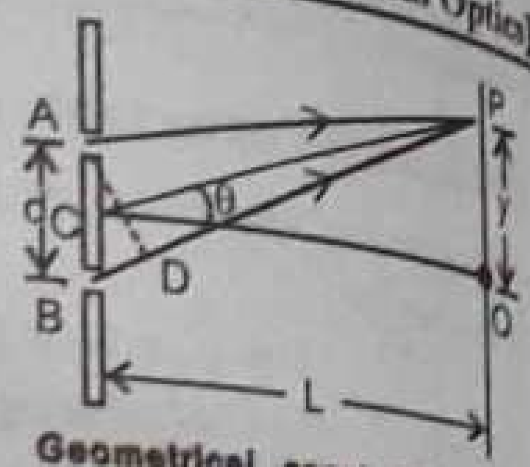
If the point P is to have bright fringe, the path difference BD must be an integral multiple of wavelength. So,

$$BD = m\lambda$$

Thus $\quad d \sin\theta = m\lambda$ __________ (2)

Where m = 0,1,2,3,.......

Where m is called the order of the fringe. At central point O, the path difference BP-AP=0, so there will be a bright fringe at O corresponding to m=0. This is called zeroth order fringe.



Geometrical construction of Young's double slit experiment

Destructive Interference:

If a dark fringe is formed at P, the path difference BD must contain half integral multiple of wavelengths. According to the condition for dark fringe.

$$BD = [m + \frac{1}{2}]\lambda$$

Thus $\quad d \sin\theta = [m + \frac{1}{2}]\lambda$ __________ (3)

Where $\quad$ m = 0, 1,2,3,.....

For Your Information

θ	sinθ	tanθ
2	0.035	0.035
4	0.070	0.070
6	0.104	0.105
8	0.139	0.140
10	0.174	0.176

Position of Dark and Bright Fringe:

Let y is the distance of point P from the central point O and a bright fringe is formed at P. Now from figure, we can write from trianglePOC

$$\tan\theta = \frac{OP}{OC} = \frac{y}{L}$$

$$y = L\tan\theta$$

(for small θ $\sin\theta \approx \tan\theta$)

$$y = L\sin\theta \quad \_\_\_\_\_\_\_\_\_\_ (4)$$

Position of Bright Fringe:

From equation (2)

$$\sin\theta = \frac{m\lambda}{d}$$

Thus equation (4) becomes

$$y = m\frac{\lambda L}{d} \qquad m = 0,1,2,3....$$

Position of Dark Fringe:

From equation (3)

$$\sin\theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{d}$$

Thus equation (4) becomes

$$y = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \qquad m = 0,1,2,3....$$

Fringe Spacing:

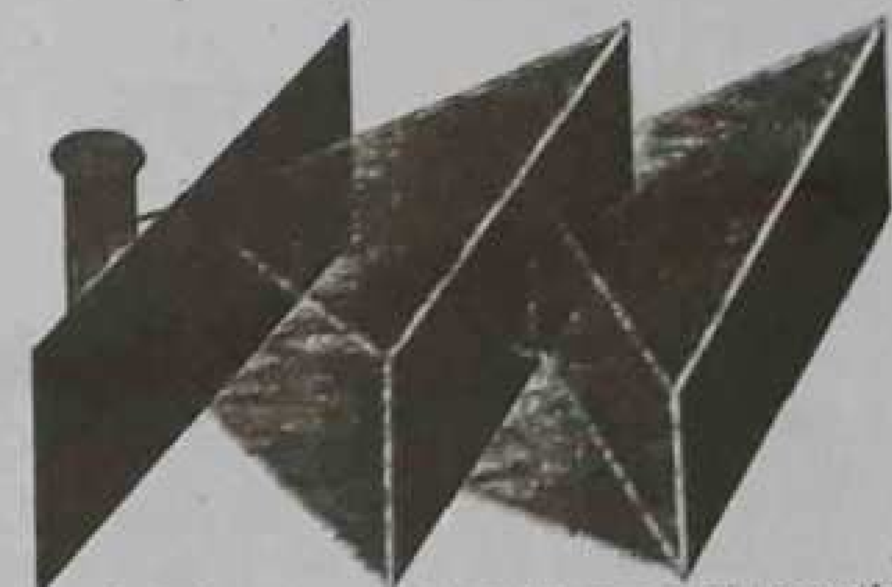
The distance between the centers of two consecutive bright or dark fringes is called fringe spacing.

For Two Bright Fringes:

In order to find the distance between two adjacent bright fringes on the screen mth and (m+1)th fringes are considered

Position of the mth fringe $y_m = (m)\left(\frac{\lambda L}{d}\right)$

Position of the (m+1) th fringe = $y_{m+1} = (m+1)\left(\frac{\lambda L}{d}\right)$


An interference pattern by monochromatic light in Young,s double slits experiment.

Then,

$$\Delta y = y_{m+1} - y_m$$

$$\Delta y = (m+1)\left(\frac{\lambda L}{d}\right) - (m)\left(\frac{\lambda L}{d}\right)$$

$$\Delta y = (\cancel{m}+1-\cancel{m})\frac{\lambda L}{d}$$

$$\Delta y = \frac{\lambda L}{d}$$

For Two Dark Fringes:

In order to find the distance between two adjacent dark fringes on the screen $(m+\frac{1}{2})$ th and $(m+\frac{3}{2})$ th fringes are considered.

Position of $(m+\frac{1}{2})$ th fringe = $y_m = (m+\frac{1}{2})\left(\frac{\lambda L}{d}\right)$

Position of $(m+\frac{3}{2})$ th fringe = $y_{m+1} = (m+\frac{3}{2})\left(\frac{\lambda L}{d}\right)$

Then,

$$\Delta y = y_{m+1} - y_m$$

$$\Delta y = (m+\frac{3}{2})\left(\frac{\lambda L}{d}\right) - (m+\frac{1}{2})\left(\frac{\lambda L}{d}\right)$$

$$\Delta y = (\cancel{m} + \frac{3}{2} - \cancel{m} - \frac{1}{2})\left(\frac{\lambda L}{d}\right)$$

$$\Delta y = \frac{\lambda L}{d}$$

Hence the bright and dark fringes are equal width and equally spaced. The fringe spacing varies directly with distance 'L' between the slits and screen and inversely with the separation 'd' of the slits. The formula for the fringe spacing can be used for the determination of the wavelength.

Q.5 Explain the phenomena of interference of light in a thin film?

Ans

Interference in a Thin Film

A transparent medium whose thickness is very small (Comparable with the wavelength of light), is called thin film.

Examples:

(i) oil film on the surface of water,
(ii) surface of soap bubble,
(iii) cracks in glass plate.

Explanation:

Consider a thin film of a refracting medium. A beam AB of monochromatic light of wavelength λ is incident on its upper surface. It is partly reflected along BC and partly refracted into the medium along BD. At D it is again partly reflected inside the medium along DE and then along EF as shown in fig. The distance between the beams BC and EF will be very small, and they will superpose and the result of their interference will be detected by the eye.

It can be seen from fig. that the original beam splits into two parts BC and EF due to thin film which enters the eye after covering different lengths of path. The path difference depends upon;

1. Thickness of the film
2. Nature of the film
3. Angle of incidence

If the two reflected waves reinforce each other, then the film will look bright. However, if the thickness of the film and the angle of incidence are such that the two reflected waves cancel each other, the film will look dark.

Interference of White Light:

If white light is incident on a film of irregular thickness at all possible angles, we should consider the interference pattern due to each spectral colour separately.

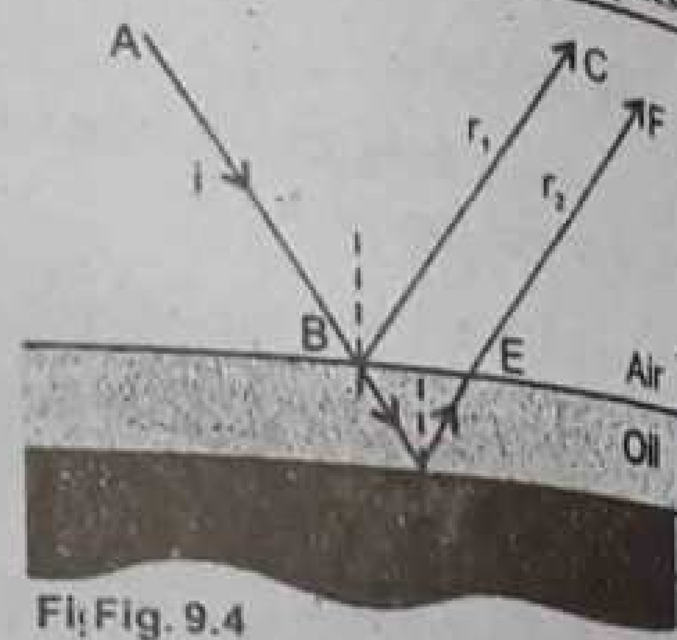
But if the thickness of the film and the angle of incidence are such that the destructive interference takes place from one colour. Then the remaining colour of the white light will make appearance on the film.

Q.6 Discuss the formation of Newton's rings. Why does the central spot of Newton's ring look dark?

Ans

Newton's Ring

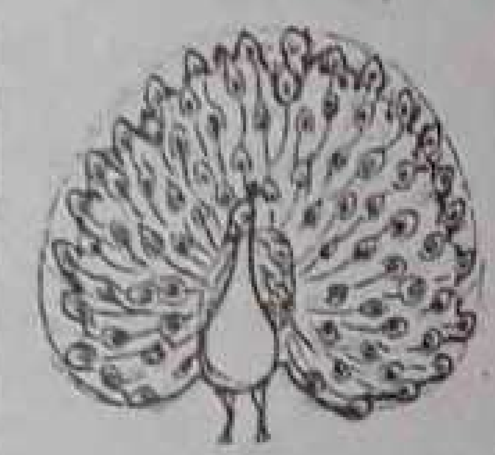
When a plano-convex lens of long focal length is placed in contact with a plane glass plate, a thin air film is enclosed between them to form circular dark and bright fringes known as Newton's rings.



Fi Fig. 9.4

Geometrical construction of interference of light due to a thin oil film.

Do You Know?



The vivid iridescence of peacock feathers due to interference of the light reflected from its complex layered surface.

Interesting Information

Colours seen on oily water surface are due to interference of incident white light.

Experimental arrangement:

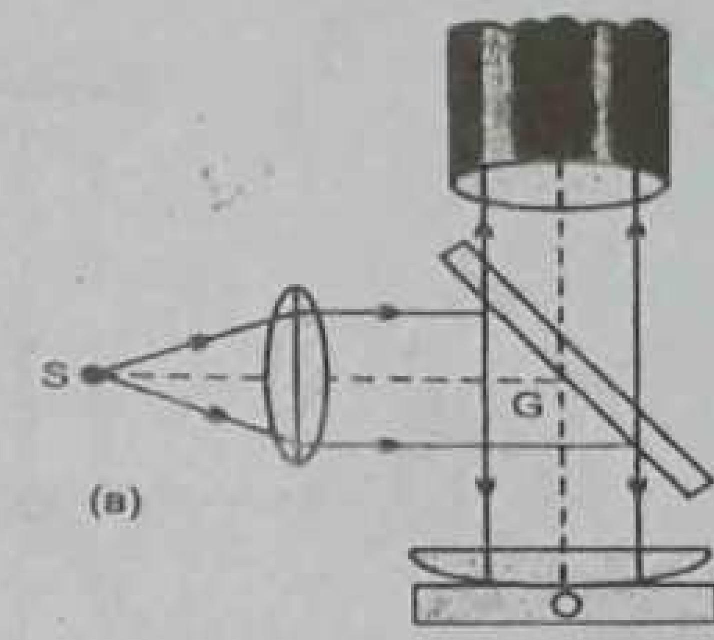
The thickness of the air film between plano-convex lens and glass slit is almost zero at the point of contact 'O' and it increases gradually as we proceed towards the periphery of the lens. Thus the point where the thickness of the air film is constant will lie on the circle with O as center.

Light beam from a monochromatic source 'S' becomes parallel after passing through the convex lens 'L'. This beam of light falls on the glass plate G. Some rays are partly reflected normally towards the air film and partly refracted through G. When light rays falls normally on the lens, these rays are reflected by the top and bottom surfaces of the air film. As these rays are coherent and interfere each other constructively or destructively.

When the light reflected upward is observed through a microscope. "M" focused at the glass plate G, a series of dark and bright circular rings are observed, as shown in figure. These concentric rings are called Newton's rings.

Dark Central Spot:

At the point of contact of the lens and the glass plate, the thickness of the film is effectively zero but due to reflection at the lower surface of air film from denser medium, an additional path difference of $\lambda/2$ is (or phase change of 180°) introduced. Consequently the center of Newton rings is dark due to destructive interference.



(a)

Fig. 9.6 (a)

Experimental arrangement for observing Newton's rings.

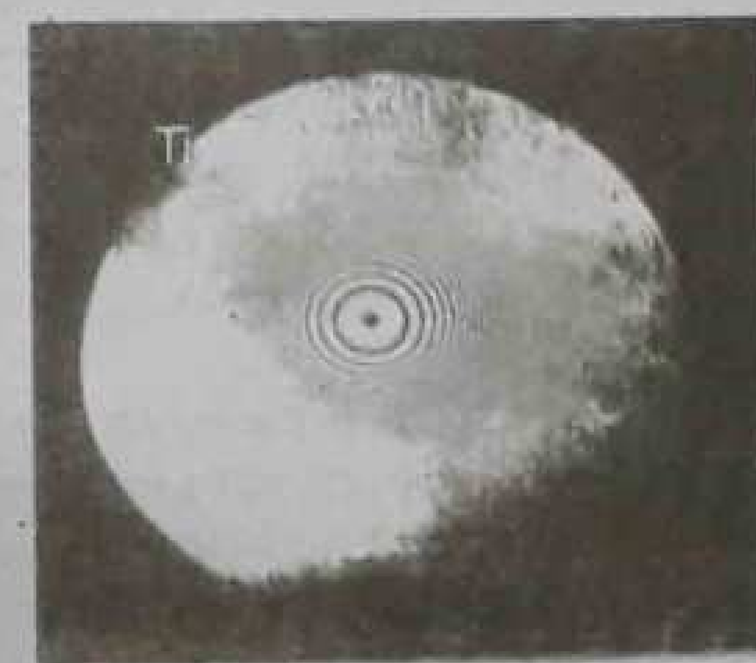


Fig. 9.6 (b)

A pattern of Newton's rings due to interference of monochromatic light.

Q.7 Describe the principle, construction and working of Michelson's interferometer. How can you find the wave length of light used?

Ans

Michelson's Interferometer

Michelson's interferometer is an instrument that can be used to measure distance with extremely high precision. Albert A. Michelson devised this instrument in 1881, using the idea of interference of light rays.

Principle:

Its working is based on interference. When light from a single source is spitted into two parts and then interfere it forms an interference pattern.

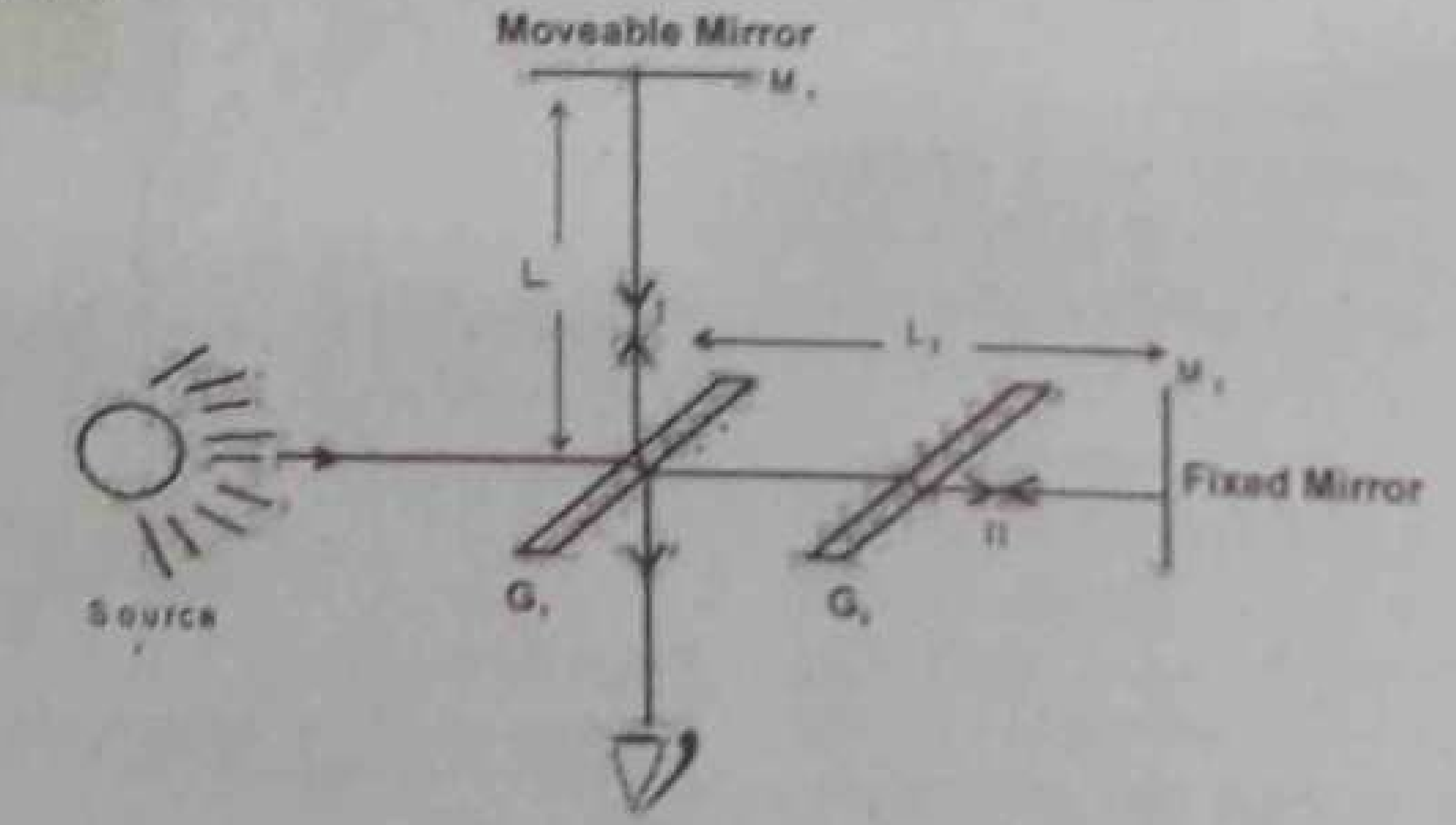
Construction and working:

The essential features of a Michelson's interferometer are shown schematically in Figure.

For Your Information

A photograph of Michel Interferometer.

Ray Diagram



Schematic diagram of a Michelson's interferometer

Working

Monochromatic light from an extended source falls on a half silvered glass plate G_1 that partially reflects it and partially transmits it. The reflected portion labeled as I in the figure travels a distance L_1 to mirror M_1, which reflects the beam back towards G_1

The half silvered plate G_1 partially transmits this portion that finally arrives at the observer's eye. The transmitted portion of the original beam labeled as II, travels a distance L_2 to mirror M_2 which reflects the beam back toward G_2 cut from the same piece of glass as G_1 is introduced in the path of beam II as a compensator plate. G_2 therefore equalizes the path length of the beam I and II in glass. The two beams having their different paths are coherent. They produce interference effects when they arrive at observer's eyes.

The observer then sees a series of a parallel interference fringes. In a practical interferometer, the mirror M_1 can be moved along the direction perpendicular to its surface by means of a precision screw. As the length L_1 is changed, the pattern of interference fringes is observed to shift.

If M_1 is displaced through a distance equal $\lambda/2$, a path difference of double of this displacement (i.e. $\frac{\lambda}{2}+\frac{\lambda}{2}=\lambda$) is produced, i.e. equal to λ. Thus a fringe is seen shifted forward across the line of reference of cross wire in the eye piece of the telescope used to see the fringes. A fringe is shifted each time the mirror is displaced through $\lambda/2$. Hence, by counting the number m of the fringes which are shifted by the displacement L of the mirror. We can write the equation.

$$L = m\frac{\lambda}{2}$$

or $$\lambda = \frac{2L}{m}$$

Very precise length measurements can be made with an interferometer.

Uses:

Michelson's interferometer is used for the following purposes:

(1) It is used for the determination of wavelength of light.

For Your Information

Interference fringes in the Michelson interferometer

(2) Michelson measured the length of the standard meter in terms of wavelength of red cadmium light and proved that Standard meter = 1553163.5 wavelength of light

(3) If light of wavelength λ = 400 nm is used, then it can measure the thickness upto 10^{-4} mm (or 100 nm).

(4) It is used to observe the interference of light.

Q.8 What is meant by diffraction of light?
Also discuss the diffraction of light through a narrow slit?

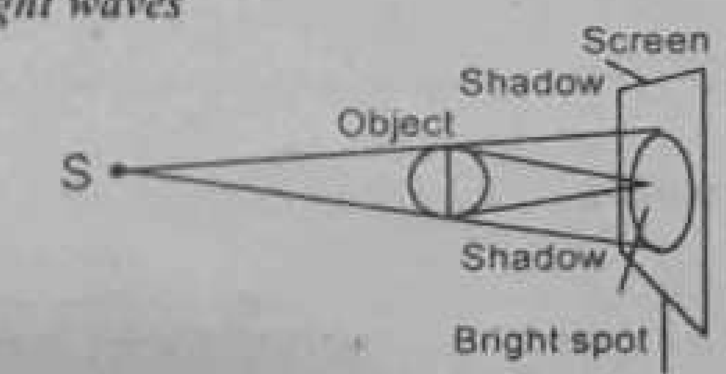
Ans.

Diffraction of Light

The property of bending of light around obstacles and spreading of light waves into the geometrical shadow of an obstacle is called diffraction.

Explanations:

Consider a small and smooth steel ball of about 3 mm in diameter is illuminated by a point source of light 'S'. The shadow of the object is received on a screen as shown in figure. The shadow of the spherical object is not completely dark but has a bright spot at its center. It happens only if the light bends around the obstacle.



Bending of light caused by its passage past a spherical object.

Conclusion:

This phenomenon becomes prominent when the wavelength of light is large as compared with the size of the obstacle or aperture of the slit.

The diffraction of light occurs, in effect, due to the interference between rays coming from ***different parts of the same wavefront.***

Diffraction of light through a narrow slit:

The experimental arrangement for studying diffraction of light due to narrow slit is shown in figure.

Let us consider a slit AB of width d is illuminated by a parallel beam of monochromatic light of wavelength 'λ'. A small portion of the incident wavefront passes through the narrow slit. Each point of this section of wavefront sends out secondary wavelets to the screen. These wavelets then interfere to produce the diffraction pattern. It becomes simple to deal with rays instead of wave fronts. Only nine rays have been drawn where as actually there are a large number of rays.

Let us consider ray 1 and 5 which are in phase when in the wavefront AB. After these reach the wave front AC, ray 5 would have a path difference 'ab' say equal to $\lambda/2$. Thus, when these two rays reach point P on the screen, they will interfere destructively. Similarly, each pair 2 and 6, 3 and 7, 4 and 8 differ in path by $\lambda/2$ and will do the same.

In order to find the value of path difference ab, we consider the right angle triangle aAb, as shown in figure.

$$\text{Sin}\theta = \frac{ab}{AB/2}$$

Point to Ponder

Hold two fingers close together to Form a slit. Look at a light bulb Through the slit. Observe the Pattern of light being seen and Think why it is so.

EXPLANATION:

The sending of light rays around the small openings or obstacles is called diffraction. We see the diffraction fringes through the two fingers close together like slit.

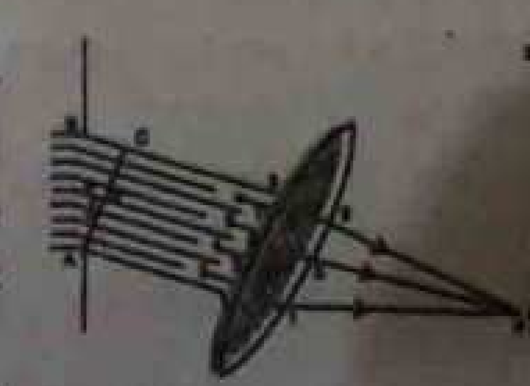


Fig. 9.9

Diffraction of light due to a narrow slit AB. The dots represent the sources of secondary wavelets

or $\quad ab = \dfrac{AB}{2}\sin\theta \qquad$ But AB = d

thus $\quad ab = \dfrac{d}{2}\sin\theta$

For destructive interference (first minimum), $ab = \dfrac{\lambda}{2}$

Hence equation for first minimum can be written as

$$\frac{d}{2}\sin = \frac{\lambda}{2}$$

Or $$d\sin\theta = \lambda$$

In general, the conditions for different orders of minima observed on either side of centre of screen are given by

$$d\sin\theta = m\lambda$$

where $m = \pm1, \pm2, \pm3, \ldots$

The region between any two consecutive minima both above and below the centre be bright i.e., maxima. In this way we may obtain series of alternate dark and bright fringes with central bright.

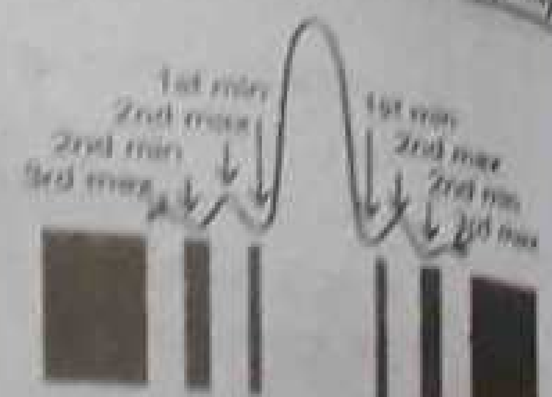


Fig. 9.10 (a)

Diffraction pattern of mono chromatic light produced due to a single slit; graphical representation and photograph of the pattern

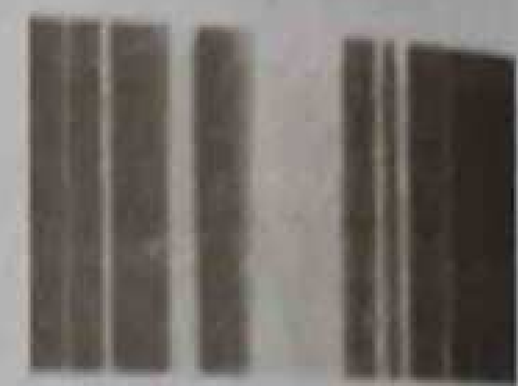


Fig. 9.10 (b)

Diffraction pattern produced by white light through a single slit

Q.9 What is diffraction grating and obtain the grating equation to find the wavelength of light?

Ans.

Diffraction Grating

"A diffraction grating consists of a glass plate on which very fine equidistant parallel lines (scratches) are drawn by mean of ruling engine with fine diamond point. The transparent spacing between the scratches on the glass plate acts as slits".

A typical diffraction grating has about 400 to 5000 lines per centimeter.

Grating element:

"The distance between the centers of two adjacent lines is called grating element."

Its value is obtained by dividing the length L of the grating by the total number N of the lines ruled on it.

So the grating element = d = L/N.

If we consider the unit length of the diffraction grating then d = 1/N.

Working and Theory:

Consider the parallel beam of monochromatic light illuminating the grating at normal incidence. The parts of the wave front that pass through the slits behaves as source of secondary wave lets according to Huygen's principle.

Consider the parallel rays which make an angle θ with AB after diffraction. They are then brought to focus on the screen at P by convex lens. If the path difference between ray 1 and 2 is one wavelength λ, they will reinforce each other at P. As the incident beam consists of the

Diffraction of light due to grating

Interesting Information

The fine rulings, each 0.5 μm wide, on a compact disc function as a diffraction grating. When a small source of white light illuminates a disc, the diffracted light forms colored "lanes" that are composite of the diffraction patterns from the rulings.

parallel rays, the rays from any two consecutive slits will differ in path by λ when they arrive at P. They will therefore, interfere constructively.

Grating Equation:

The path difference for constructive interference between two consecutive rays should be integral multiple of λ, i.e.,

$$\text{Path diff} = ab = \lambda \quad \ldots(1)$$

From figure, acb is a right angled triangle

$$ab/ac = \sin\theta$$

or $$ab = ac\sin\theta$$

$$ab = d\sin\theta \quad \text{\_\_\_\_\_ (2)}$$

Comparing (1) and (2)

$$\lambda = d\sin\theta \quad \ldots\ldots (3)$$

According to equation (2), if θ = 0°, then path difference = 0, so we will get a bright fringe. This is known zero order image formed by the grating. If we increase θ on either side of this direction, a value of θ will be arrived at which λ = d Sin θ and we again get bright image. If path difference is the integral multiple of λ, we will get second, third order etc. images.

Therefore in general we can write

$$d\sin\theta = n\lambda \qquad \text{Where } n = 0, 1, 2, 3, \ldots$$

This is called the equation of granting and 'n' is called the order of diffraction n = 0, is the central maxima, n = 1 is the first order maxima and so on. The separate images are obtained corresponding to each wavelength or colour.

Q.10 Describe the diffraction of x-rays through crystals? Also describe the Braggs equation and its different uses?

Ans.

Diffraction of X-Rays by Crystals

X-rays is type of electromagnetic radiation of much shorter wavelength, of the order of 10^{-10} m.

In order to observe the effect of diffraction, the grating spacing must be of the order of the wavelength of the radiation used. The regular array of atoms in a crystal forms a natural diffraction grating.

The study of atomic structure of crystals by X-rays was initiated in 1914 by W.H. Bragg and W.L Bragg with remarkable achievements. They found that a monochromatic beam of X-rays was reflected from a crystal plane as if it acted like mirror. To understand this effect, a series of atomic planes of constant inter planar spacing 'd' parallel to a crystal face are shown by lines PP′, P_1P_1', P_2P_2' shown in fig. and so on.

Bragg's equation:

Suppose an x-ray beam is incident at an angle 'θ' on one of the planes. The beam reflected from the lower plane travels some extra distance (BC+CB′) as compare to the beam reflected from the upper plane. Thus effective path difference between the two reflected beams is BC+CB′.

For your Information

Diffraction of white light by a fine diffraction grating

Interesting Application

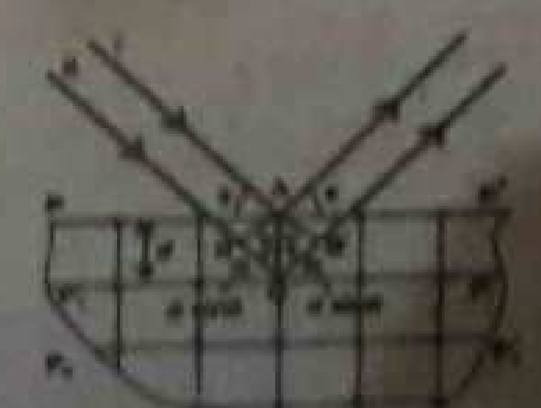
Diffraction of radio waves

Interesting Information

The spectrum of white light due to diffraction grating of 100 slits

The spectrum of white light due to diffraction grating of 2000 slits

EXPLANATION:
For 100 slits, the separation between the fringes is greater (Fig a) whereas for 2000 slits the separation between the fringes is smaller (fig b) and they overlap each other.



Direction of X-rays from the lattice planes of crystal.

From triangle ABC, we have

$$\frac{BC}{AC} = \sin\theta$$

Or $\quad BC = AC\sin\theta$

Or $\quad BC = d\sin\theta \qquad$ As AC=d

Similarly from triangle ACB′, we have

$$\frac{CB'}{AC} = \sin\theta$$

Or

$$CB' = AC\sin\theta$$

Or

$$CB' = d\sin\theta$$

Thus, path difference = BC+CB′

$$= d\sin\theta + d\sin\theta \text{ ....(1)}$$

Path difference $= 2d\sin\theta$

Both reflected beams will reinforce (constructive interference) if difference is equal to integral multiple of wavelength. Hence for constructive interference

$$\text{Path difference} = n\lambda \text{ .... (2)}$$

Comparing 1 and 2, we get

$$2d\sin\theta = n\lambda \quad \text{where } n = 1,2,3, \text{.........}$$

where n is the order of reflection. This is known as Bragg's equation, or Bragg's law

Uses:

1. Bragg's equation can be used to determine the interplaner spacing between similar parallel planes of a crystal i.e., $d = \dfrac{n\lambda}{2\sin\theta}$
2. X-rays diffraction is very useful in determining the structure of biologically important molecules such as hemoglobin, which is an important constituent of blood, and double helix structure of DNA.
3. Bragg's equation can be used to determine the wave length of light. $\left(\text{i.e. by } \lambda = \dfrac{2d\sin\theta}{n}\right)$

A multi-aperture diffraction pattern. This is a picture of a white-light point source shot through a piece of tightly woven cloth.

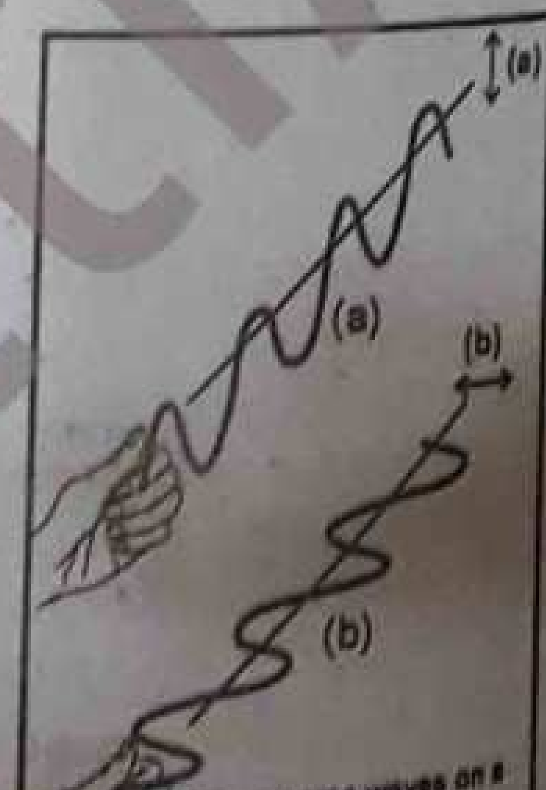
Diffraction pattern of a single human hair under laser beam illumination.

Q.11 Explain the phenomenon of polarization. How plane polarized light is produced and detected?

Ans.

Polarization

The phenomenon of interference and diffraction have proved that light has wave nature, but these phenomenon do not show whether light waves are longitudinal or transverse. The phenomenon of polarization shows that light waves are transverse.

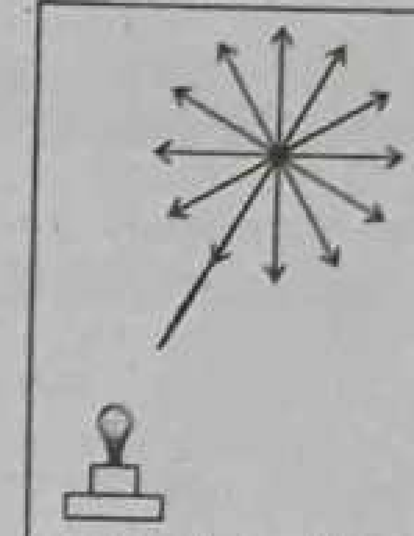


Transverse waves on a string polarized (a) in a vertical plane and (b) in a horizontal plane

In transverse mechanical waves, the vibration can be oriented along vertical, horizontal or any other direction. In each of these cases, the wave is said to be polarized.

The plane of polarization is the plane containing the direction of vibration of the particles of the medium and the direction of propagation of wave.

A light wave produced by oscillating charge consists of a periodic variation of electric field vector along with magnetic field vector at right angle to each other. The direction of polarization in a plane polarized light wave is taken as the direction of electric field vector.



An unpolarized light, due to incandescent bulb, has vibrations in all directions.

Unpolarized light:

A beam of ordinary light consisting of large number of planes of vibration is called unpolarized light.

Polarized light:

The beam of light in which all vibrations are confined to a single plane of vibration is called polarized light.

Production and Detection of plane polarized light

The light emitted by an ordinary incandescent bulb is unpolarized, because its vibrations are randomly oriented in space. It is possible to obtain plane polarized beam of light from un-polarized light by removing all waves from the beam except those having vibrations along one particular direction. This can be achieved by various method as given below:

(1) Selective absorption
(2) Reflection from different surfaces
(3) Scattering by small particles
(4) Refraction through crystals

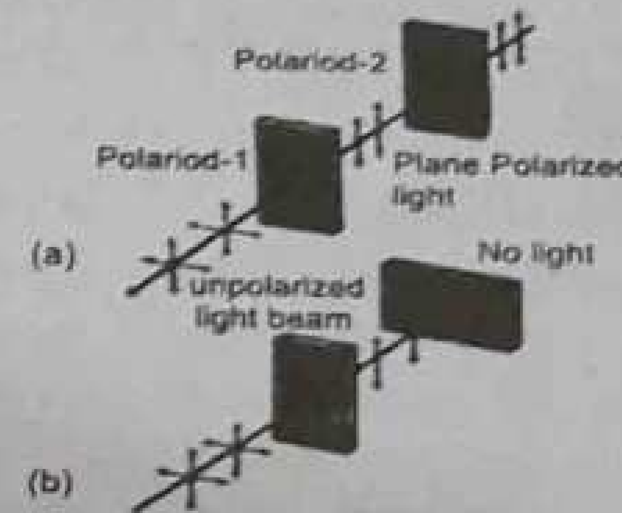


Fig. 9.15
Experimental arrangement to show that light waves are transverse. The lines with arrows indicates electric vibrations of light waves.

Selective absorption method:

Selective abortion method is the most common method to obtain plane polarized light by using certain types of materials called ***dichoric substances***. These transmit only those waves, whose vibration are parallel to the particular direction and will absorb those waves whose vibration are in other directions. One such commercial polarizing material is Polaroid.

If the un-polarized light is made incident on the sheet of Polaroid, the transmitted light will be plane polarized. If a second sheet of Polaroid is placed in such a way that the axes of the Polaroid, as shown by the straight lines drawn on them, are parallel, the light is transmitted through the second Polaroid also. If the second Polaroid is slowly rotated about the beam of light, as axis o rotation, the light emerging out of the second Polaroid gets dimmer and dimmer and disappears when the axes become mutually perpendicular. The light reappears on further rotation and become brightest when the axes are again parallel to each other.

For Your Information

Looking through two polarizers. When they are "crossed" very little light passes through.

EXPLANATION:
Little light is due to polarization.

Transverse Nature of Light:

This experiment proves that light waves are transverse waves. If the light waves were longitudinal, they would never disappear even if the two Polaroids were mutually perpendicular. Sunlight also becomes partially polarized because of scattering by air molecules of the Earth's atmosphere or by reflection we can obtain the partially polarized light instead of glare of light.

Reflection from different surfaces:

- Reflection of light from water, glass, snow and rough road surfaces, for larger angles of incidences, produces glare. since the reflected light is partially polarized, glare can considerably be reduced by using polarized sunglasses.

Scattering by small particles:

Sunlight also becomes partially polarized due to scattering by air molecules of earth's atmosphere. This effect can be observed by looking directly up through a pair of sunglasses made of polarizing glass, a certain directions of the lens, less light passes through than at others.

Polaroid:

A synthetic doubly refracting substance, that strongly absorbs polarized light in one plane, while easily passing polarized light in another plane of right angles.

Do you know?

Light reflected from smooth surface of water is partially polarized parallel to the surface.

Q.12 What is meant by optical rotation?

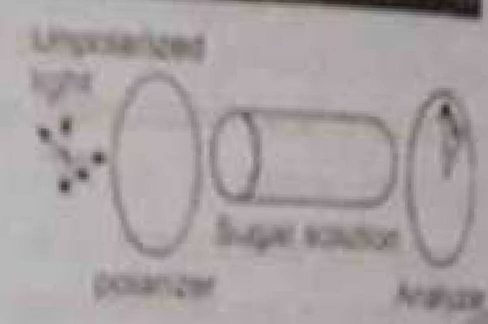
Ans

Optical Rotation

When a plane polarized light is passed through certain crystals. They rotate the plane of polarization. Quartz and sodium chlorate crystals are typical examples, which are termed optically active crystals.

A few millimeter thicknesses of such crystals will rotate the plane of polarization by many degrees. Certain organic substances, such as sugar and tartaric acid, show optical rotation when they are in solution, this property of optical active substances can be used to determine the concentration in the solutions.

Interesting Information



Sugar solution rotates the plane of polarization of incident light so that it is no longer horizontal but at an angle. The analyzer thus stops the light when rotated from the vertical (crossed) position.

FORMULAE

Path difference for constructive interference in light	$d\sin\theta = m\lambda$, where m= 0,1,2,3,...	
Path difference for destructive interference in light	$d\sin\theta = [m+\frac{1}{2}]\lambda$, where m= 0,1,2,3,...	$d\sin\theta = [2m+1]\frac{\lambda}{2}$ where m= 0,1,2,3,...
Position of mth bright fringe	$y = m\frac{\lambda L}{d}$	
Position of mth dark fringe	$y = (m+\frac{1}{2})\frac{\lambda L}{d}$	
Fringe width /spacing	$\Delta y = \frac{\lambda L}{d}$	
Grating element	$d = \frac{L}{N}$	$d = \frac{1}{N}$ [when L = 1m]
Displacement of mirror in Michelson interferometer	$L = m\frac{\lambda}{2}$	$\lambda = \frac{2L}{m}$
Bragg's Law	$2d\sin\theta = n\lambda$ Where n= 1,2,3,...	

Multiple Choice Questions

- *Four possible answers to each statement are given below. Tick (✓) the correct answer:*

1. The wave theory of light was proposed by:
   (a) Galileo (b) Huygen
   (c) Kepler (d) Newton
2. The locus of all points in the same phase of vibration is:
   (a) Wavefront (b) Interference
   (c) Diffraction (d) Polarization
3. In Newton's rings experiment the Plano convex lens used should be of:
   (a) Small focal length (b) Large focal length
   (c) Very short focal length (d) None of these
4. The air between lens and the plate in Newton's rings experiment is replaced by water. The ri pattern:
   (a) Contracts (b) Expands
   (c) Remain same (d) Becomes darker
5. Fringe spacing in young's double slit experiment increases due to increase in:
   (a) Slit separation (b) Wavelength
   (c) Order of fringe (d) Frequency of source
6. Fringe width in interference pattern will be more, when coherent sources are:
   (a) Close together (b) Too much apart
   (c) Inclined at same angle (d) Inclined at 45°
7. Which is not optically active?
   (a) Sugar (b) Tartaric acid
   (c) Water (d) Sodium chlorate
8. Longitudinal waves do not exhibit:
   (a) Reflection (b) Refraction
   (c) Diffraction (d) Polarization
9. When light falls on a ball then its shadow has bright centre, it is due to:
   (a) Interference (b) Diffraction
   (c) Polarization (d) Refraction
10. The danger signals are red while eye is more sensitive to yellow because:
    (a) Scattering in yellow colour is less than that of red
    (b) Red light is longer in wavelength than yellow colour
    (c) Scattering in red is less than in yellow (d) None of these
11. Which one of the following is nearest to monochromatic light?
    (a) Light from neon lamp (b) Light from fluorescent tube

(c) Light from sodium light (d) None of these

12. Two sources of light are said to be coherent if the waves produced by them have the same:
(a) Wavelength (b) Amplitude
(c) Wavelength and amplitude (d) Wavelength and a constant phase difference

13. For which of the following colours will the fringe width be minimum in the double slit experiment?
(a) Violet (b) Red
(c) Green (d) Yellow

14. If light of low wavelength is used in young's double slit experiment, then width of fringe will:
(a) Decrease (b) Increase
(c) No fixed (d) No change

15. Intensity of light depends on:
(a) Wavelength (b) Amplitude
(c) Velocity (d) Frequency

16. How will the diffraction pattern of single slit change when yellow light is replaced by blue light?
(a) Fainter (b) Wider
(c) Narrower (d) Bright

17. Which of the followings can not produce colours with white light?
(a) Diffraction (b) Interference
(c) Polarization (d) Dispersion

18. In a Michelson interferometer by moving the mirror through a distance of $\frac{\lambda}{4}$, the path difference changes by
(a) $\frac{\lambda}{2}$ (b) λ
(c) $\frac{\lambda}{4}$ (d) 2λ

19. On dividing the length of the grating 'L' by total number 'N' of the lines ruled on it, we get
(a) Grating element (b) Total length of slits
(c) Total height of slits (d) None of these

20. Polarization of light shows that light waves are
(a) longitudinal waves (b) transverse waves
(c) sound waves (d) all of these

ANSWERS

. b	2. a	3. b	4. a	5. b	6. a	7. c	8. d	9. b	10. c
1. c	12. d	13. a	14. a	15. b	16. c	17. c	18. a	19. a	20. b

Short Questions of Exercise

Q.9.1 Under what conditions, the two sources of light behave as coherent sources?

(Fsd 2005, Grw 2005-2009, Lhr 2008, Lhr 2009, Lhr 2010-2011)

Ans. Conditions for coherent sources

The two sources are said to be coherent, if

(i) the sources must emit waves of *same* wave length. (mono chromatic)

(ii) the waves emitted by the sources must have *constant* phase difference.

Q.9.2 How is the distance between interference fringes affected by the separation between the slits of Young's experiment? Can fringes disappear?

Ans. By increasing the separation between slits fringe spacing is *decreased* and vice versa.

Explanation

The fringe spacing is given by,

$$\Delta y = \frac{\lambda L}{d}$$

This equation shows that the distance between fringes(Δy) is *inversely* proportional to the separation between the slits (d).

Fringes can disappear

When separation between the slits is made *large enough*, the fringes will be so close that they cannot be distinguished from one another and pattern will disappear.

Q.9.3 Can the visible light produce interference fringes? Explain.

(Mtn 2003-2006, Bwp 2006, Fsd 2005, Sgd 2005, Grw 2009-2011)

Ans. *Yes*, visible light (white light) can produce the interference fringes.

Explanation

White light consists of seven colours. Each spectral colour produces its *own* interference fringe pattern. These patterns *overlap* to give rise a *resultant diffused coloured* interference pattern.

Q.9.4 In Young's experiment, one of the slits is covered with blue filter and other with red filter. What would be the pattern of light intensity on the screen?

Ans. *No interference pattern* of bright and dark fringes is formed on screen.

Reason

We know that for detectable interference, the two sources must be monochromatic having constant phase difference. Since the blue and red lights have *different wavelengths*, So, the condition for interference is not satisfied.

Q.9.5 Explain whether Young's experiment is an experiment for studying interference or diffraction effects of light. (Grw 2008)

Ans. Basically, it is an experiment to study the ***interference*** of light though it involves diffraction.

Explanation

As the light passes through the slits it ***bends around*** the slit (diffraction).Then these diffracted rays superpose each other to produce the interference pattern effect of light. But in this experiment we only study the interference effect of light.

Q.9.6 An oil film spreading over a wet footpath shows colours? Explain how does it happen?

(Bwp 2003, D.G.Khan 2005-2006, Rwp 2005, Fsd 2005, Bwp 2008, Grw 2009, Lhr 2010-2011,Grw2011)

Ans. This happens due to the ***interference*** of light waves

Explanation

- Oil film spread over a wet foot path acts like a ***thin film***.
- A light beam is incident on the upper surface.
- It is partly reflected from ***upper*** surface and partly reflected from the ***lower*** surface of thin film of oil.
- The two reflected coherent beams ***superpose*** and an diffuse interference pattern of different colours is obtained.

Q.9.7 Could you obtain Newton's rings with transmitted light? If yes, would the pattern be different from that obtained with reflected light?

(Lhr 2006, Bwp 2008, Mir Pur 2009)

Ans. ***Yes***, the Newton's rings can be obtained by transmitted light.

Pattern of transmitted light:

The pattern obtained from transmitted light is exactly ***opposite*** to that of reflected light. There is ***no phase change*** in this case. Every dark ring is converted into Bright ring. So the centre of Newton's rings is ***bright.***

Q.9.8 In white light spectrum obtained with diffraction grating, the third order image of wave length coincides with the fourth order image of second wave length. Calculate the ratio of the two wave lengths?

Ans. Ratio of two wavelengths

Grating equation is given by

$$d\sin\theta = n\lambda$$

For first wavelength λ_1, $n = 3$

So $\quad d\sin\theta = 3\lambda_1$ ______(1)

For second wavelength λ_2, $n = 4$

So $\quad d\sin\theta = 4\lambda_2$ ______(2)

Comparing equations (1) and (2) we have

$$3\lambda_1 = 4\lambda_2$$

OR
$$\frac{\lambda_1}{\lambda_2} = \frac{4}{3}.$$

Hence, the ratio is 4: 3.

Q.9.9 How would you manage to get more orders of spectra using a grating?

(Federal 2005, Grw 2005, Mir Pur 2006, Rwp 2006, Lhr 2006, Bwp 2008, Mir Pur 2009, Lhr 2009, Grw 2010-2011,Lhr 2010)

Ans. Orders of spectra (n)

The formula for diffraction grating is,

$$d\sin\theta = n\lambda$$

OR
$$n = \frac{d\sin\theta}{\lambda}$$

for maximum value of n, $\sin\theta = 1 \Rightarrow \theta = 90^\circ$

So

This equation shows that we can get more orders of if;

For a given diffraction grating

Its grating element is constant, so

$$n \propto \frac{1}{\lambda}$$

Hence, by decreasing the wave length we can obtain more order of spectra.

Q.9.10 Why Polaroid sunglasses are better than ordinary sun glasses?

(Grw 2003, Sgd 2003-2005, D.G.Khan 2005, Lhr 2005, Fsd 2005, Mir Pur 2009, Lhr 2009,Grw 2010)

Ans.

- Polaroid sunglasses pass small light having specific planes of vibration.
- Polarized sunglasses reduce ***glare***.
- They ***protect*** the eyes from bright rays of sun light.
- Polaroid sun glasses blot out only the ***harmful glares***.

Q.9.11 How would you distinguish between unpolarized and plane polarized light?

(Bwp 2004, Mir Pur 2004, Rwp 2004-2005, Mtn 2005, Grw 2006)

Ans. Plane polarized unpolarized and light

Suppose light is incident on a polaroid. The polaroid is slowly rotated about an axis which is along the direction of incident ray.

- If the transmitted light is plane polarized, it becomes dimmer and dimmer and ***disappears*** at certain orientation.
- If the transmitted light is unpolarized, it becomes dim but ***not completely*** blocked at any orientation.

Q.9.12 Fill in the blanks.

(i) According to ***Huygen's principle***, each point on a wave front acts as a source of secondary ***wavelets***.

(ii) In Young's experiment, the distance between two adjacent bright fringes for violet light is ***smaller*** than that for green light.

(iii) The distance between bright fringes n the interference pattern ***increases*** as the wavelength of the light used increases.

(iv) A diffraction grating is used to make a diffraction pattern for yellow light and then for red light. The distances between the red spots will be greater than that for yellow light.

(v) The phenomenon of polarization of light reveals that light waves are transverse waves.

(vi) A polariod is a commercial polarizer.

(vii) A polariod glass reduces the glare of light produced at the road surface.

Solved Examples

Example 9.1

The distance between the slits in Young's double slit experiment is 0.25 cm. Interference fringes are formed on a screen placed at a distance of 100cm from the slits. The distance of the third dark fringe from the central bright fringe is 0.059 cm. Find the wavelength of the incident light.

Given Data:

Distance between slits = $d = 0.25\ cm = 2.5 \times 10^{-3}m$

Distance of screen from slits = $L = 100\ cm = 1m$

Distance of third dark fringe = $y = 0.059\ cm = 5.9 \times 10^{-4}m$

For the third dark fringe, $m = 2$

To Find:

Wavelength of incident light = $\lambda = ?$

Calculation:

As the formula for dark fringe is

$$y = (m + \frac{1}{2})\frac{\lambda L}{d}$$

Putting values, we get

$$5.9 \times 10^{-4} = (2 + \frac{1}{2})(\frac{\lambda \times 1}{2.5 \times 10^{-3}})$$

$$5.9 \times 10^{-4} = \left(\frac{5}{2}\right)\left(\frac{\lambda}{2.5 \times 10^{-3}}\right)$$

Or $\lambda = \dfrac{2 \times 5.9 \times 10^{-4} \times 2.5 \times 10^{-3}}{5}$

$\lambda = 5.9 \times 10^{-7}m$

Or $\lambda = 590 \times 10^{-9}m$

Or $\boxed{\lambda = 590\ nm}$

Example 9.2

Yellow sodium light of wavelength 589 nm, emitted by a single source passes through two narrow slits 1.00 mm apart. The interference pattern is observed on a screen 225 cm away. How far apart are two adjacent bright fringes?

Given Data:

Wavelength of sodium light = $\lambda = 589\ nm = 589 \times 10^{-9}m$

Separation between the slits = $d = 1.00\ nm = 1.00 \times 10^{-3}m$

Distance of screen from slits = $L = 225cm = 2.25m$

To Find:

Distance between two bright fringes = fringe spacing = $\Delta y = ?$

Calculation:

As formula for fringe spacing is

$$\Delta y = \frac{\lambda L}{d}$$

Putting values, we get

$$\Delta y = \frac{589 \times 10^{-9} \times 2.25}{1.0 \times 10^{-3}}$$

$\Delta y = 1325.25 \times 10^{-6}m$

Or $\Delta y = 1.33 \times 10^{-3}m$

$\boxed{\Delta y = 1.33\ mm}$

Example 9.3

Light of wavelength 450 nm is incident on a diffraction grating on which 5000 lines/cm have been ruled.

(i) How many orders of spectra can be observed on either side of the direct beam?

(ii) Determine the angle corresponding to each other.

Given Data:

Wavelength of light = $\lambda = 450nm = 450 \times 10^{-9}m$

Lines ruled on diffraction grating = $N = 5000$ lines per cm

Grating element = $d = \frac{1}{N} = \frac{1}{5000}cm = \frac{1}{500000}m$

To Find:

(i) Orders of Spectra = $n = ?$

(ii) Angle = $\theta = ?$

Calculation:

(i) For maximum number of order of spectra, $\sin\theta = 1$

As maximum possible value of angle of diffraction is $\theta = 90°$, therefore $\sin 90° = 1$

Using the formula

$$d \sin\theta = n\lambda$$

Putting values, we get

$$\left(\frac{1}{500000}\right) \times 1 = n \times 450 \times 10^{-9}$$

$$n = \frac{1}{500000 \times 450 \times 10^{-9}}$$

$$n = \frac{10^9}{5 \times 45 \times 10^6} = \frac{1}{225} \times 10^3$$

$n = 0.0044 \times 10^3$

$n = 4.4$

Hence, maximum order of spectrum = 4

$\boxed{n = 4}$

(ii) For first order spectrum, $n = 1$

Using $d \sin\theta = n\lambda$

Putting values, we get

$$\frac{1}{500000}\ \sin\theta = 1 \times 450 \times 10^{-9}$$
$$\sin\theta = (500000) \times (450 \times 10^{-9})$$
$$\sin\theta = 225 \times 10^{6} \times 10^{-9}$$
$$\sin\theta = 225 \times 10^{-3}$$
$$\sin\theta = 0.225$$
$$\theta = \sin^{-1}(0.225)$$
$$\boxed{\theta = 13^\circ}$$

For second order spectrum, n = 2

$$d\sin\theta = n\lambda$$

Putting values we get

$$\frac{1}{500000}\ \sin\theta = (2)\,(450 \times 10^{-9})$$
$$\sin\theta = (500000)\,(2 \times 450 \times 10^{-9})$$
$$\sin\theta = 450 \times 10^{6} \times 10^{-9}$$
$$\sin\theta = 0.45$$
$$\theta = \mathrm{Sin}^{-1}(0.45)$$
$$\boxed{\theta = 26.7^\circ}$$

For third order spectrum , n = 3

$$d\,\mathrm{Sin}\,\theta = n\lambda$$

Putting values we get

$$\frac{1}{500000}\ \sin\theta = (3)\,(450 \times 10^{-9})$$
$$\sin\theta = (500000)\,(3 \times 450 \times 10^{-9})$$
$$\sin\theta = 675 \times 10^{6} \times 10^{-9}$$
$$\sin\theta = 0.675$$
$$\theta = \mathrm{Sin}^{-1}(0.675)$$
$$\boxed{\theta = 42.5^\circ}$$

For fourth order spectrum , n = λ

$$d\,\mathrm{Sin}\theta = n\lambda$$

Putting values, we get

$$\frac{1}{500000}\ \sin\theta = (4)\,(450 \times 10^{-9})$$
$$\sin\theta = (500000)\,(4 \times 450 \times 10^{-9})$$
$$\sin\theta = 900 \times 10^{6} \times 10^{-9}$$
$$\sin\theta = 0.9$$
$$\theta = \sin^{-1}(0.9)$$
$$\boxed{\theta = 62.2^\circ}$$

Exercise Problems

9.1 Light of wavelength 546 nm is allowed to illuminate the slits of Young's experiment. The separation between the slits is 0.10 mm and the distance of screen from the slits where interference effects are observed is 200cm. At what angle the first minimum will fall? What will be the linear distance on the screen between adjacent maxima?

Given data:

Wavelength of light = $\lambda = 546\,nm = 546 \times 10^{-9}\,m$

Separation between slits = $d = 0.10$ mm $= 0.10 \times 10^{-3}\,m$

Distance of screen from the slits = L = 20cm = $20 \times 10^{-2}\,m$

To find:

Angle for first minimum = θ = ?

Fringe spacing = liner distance between adjacent maxima = $\Delta y = ?$

Calculation:

As the formula for minima is

$$d\sin\theta = (m + \frac{1}{2})\lambda$$

For first minima, m = 0

Putting values, we get

$$\sin\theta = \frac{546 \times 10^{-9}}{2 \times 0.10 \times 10^{-3}}$$
$$\sin\theta = 2730 \times 10^{-6}$$
$$\sin\theta = 0.00273$$
$$\theta = \sin^{-1}(0.00273)$$
$$\boxed{\theta = 0.16^\circ}$$

For fringe spacing, using the expression

$$\Delta y = \frac{L\lambda}{d}$$

Putting values, we get

$$\Delta y = \frac{20 \times 10^{-2} \times 546 \times 10^{-9}}{0.10 \times 10^{-3}}$$
$$\Delta y = 1.092 \times 10^{-3}\mathrm{m}$$
$$\Delta y = 1.092\mathrm{mm}$$
$$\boxed{\Delta y = 1.1\mathrm{mm}}\ \textit{approx}$$

9.2 Calculate the wavelength of light , which illuminates two slits 0.5 mm apart and produces an interference pattern on a screen placed 200 cm away from the slits. The first bright fringe is observed at a distance of 2.40 mm from the central bright image.

Given data:

Distance between slits = $d = 0.5\mathrm{mm} = 0.5 \times 10^{-3}\,\mathrm{m}$

Distance of screen from slits = L = 200cm =2m

Distance of first bright fringe = y = 2.40mm = 2.40×10^{-3} m

For first order maximum = m = 1

To find:

Wavelength of light = λ = ?

Calculation:

Using the formula

$$y=m\frac{\lambda L}{d}$$

or $$\lambda=\frac{yd}{mL}$$

Putting values, we get

$$\lambda=\frac{2.40\times10^{-3}\times0.5\times10^{-3}}{1\times2}$$

As for first fringe m=1

$$\lambda=1.20\times0.5\times10^{-6}$$

$$\lambda=0.6\times10^{-6}m$$

$$\boxed{\lambda=600\ nm}$$

9.3 In a double slit experiment the second order maximum occurs at $\theta=0.25^\circ$. The wavelength is 650 nm. Determine the slit separation.

Given data:

For second order maximum = m = 2

Angle = θ = 0.25°

Wavelength of light = $\lambda=650nm=650\times10^{-9}m$

To find:

Slit separation = d = ?

Calculation:

Using the formula

$$d\sin\theta=m\lambda$$

Or $$d=\frac{m\lambda}{\sin\theta}$$

Putting values, we get

$$d=\frac{2\times650\times10^{-9}}{\sin(0.25^\circ)}$$

$$d=2.979\times10^{-4}$$

$$d=0.3\times10^{-3}m$$

$$\boxed{m=0.3mm}$$

9.4 A monochromatic light of λ=588 nm is allowed to fall on the half silvered glass plate G_1, in the Michelson interferometer. If mirror M_1 is moved through 0.233 mm, how many fringes will be observed to shift?

Given data:

Wavelength of light = $\lambda=588nm=588\times10^{-9}m$

Distance through which mirror M_1 is moved = L =0.233mm = 0.233×10^{-3} m

To find:

Number of fringes shifted = m = ?

Calculation: As for Michelson-interferometer, the expression is

$$L=m\left(\frac{\lambda}{2}\right)$$

Or $$m=\frac{2L}{\lambda}$$

Putting values, we get

Or $$m=\frac{2\times0.233\times10^{-3}}{588\times10^{-9}}$$

$$\boxed{m=792}$$

9.5 A second order spectrum is formed at an angle of 38.0° when light falls normally on a diffraction grating having 5400 lines per centimeter. Determine wavelength of the light used.

Given data:

Angle of diffraction = θ = 38°

Number of lines on grating = N = 5400 lines per cm =540000 lines per meter

Order of spectrum = n = 2

To find:

Wavelength of light = λ = ?

Calculation:

Using the formula

$$d\sin\theta=n\lambda$$

Or $$\lambda=\frac{d\sin\theta}{n}$$

As, $$d=\frac{1}{N}$$

So, $$\lambda=\frac{\sin\theta}{N\times n}$$

Putting values, we get

$$\lambda=\frac{\sin38^\circ}{540000\times2}$$

$$\lambda=5.70\times10^{-7}$$

$$\lambda=570\times10^{-9}m$$

$$\boxed{\lambda=570nm}$$

9.6 A light is incident normally on a grating which has 2500 lines per centimeter. Compute the wavelength of a spectral line for which the deviation in second order is 15.0°.

Given data:

Number of liner on grating = N = 2500 lines per cm = 250000 lines per meter
Angle of deviation = $\theta = 15^\circ$
Order of spectrum = n = 2

To find:

Wavelength of light = λ = ?

Calculation:

As grating equation is

$$d\sin\theta = n\lambda$$

Or $\frac{1}{N}\sin\theta = n\lambda$ As $d = \frac{1}{N}$

Or $\lambda = \frac{\sin\theta}{N\times n}$

Putting values, we get

Or $\lambda = \frac{\sin 15^\circ}{250000\times 2}$

Or $\lambda = 5.176\times 10^{-7}$

$\lambda = 517.6\times 10^{-9}$ m

$\boxed{\lambda = 518\text{nm}}$

9.7 Sodium light (λ = 589nm) is incident normally on a grating having 3000 lines per centimeter. What is the highest order of the spectrum obtained with this grating?

Given data:

Wavelength of sodium light = $\lambda = 589$ nm $= 589\times 10^{-9}$ m
Number of lines on grating = N = 3000 lines per cm = 300000 lines per meter
Angle of highest order = $\theta = 90^0$

To find:

Order of the spectrum = n = ?

Calculation:

As grating equation is

As $d\sin\theta = n\lambda$ As, $d = \frac{1}{N}$

or $\frac{1}{N}\sin\theta = n\lambda$

$n = \frac{\sin\theta}{N\lambda}$

Putting the values, we get

$n = \frac{\sin 90^\circ}{300000\times 589\times 10^{-9}}$

$n = \frac{1}{300000\times 589\times 10^{-9}}$

$n = 5.659$

$n = 5.66$

Hence $\boxed{n = 5\text{th}}$

9.8 Blue light of wavelength 480 nm illuminates a diffraction grating. The second order image is formed at an angle of 30° from the central image. How many lines in a centimeter of the grating have been ruled?

Given data:

Wavelength of light = λ = 480nm = 480×10^{-9} m
For second order image = n = 2
Angle of diffraction = $\theta = 30^0$

To find:

Number of line per cm = N = ?

Calculation:

Using the formula

$$d\sin\theta = n\lambda$$

$\frac{1}{N}\sin\theta = n\lambda$ As $d = \frac{1}{N}$

$N = \frac{\sin\theta}{n\lambda}$

Putting the values, we get

$N = \frac{\sin 30^0}{2\times 480\times 10^{-9}}$

$N = \frac{0.5}{2\times 480\times 10^{-9}}$

$N = 5.2\times 10^{5}$ lines per meter

$\boxed{N = 5.2\times 10^{3}\text{lines per cm}}$

9.9 x-rays of wavelength 0.150 nm are observed to undergo a first order reflection at a Bragg angle of 13.3° from a quartz (SiO_2) crystal. What is the interplanar spacing of the reflecting planes in the crystal?

Given data:

Wavelength of x-rays = λ = 0.150nm = 0.150×10^{-9} m
For 1^{st} order reflection = m = 1
Bragg ,s angle = $\theta = 13.3^0$

To find:

Interplanar spacing = d = ?

Calculation:

Using Bragg's equation

$$2d\sin\theta = m\lambda$$

$d = \frac{m\lambda}{2\sin\theta}$

Putting values, we get

$d = \frac{1\times 0.150\times 10^{-9}}{2\times \sin 13.3^\circ}$

$d = \frac{0.150\times 10^{-9}}{2\times 0.23}$

$d = 0.326\times 10^{-9}$ m

$\boxed{d = 0.326\text{nm}}$

9.10 An X-ray beam of wavelength λ undergoes a first order reflection from a crystal when its angle of incidence to a crystal face is 26.5°, and an X-ray beam of wavelength 0.097 nm undergoes a third order reflection when its angle of incidence to that face is 60.0°. Assuming that the two beams reflect from the same family of planes calculate (a) interplanar spacing of the planes and (b) the wavelength λ.

Given data:

For first wavelength λ_1

Angle of incidence = $\theta_1 = 26.5^0$

For 1st order reflection = $n_1 = 1$

For second wavelength λ_2

Wavelength of x-ray beam = 0.097nm

$$\lambda_2 = 0.097\times10^{-9} m$$

For 3rd order reflection = $n_2 = 3$

Angle of incidence = $\theta_2 = 60^0$

To find:

Inter plane spacing = d = ?

Wavelength of first beam = λ_1 = ?

Calculation:

Using Bragg's equation

$$2d\sin\theta = n\lambda$$

For 2nd wavelength

$$2d\sin\theta_2 = n_2\lambda_2$$

Putting values, we get

$$2d\sin60^0 = 3\times0.097\times10^{-9}$$

$$2d\times0.866 = 0.291\times10^{-9}$$

$$d = \frac{0.291\times10^{-9}}{2\times0.866}$$

$$d = 0.168\times10^{-9}m$$

$$\boxed{d = 0.168nm}$$

For calculating wavelength λ_1

$$2d\sin\theta_1 = n_1\lambda_1$$

$$\lambda_1 = \frac{2d\sin\theta_1}{n_1}$$

Putting values, we get

$$\lambda_1 = \frac{2\times0.168\times10^{-9}\sin26.5^0}{1}$$

$$\lambda_1 = 2\times0.168\times10^{-9}\times0.4462$$

$$\lambda_1 = 0.149\times10^{-9}m$$

$$\boxed{\lambda_1 = 0.15nm}$$

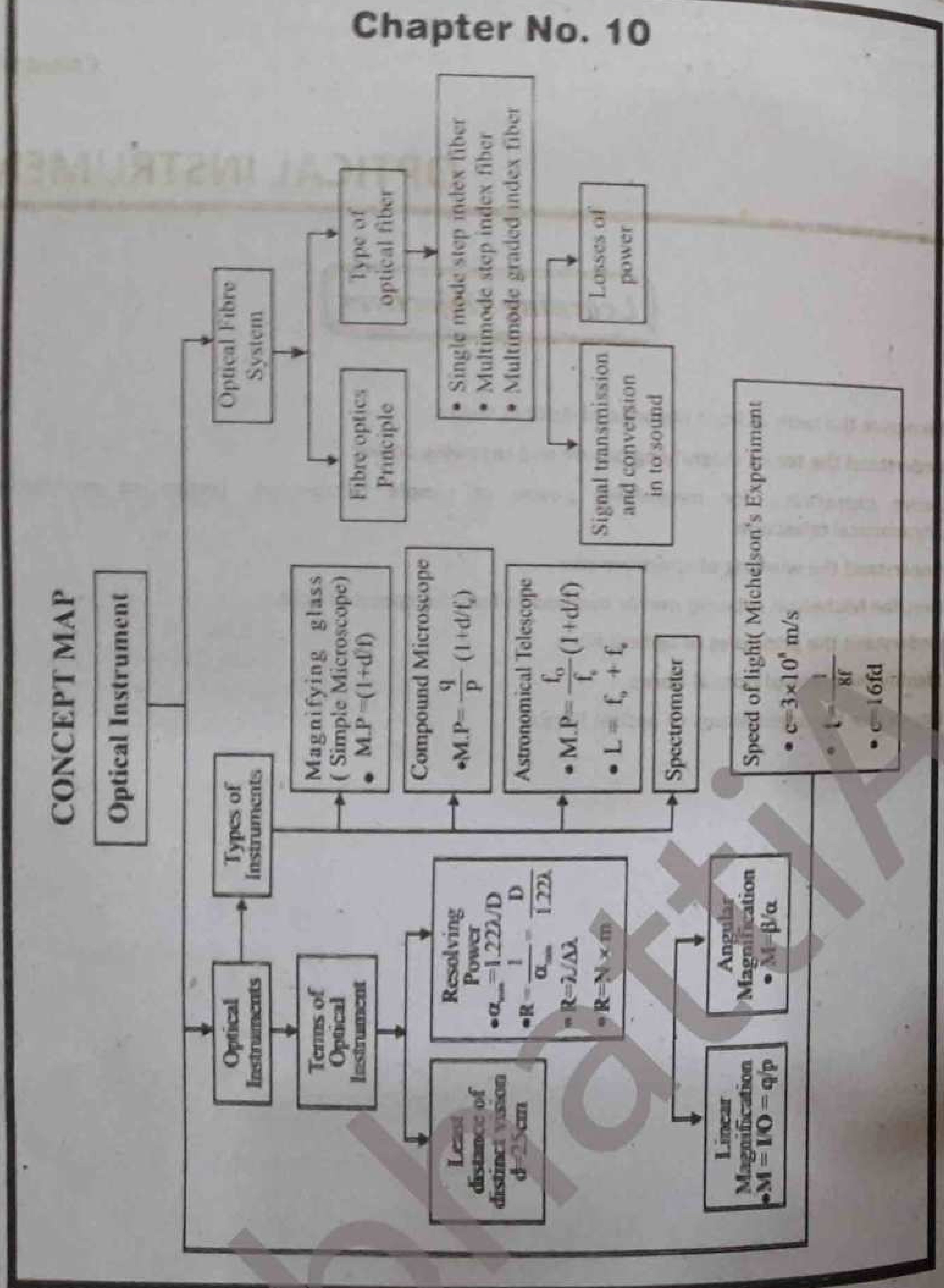
Chapter 10

OPTICAL INSTRUMENTS

Learning Objectives

1. Recognize the term of least distance of distinct vision.
2. Understand the terms magnifying power and resolving power.
3. Derive expressions for magnifying power of simple microscope, compound microscope and astronomical telescope.
4. Understand the working of spectrometer.
5. Describe Michelson rotating mirror method to find the speed of light.
6. Understand the principles of optical fibre.
7. Identify the types of optical fibres.
8. Appreciate the applications of optical fibres.

Chapter No. 10



Optical Instruments

The instruments which are based on the principles of reflection and refraction are called optical instruments.

We use wide variety of optical instruments for practical application like simple microscope, compound microscope telescope, spectrometer, optical fibers etc.

Visual angle

The angle made by on object at the eye is called visual angle.

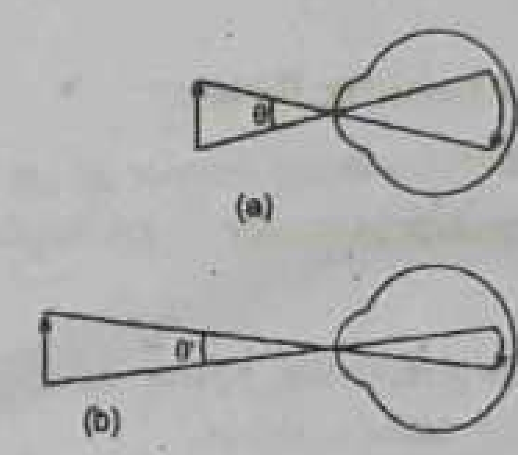


Fig. 10.2
When the same object is viewed at a shorter distance, the image on the retina of the eye is greater; so the object appears larger and more details can be seen. The angle θ the object subtends in (a) is greater than θ' in (b).

Least Distance of Distinct Vision (Near Point)

The minimum distance from the eye at which an object appears to be distinct is called the least distance of distinct vision or near point. It is denoted by d

Explanation

For a human eye, a sharp image is formed when the object is placed at a distance beyond a minimum limit. But when the object is within this limit, the image appears to be blurred and fuzzy. The distance of distinct vision is about 25 cm from the eye.(for young) The location of near point increases with age.

Linear Magnification

The size of the image goes on increasing, when the object brought from a far off point to the focus of the lens. So the magnification takes place.

The ratio of the size of the image to the size of the object is called magnification or linear magnification .i.e.

$$\text{Magnification} = \frac{\text{size of image}}{\text{size of object}}$$

OR

$$M = \frac{I}{O} \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

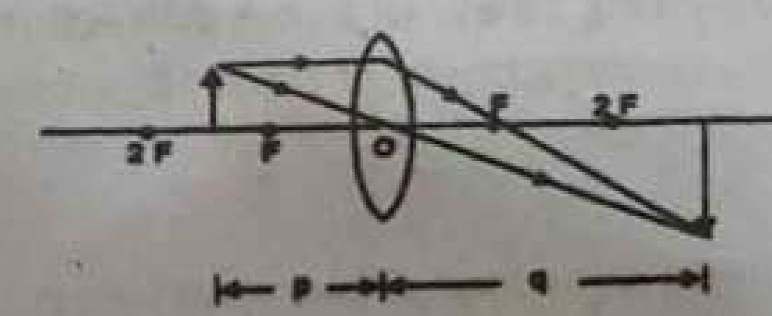


Fig. 10.1

Another Definition

The ratio of distance of image from the lens to the distance of the object from lens is called magnification.

$$\text{Magnification} = \frac{\text{image distance}}{\text{object distance}}$$

OR

$$M = \frac{q}{p} \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

Thus by comparing (1) and (2), we have

$$M = \frac{I}{O} = \frac{q}{p}$$

Angular Magnification

The magnifying power or angular magnification can be defined as ***the ratio of angles subtended by the image as seen through the optical device to that subtended by the object at the unaided eye.***

Let α = the angle subtended by the object form near point at unaided eye and

β = angle subtended by the object with optical device at near point.

Then

$$\text{Magnifying Power} = \frac{\text{angle subtended by image}}{\text{angle subtended by object}}$$

$$M = \frac{\beta}{\alpha}$$

OR $$M = \frac{\theta_i}{\theta_o}$$

Resolving Power

The resolving power of an instrument is it ability to reveal the minor details of the object under examination.

OR

The resolving power of an instrument is it ability to separate the image of two very close objects.

Mathematically

The resolving power is the reciprocal of minimum angle of resolution α_{min}

Raleigh showed that for light of wave length λ used to see two points separately by using a lens of diameter D,

$$\text{Angle of minimum resolution} = \alpha_{min} = 1.22\frac{\lambda}{D}$$

OR $$\text{Resolving Power} = \frac{1}{\alpha_{min}} = \frac{D}{1.22\lambda}$$

Resolving power of plane diffraction grating

The resolving power of plane diffraction grating is

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda}$$

where λ= average of two wave length λ_1 and λ_2)

And $\lambda_1 \approx \lambda_2 \approx \lambda$

Also $\Delta\lambda = \lambda_2 - \lambda_1$

If N is the total number of lines ruled on grating then

$$R = N \times m$$

Where m is the order of diffraction.

> **Tit-Bits**
> If you find it difficult to read small print, make a pinhole in a piece of paper and hold it in front of your eye close to the page. You will see the print clearly.
> **EXPLANATION:**
> It is the rectilinear propagation of light through the pinhole which makes the clear image as in a pinhole camera.

Q.1 What is a simple microscope? Calculate its magnifying power.

Ans

Microscope

چھوٹی چیز کو بڑا دکھانا

A microscope is a device which is used to see the magnified image of very small and near object.

Simple Microscope

A convex lens can be used for magnification

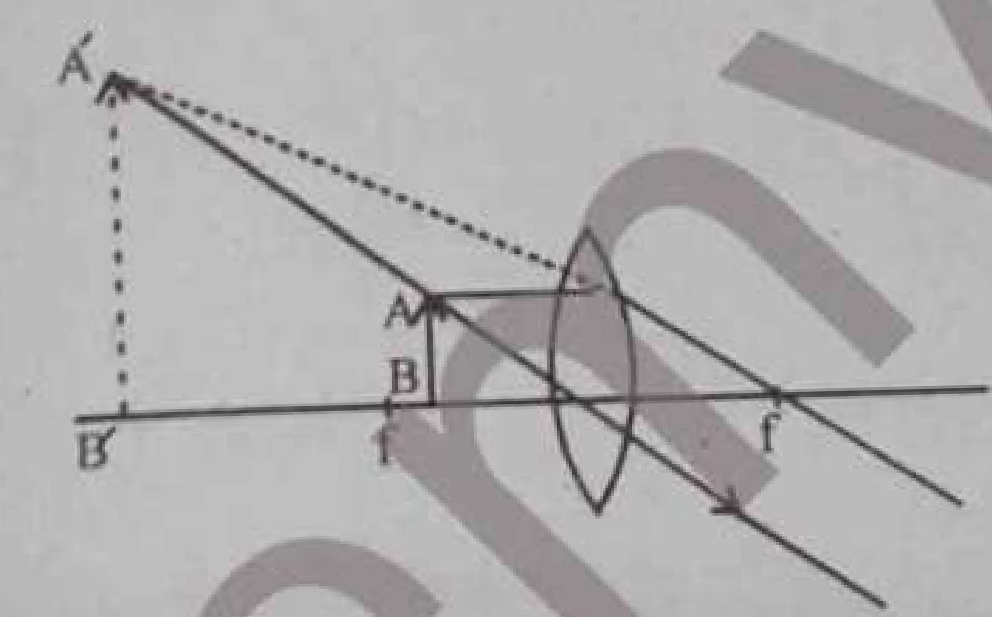
Working principle

When the object is placed between focal point and optical center of biconvex lens then an erect, virtual and enlarged image is obtained.

Construction

It consists of a magnifying glass (i.e. biconvex lens).

Ray Diagram



Working

Consider an object of size O at near point d. The angle subtended by the object at eye is α. When we place the object within focal length of convex lens, then a magnified and virtual image is obtained at near point d making an angle β at eye.

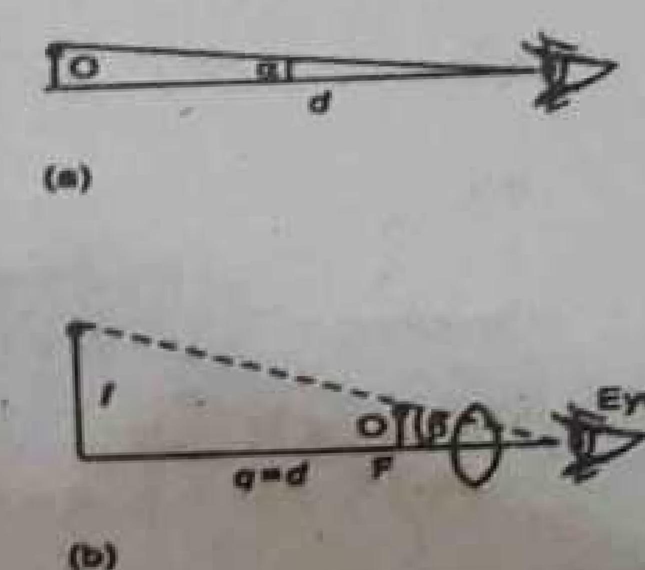


Fig. 10.3
Simple Microscope

Magnifying Power

The angular magnification is

$$M = \frac{\beta}{\alpha} \quad \text{\_\_\_\_\_\_\_\_(1)}$$

By using the figure (I)

$$\tan\alpha = \frac{\text{size of object}}{\text{distance of the object}}$$

$$\tan\alpha = \frac{O}{d}$$

For small value of α , tan α ≈ α

So $$\alpha = \frac{O}{d} \quad \text{\_\_\_\_\_\_\_\_(2)}$$

By using the figure (2)

$$\tan\beta = \frac{\text{Size of image}}{\text{distance of image}}$$

$$\tan\beta = \frac{I}{q}$$

For small value of β, tanβ ≈ β, Also q = d

So $$\beta = \frac{I}{d} \quad \text{\_\_\_\_\_\_\_\_(3)}$$

Putting the values of α and β from equation (2) and (3) in equation (1)

$$M = \frac{I/d}{O/d}$$

$$M = \frac{I}{O}$$

Also $$\frac{I}{O} = \frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{Distance of image}}{\text{Distance of object}} = \frac{q}{p}$$

$$M = \frac{q}{p}$$

Therefore, $M = \dfrac{d}{p}$ __________ (4) $[\because q = d]$

Now, using the lens formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Since the image is virtual, so q = -d

$$\frac{1}{f} = \frac{1}{p} - \frac{1}{d}$$

Multiplying both sides by d, we get

$$\frac{d}{f} = \frac{d}{p} - \frac{d}{d}$$

$$\frac{d}{f} = \frac{d}{p} - 1$$

OR $$\frac{d}{p} = 1 + \frac{d}{f}$$

But $$\frac{d}{p} = M$$

Therefore, $$\boxed{M = 1 + \frac{d}{f}}$$

Which is the formula for magnification; it shows that focal length should be small for high angular magnification.

Q.2 What is compound microscope? Describe its construction and working. Also calculate its magnifying power.

Ans.

Compound Microscope

A compound microscope is used when high magnification is required.

A Compound Microscope.

Construction

It consists of two convex lenses.

(i) Objective

It is of short focal length and small aperture.

(ii) Eye piece

It is of large local length and large aperture.

Principle

When the image formed by the objective of small focal length is within the focal length of the eye piece of large focal length then a virtual, inverted and magnified image is obtained.

Ray Diagram

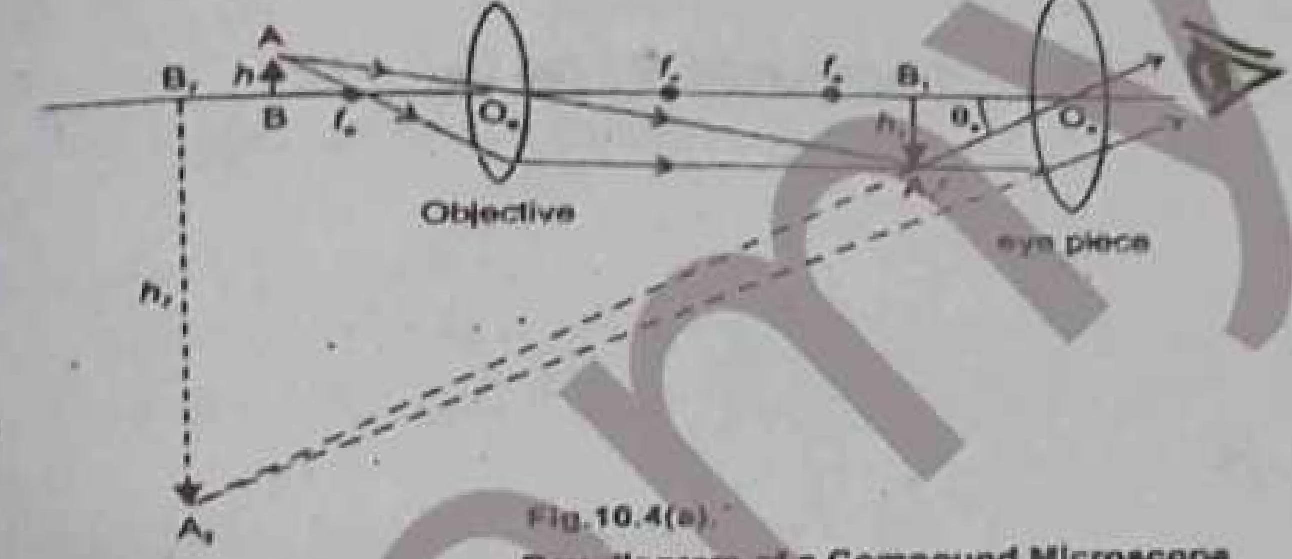


Fig.10.4(a). Ray diagram of a Compound Microscope

For Your Information

A seventeenth century microscope which could be moved up and down in its support ring (Courtesy of the Museum of the History of Science, Florence)

Working

Image formed by objective

The object of height h is placed **just beyond** the principle focus of objective. A **real, inverted** and magnified image of height h_1 is formed. This image acts as an object for eyepiece.

Image formed by eye piece

The image formed by objective lies within the focal length of the eyepiece. Eye-piece makes a **virtual** and **magnified** image of height h_2 at near point.

Magnifying Power

If

β = angle subtended by the final image of height h_2 at the eye

α = angle subtended by the object of height h at the eye.

Then

$$M = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha}$$

Since $\tan\alpha = \dfrac{h}{d}$ and $\tan\beta = \dfrac{h_2}{d}$

So $$M = \frac{h_2/d}{h/d}$$

$$M = \frac{h_2}{h}$$

Multiplying and dividing by h_1

$$M = \frac{h_2}{h} \times \frac{h_1}{h_1}$$

OR $$M = \frac{h_1}{h} \times \frac{h_2}{h_1}$$

where

$\dfrac{h_1}{h}$ = linear magnification of the objective = M_1

$\dfrac{h_2}{h_1}$ = linear magnification of the age piece = M_2

Thus $M = M_1 \times M_2$ __________ (1)

Now M_1 can be written as,

$$M_1 = \frac{q}{p} \quad \text{—————— (2)}$$

As eye piece acts as a simple microscope, hence its magnification M_2 can be expressed as.

$$M_2 = 1 + \frac{d}{f_e} \quad \text{—————— (3)}$$

So, equation (1) becomes

$$M = \frac{q}{p}\left[1 + \frac{d}{f_e}\right] \quad \text{—————— (4)}$$

For higher magnification

We should use the eye piece of **shorter** focal lengths.

For higher resolving power

With **wider objective** we use blue light (of **shorter wave length**) to produce less diffraction and to give more detail of the object.

Q.3 What is an astronomical telescope? Find its magnifying Power.

Ans.

Astronomical Telescope

Telescope

Telescope is an optical instrument used for viewing the **distant** objects. It is of many kinds.

Astronomical Telescope

The telescope used to see the distinct image of distant **heavenly objects** like planets or moon, is called astronomical telescope.

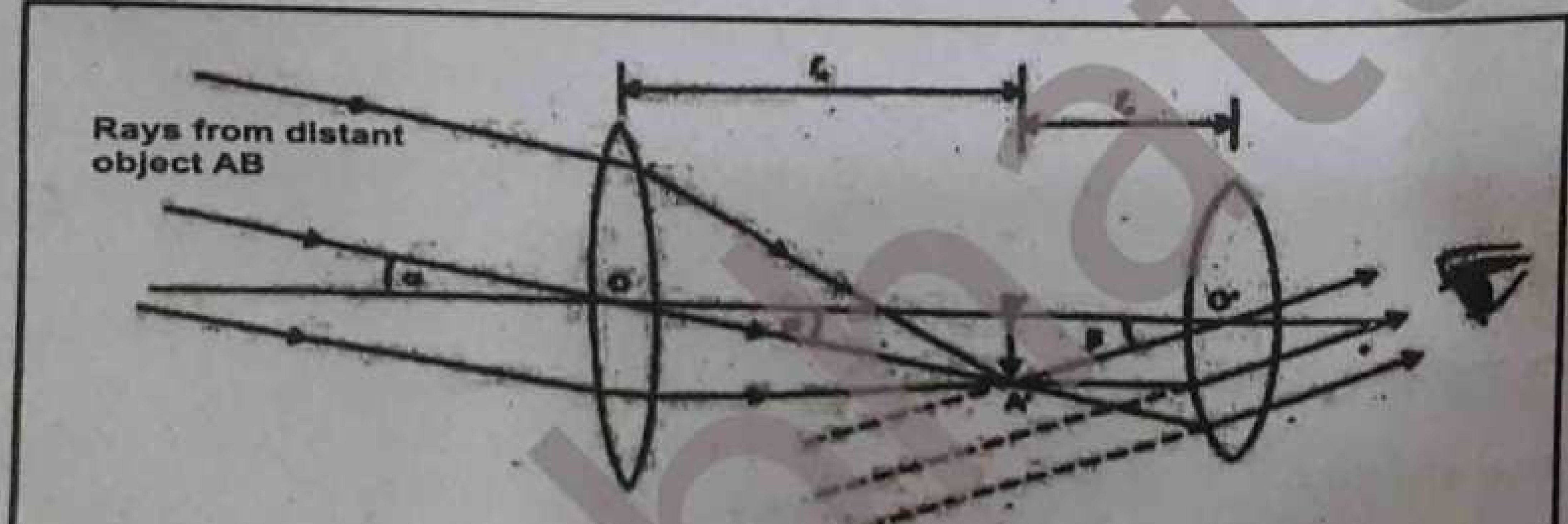
Construction

A simple astronomical telescope consists of two parts.

(i) Objective: It is of large focal length f_o and large aperture.

(ii) Eye piece: It is of short focal length f_e and short aperture.

Ray Diagram



Ray Diagram of Astronomical Telescope

Principle

A real, inverted and diminished image formed by the objective serves as an object for eyepiece which is at the focal point of both the lenses then a virtual and magnified image is formed at infinity.

Working

Image formed by Objective lens

The parallel beam light rays coming from distinct object forms an image A' B' at focal point of the objective. The image formed is a **real, inverted** and **diminished**.

Image formed by eye-piece

The real image A'B' acts as object for eye piece. Image A' B' is formed at the focus of the eye-piece. The eye piece forms the final image at infinity. The final image formed is **virtual, inverted** and **magnified.**

Focusing for Infinity

When the image formed by the objective is at the focus of objective as well as eye piece. Then the telescope is in its normal adjustment or focused for infinity.

Length of Telescope: (in normal adjustment)

In normal adjustment of telescope, the distance between eye piece and objective is called the length of telescope. So

$$L = f_o + f_e$$

Magnifying Power

For a telescope, it can be defined as angle subtended by the image at the eye as seen through telescope to the angle subtended by the object as seen by unaided eye. So

$$M = \frac{\beta}{\alpha} \quad \text{———— (1)}$$

In right triangle OA 'B'

$$\tan\alpha = \frac{A'B'}{OB'} = \frac{A'B'}{f_o}$$

$$\tan\alpha \approx \alpha = \frac{A'B'}{f_o}$$

In right triangle O'A'B'

$$\tan\beta \approx \beta = \frac{A'B'}{O'B'} = \frac{A'B'}{f_e}$$

Putting values of α and β in equation (1), we get

So, equation (1) becomes

$$M = \frac{A'B'/f_e}{A'B'/f_o}$$

$$M = \frac{A'B'}{f_e} \times \frac{f_o}{A'B'}$$

OR $$M = \frac{f_o}{f_e}$$

$$M = \frac{\text{focal length of objective}}{\text{focal length of eye piece}}$$

For Your Information

Reflecting Telescope

Note

A good telescope has an objective of large focal length and large aperture. The brightness of final image depends upon intensity of incident light on the objective. We use the objective of large aperture for this purpose. It may magnify as well as make image to be bright.

Q.4 What is a spectrometer? Describe its construction and working; also write down its uses.

Ans.

Spectrometer

The optical device which is used to study the **spectrum** of various sources of light is called a spectrometer. Spectrum of light beam can be obtained either by refraction by prism or by diffraction from a grating.

Construction

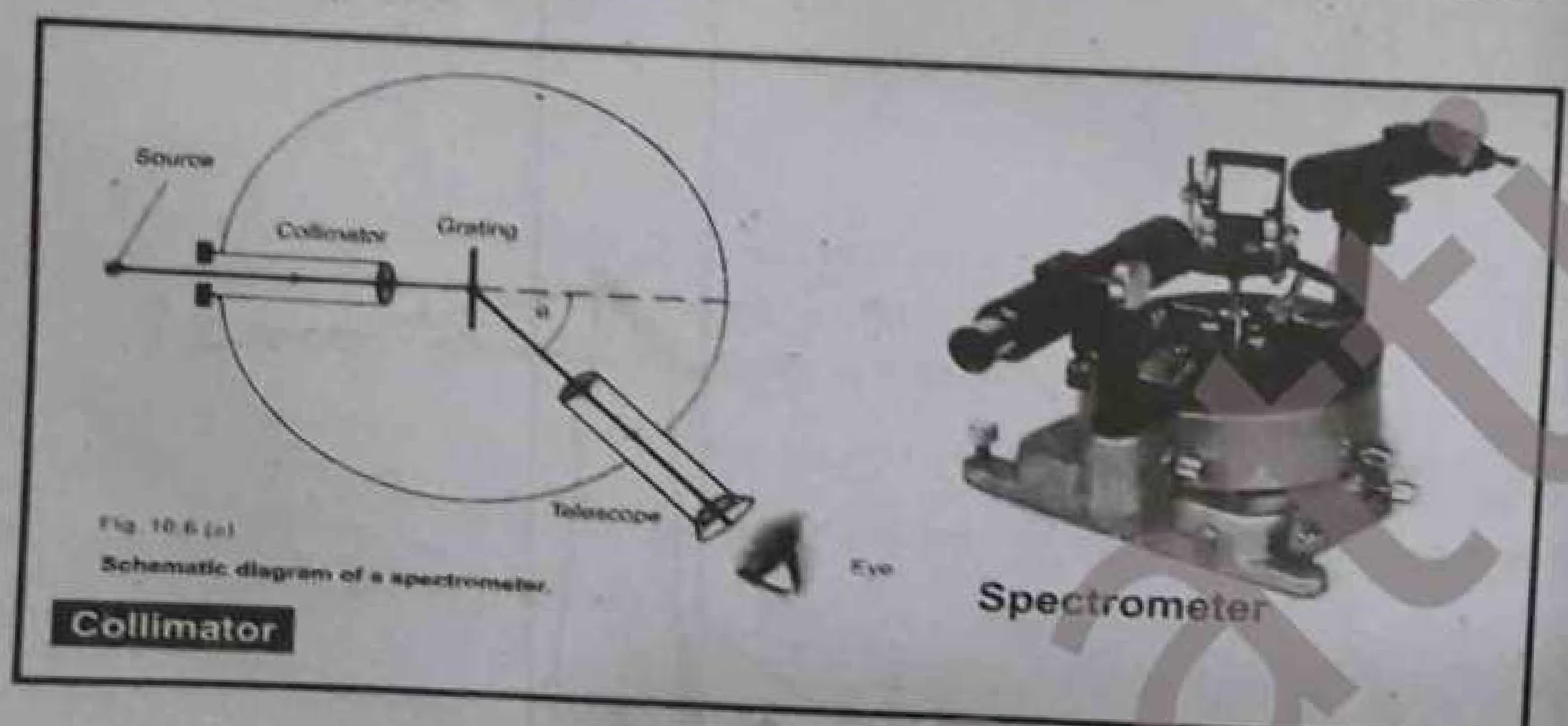
It has three main components:

(i) Collimator

The function of is to make the rays coming from a near by source ***parallel***. At one end of the tube a convex lens is fixed and on the other end, an adjustable slit is provided. When slit is just at the focus of convex lens then light rays entering from slit become parallel after passing through the lens.

(ii) Turn Table

Turntable is capable to rotate about the fixed vertical axis. A circular scale is also attached at the bottom of turn table. A prism or grating is place on the turntable to observe the spectra.



(iii) Telescope

A telescope fixed on a stand and is rotatable about the same axis as that of turntable. A vernier scale is also attached along with the telescope.

Working

Before using the spectrometer, one should carefully adjust the turn table horizontally by leveling screws. The collimator is adjusted to get the parallel rays of light.

The telescope should focus so that the parallel rays entering it are focused at the crosswire near the eye piece.

Uses of Spectrometer

It is used to;

- Study the spectra of different light sources.
- Study the **deviation** of light by glass prism.
- Calculate the **refractive index** of material of prism.
- Measure the **wave length** of light by grating.

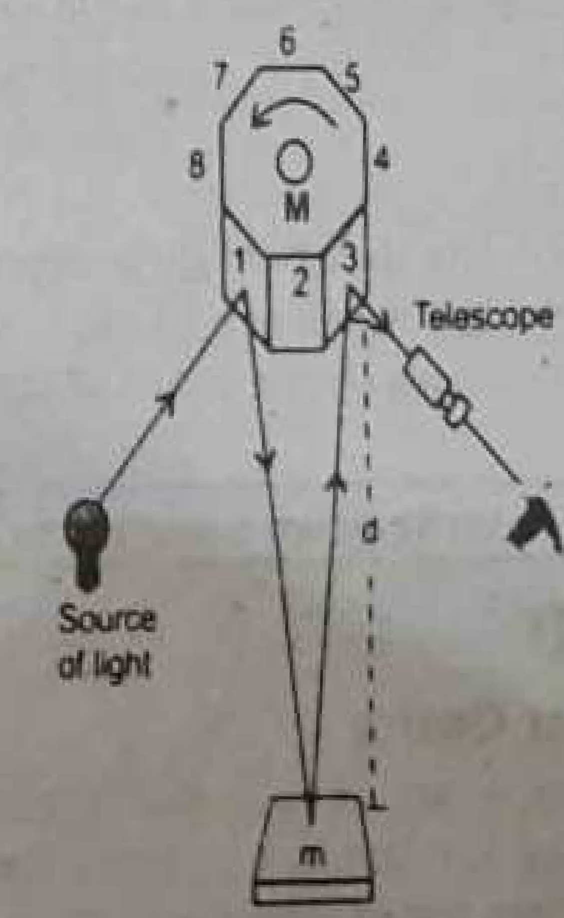
Q.5 Describe the Michelson's experiment to calculate the speed of light.

Ans.

Speed of Light

Light travels so rapidly that it is very difficult to measure its speed. Galileo was the first who tried to measure the speed of light. Although he did not succeed in measurement of speed of light yet he was convinced that it takes some time to travel form one place to another.

In 1926, Michelson made observations for the calculating of speed of light in air. The apparatus and experimental setup is shown in figure.



Michelson's method for measurement of speed of light.

Experimental arrangement

An octagonal (eight – sided) polished mirror M is mounted on the shaft of a motor. The speed of motor is adjustable. Suppose that the mirror in stationary position. The light from source S falls on face 1 of mirror M. The reflected light from M falls on plane mirror m. The light reflected from mirror m falls again on face 3 of M. On reflection through face 3 it enters the telescope T.

Working

When the mirror rotates anti – clockwise, initially the source of light will not be visible but when mirror M gains a certain speed, the source S becomes visible. This happen when the time taken by light in moving from M to m and back to M is equal to the time taken by face 2 to move to the position of face 3.

Expression for speed of light

The angle subtended by any side of eight sided mirror at the center is $2\pi/8$ radian.

Thus $\theta = \frac{2\pi}{8}$ radian ________(i)

Also $\theta = \omega t$ OR $\omega = \frac{2\pi}{T}$

$\theta = (2\pi f)\, t$ $\omega = 2\pi f$

$\theta = 2\pi f\, t$ ________(ii)

Comparing equations (i) and (ii), we get

$$\frac{2\pi}{8} = 2\pi f t$$

or $\frac{1}{8} = ft$

or $t = \frac{1}{8f}$

The distance covered by the light in this time is 2d i.e., S = 2d

As Speed of light = $\frac{\text{distance}}{\text{time}}$

$$c = \frac{S}{t}$$

Putting values of S and t in it, we get

$$c = \frac{2d}{1/8f}$$

OR $c = 16\,fd$

He calculated the value of c in vacuum and is

$$\boxed{c = 2.99792458 \times 10^{8}\ \text{m/sec}}$$

OR $c = 3 \times 10^{8}$ m/sec

Note

- The speed of light in other materials is always less than c.
- The speed of light depends upon the nature of medium (refractive index)
- However the speed of light in vacuum and air is nearly equal.

Point to Ponder

Each of the thin optical fibres is small enough to fit through the eye of a needle. Why is the size of the fibre important?

Q.6 Write down a short note on optical fibers. Also discuss its advantages?

Ans.

Fiber Optics

For hundreds of years man has communicated using flashes of reflected sun light by day and lanterns by night. Navy signalmen still use powerful blinker lights to transmit coded message to other ships during periods of the radio-silence.

Graham Bell invented photo phone after the invention of telephone. His Photo phone used a modulated beam of reflected sunlight focused upon a selenium detector placed at a distance of several hundred meters away.

In this way bell succeeded to transmit a voice message through a beam of light. In recent past the idea of transmission of light through thin fibers optic has been developed. In these days it use in communicational technology.

Advantages of Fibre optics:

It has large advantages due to the use of optical signal (light signal instead of radio signal).

- It has wider band width of capability and safe from electromagnetic interference.
- Optical fiber is used to transmit light around the corners and into inaccessible places.
- It is possible to study the interior of lungs and other parts of human body. For this purpose, a flexible fibre optic is inserted into the body.

- The communication system based on optical fibre has large capabilities. It can be used to transmit thousands of telephone conversation, T.V program and data transferring.
- It has increased the efficiency of word processing, image transmission and reception.
- Fiber optic system consists of much smaller and light weight cables.
- A fiber optic with its protective case is about 6.0 mm, in diameter, which can replace a 7.62 cm diameter bundle of copper wires carrying the same amount of signals.

Optical fibre image

A precision diamond scalpel for use in eye surgery. The illumination is obtained by light passing through a fibre optic light guide.

EXPLANATION

A typical glass fibre is about 100-250μm in diameter. For small diameter the phenomenon of total internal reflection is observed and there will be small chance of power losses of light due to dispersion.

Q.7 What are the basic principles of Fibre optics? Explain them.

Ans.

Fibre Optic Principle

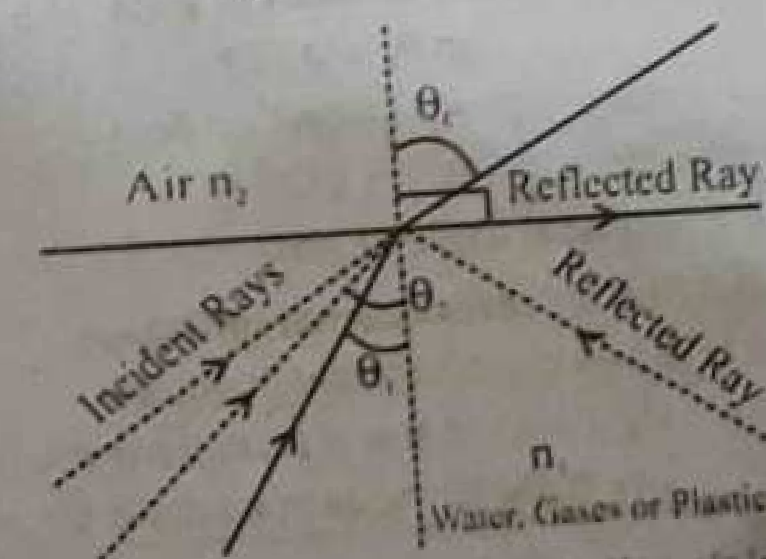
The propagation of light in an optical fibre requires that light should be totally confined within the fibre and cannot escape from it. This can be done possible by:

(1) Total Internal Reflection
(2) Continuous Refraction

Now we discuss these two different ways in detail.

(1) Total Internal Reflection

When a light ray traveling from a denser medium towards a rare medium, makes an angle of incidence greater then critical angle of the medium, then the ray is totally reflected back into the same denser medium. This phenomenon is called total internal reflection.



If the angle of reflection in the air is 90° the angle of incidence is called the critical angle.

Critical angle

The angle of incidence in denser medium for which it corresponding angle of refraction is 90° is called critical angle. It is denoted by θ_c

Refractive Index:

The transparent medium has particular value of refractive index.

The ratio of speed of light in vacuum to the speed of light in transparent medium is called index of refraction of that medium.

So, Refractive index of material = $\frac{\text{Speed of light in vacuum}}{\text{Speed of light in the medium}}$

OR $\boxed{n = \frac{c}{v}}$

Where the value of 'n' (refraction index) depends upon nature of medium. Being ratio *It has no unit.*

When light incident from one optical medium to the other then at the boundary, a part of incident light may be reflected back to the same medium while the remaining part may be refracted into the other medium.

For angles of incidence greater than the critical angle, all the lights is reflected; none is refracted into the air.

The amount of light reflected or refracted at the boundary depends upon the values of refractive indices of two optical media. It also depends upon the angle of incidence.

If θ_1 is the angle of incidence in medium '1' and θ_2 is the angle of refraction in medium '2'. Then by Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

or $$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1}$$

If light comes from denser medium and enter into rare medium then θ_2 must be greater than θ_1.

The particular value of angle of incidence θ_1 . for which angle of refraction θ_2 becomes 90º is called critical angle θ_c.

When $\theta_1 = \theta_c$ then $\theta_2 = 90º$

So $n_1 \sin\theta_c = n_2 \sin 90º$

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\sin\theta_c = \frac{1.0}{1.5}$$

where n_2 = refractive index of air = 1.0

n_1 = refractive index of glass = 1.5

$\sin\theta_c = 0.667$

Or $\theta_c = \sin^{-1}(0.667)$

Or $\theta_c = 41.8°$

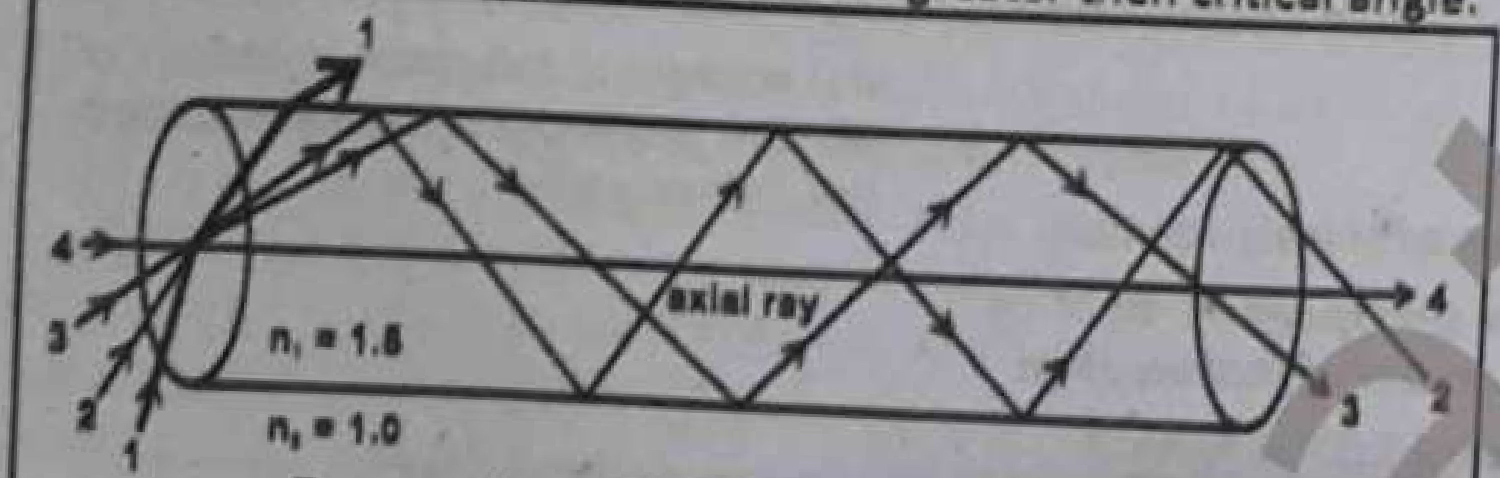
It is the critical angle of glass.

Condition for total internal reflection

For that two conditions are required

(i) Light should travel from denser to rare medium.

(ii) The angle of incidence should be greater then critical angle.



Propagation of light within a glass rod.

Fig. 10.10 (b)

Light propagation within a flexible glass fibre.

Propagation of light within glass rod (by total internal reflection)

Let us consider a glass rod of round surface. The light rays entering into the glass rod from air at an angle less than θ_c (i.e., 41.8º), will escape through the boundary of surface of rod just like ray 1 which is incident at an angle of 30°. While the ray 2 and 3 making angle 42º and 60º show the total internal reflection and so they propagate inside the glass rod due to continuous reflections from the boundary. The ray 4 is called axial ray. Its path is parallel to the axis of rod.

(2) Continuous Refraction

Another mode of propagation of light through optical fiber in which the light is continuously refracted within the fibre is continuous refraction.

If the flexible optical fibre is in such a way that central core is made up of large value of index of refraction and it is surrounded by continuously decreasing and of less index of refraction. Then such a fibre is called multimode graded index fibre.

If the cladding is of constant index of refractive and less than that the core then it is called multimode step index fibre.

As light ray passing from a denser medium to a rare medium, it bends away from normal and vice verse.

In multimode graded index fibre, the light signal has confined within the optical fibre by continuous refraction and then refraction from the boundary and then again refraction, so light remains inside it and cannot leave or escape from it.

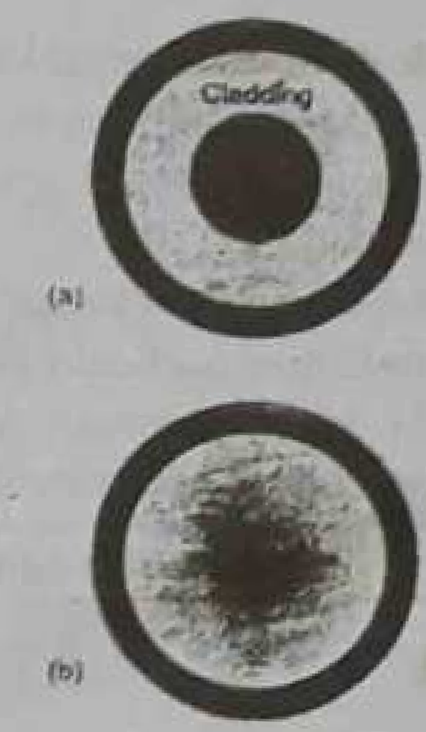
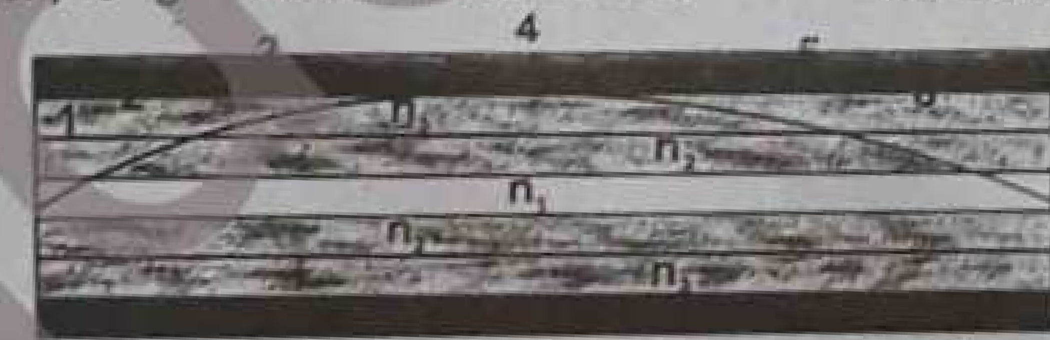


Fig. 10.11

Cross sectional view of

(a) Multi-mode step index fibre

(b) Multi-mode graded index fibre



Light propagation with a hypothetical multi layer fibre.

Q.8 Explain the different types of optical fibres?

Ans.

Types of Optical Fibers

Optical fibers may be classified into three types.

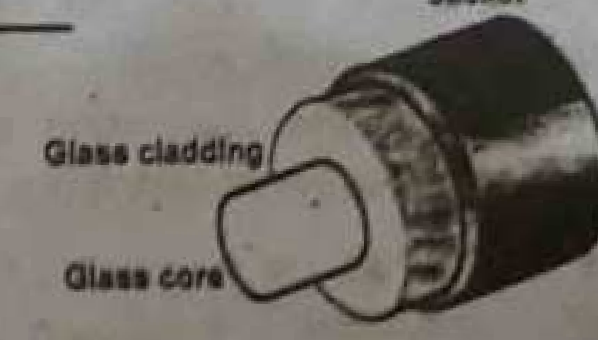
(i) Single mode step index fibre.

(ii) Multimode step index fibre.

(iii) Multimode graded index fibre.

Note:

The term mode is the method by which light is propagated within the fibre.



(a)

(b)

Single - mode step - index fibre.

(i) Single Mode Step Index Fibre

- It has very thin core about 5 μm diameter.
- It has a relatively large cladding.
- Monochromatic light source is required to send light signals through it. That is why we use a laser source.
- It can carry more than 14TV channels or 14,000 phone calls.

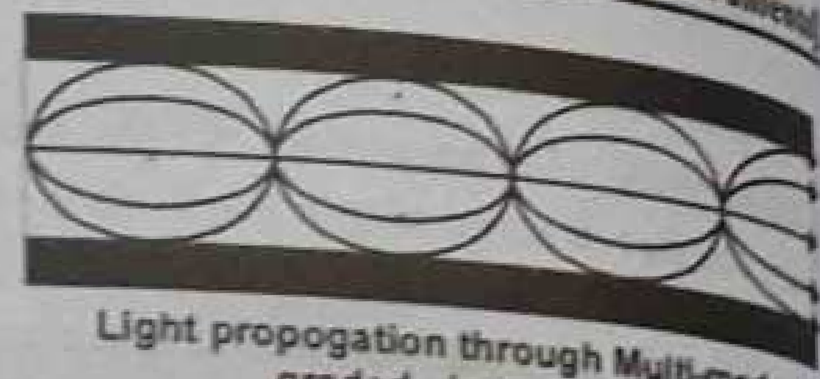
(ii) Multimode Step Index Fibre

- It is the optical fibre in which central core has a large diameter such as 50μm and high refractive index.
- The central core has a constant refractive index n_1 such as 1.52 which steps down up to 1.48 at the boundary with the cladding.
- It is useful for short distances only

Light propogation through Multi-mode step - index fibre.

(iii) Multimode Graded Index Fibre

- It is an optical fibre in which central core has high refractive index, its density gradually decreases towards its outer surface.
- The diameter of the core ranges from 50μm to 1,000μm.
- There is no particular boundary between core and cladding.
- The light is continuously refracted with in the fibre optics. It is useful for long distances.



Light propogation through Multi-mode graded - index fibre.

Q.9 Explain how the signal is transmitted and converted into sound?

Ans

Signal Transmission & Conversation to Sound

The fibre optic communication system consists of three major components.

(i) Transmitter
(ii) Optical fiber
(iii) Receiver

(i) Transmitter

The transmitter converts the electrical signal into light signal which is obtained from microphone. The light source in the transmitter is either a LED (light emitting diode) or laser. The light signal is invisible infrared of typical wave length 1.3 μm which moves faster than visible or ultraviolet light. The lasers and LEDs used in this application are tiny units (less than half the size of the thumbnail) in order to match the size of the fibres.

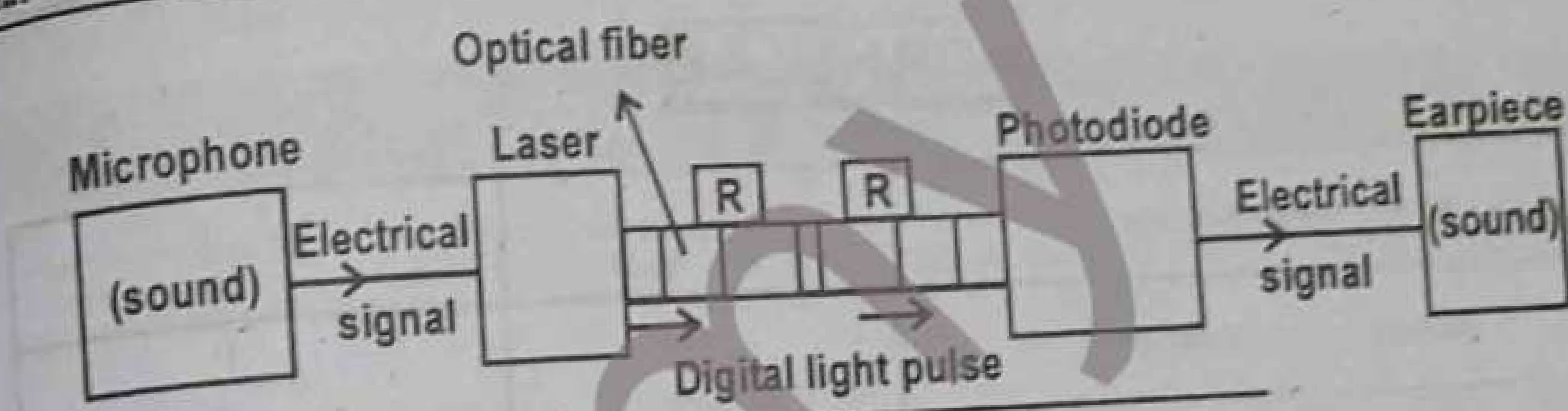
The light signals are modulated to transit the information, it flashed on and off. Digital modulation is expressed in bits or megabits per second where the bit is either 1 (on state) or 0 (off state).

(ii) Optical Fibre

The modulated pulse travel through the optical fibre by total internal reflection and continuously refraction with very faster speed despite the ultra purity (99.99 % glass) of the optical fiber. The light signals while passing through the optical fibre become dim and must be regenerated by a device call repeater. Repeaters are typically placed 30km apart, but in the newer system this separation is about 100km.

(iii) Receiver

Receiver captures the light signals at the other end, and reconverts then to electrical signals by means of photo-diode. Any communication can be represented by a particular pattern or code of 1's and 0's by using computer type equipment. The receiver is programmed to decode to 1s and 0s, and converts it into sound, picture or data as required.



Q.10 Explain the power losses in optical fibre?

Ans

Losses of Power

When light signal travel through fiber, it suffers losses of power due to given factors.

(1) Power loss by scattering and absorption

When the light travels along fibers by multiple reflections, some of light energy is absorbed by the glass medium. It is due to the impurity of the glass medium. Some part of energy of light signals is scattered by the group of atoms such as joints.

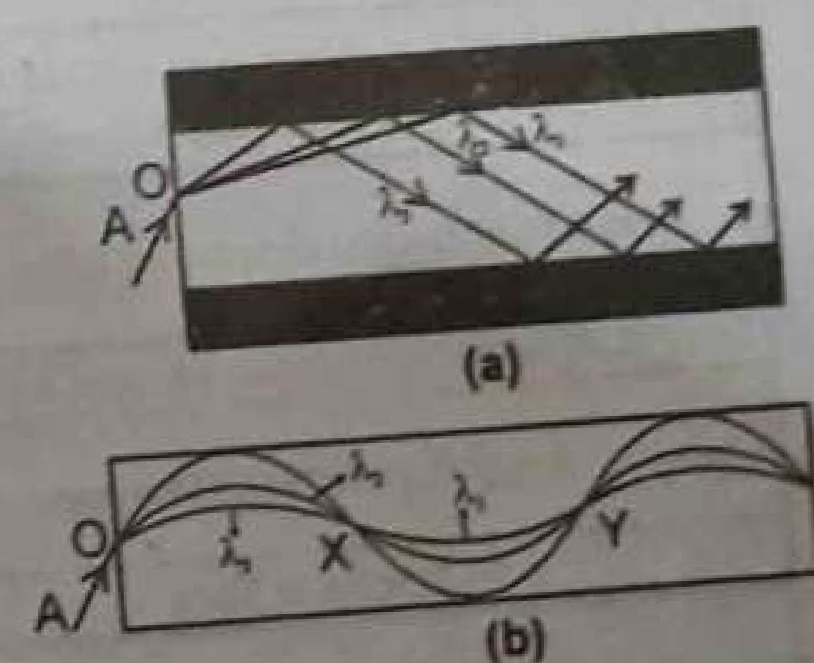
Reduction of loss

Careful manufacturing can reduce the power losses by absorption and scattering.

(2) Power loss due to dispersion

If the light signal is not perfectly monochromatic, then a narrow band of wave lengths are refracted in different directions.

So they cover different lengths of paths inside the fibre and produce phase difference and reach at different time, as shown in Fig. (a), having different wavelengths λ_1, λ_2 and λ_3. So the signal received is distorted and faulty.



Light paths in (a) step-index and (b) graded-index fibre.

Reduction of loss

Disadvantage of the step-index fibre can considerably be reduced by using a graded index fibre, as shown in figure.

The different wavelengths still take different paths and suffer total internal reflection at different layers, but still they are focused at the same point like x and y etc. It is possible, because the speed is inversely proportional to the refractive index. So the wavelength λ_1 travels a longer path than λ_2 or λ_3 but at a greater speed in a low density portion of fibre.

Time difference

In step-index fibre, the over all time difference between different wavelengths may be about 33 ns per km length of fibre. But using a graded index fibre, the time difference is reduced to about 1 ns per km.

FORMULAE

Linear magnification	$M = \frac{I}{O}$	$M = \frac{q}{p}$
Angular magnification	$M = \frac{\beta}{\alpha}$	$M = \frac{\theta_i}{\theta_o}$
Resolving power of an optical device	$\alpha_{min} = 1.22\frac{\lambda}{D}$	$R.P. = \frac{1}{\alpha_{min}} = \frac{D}{1.22\lambda}$
Resolving power of an diffraction grating	$\frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda}$	$R = N \times m$
Magnifying power of simple microscope	$M = \frac{d}{p}$	$M = 1 + \frac{d}{f}$
Magnifying power of simple microscope when focused for infinity	$M = \frac{d}{f}$	
Magnification of objective in compound microscope	$M_1 = \frac{q}{p}$	
Magnification of eye piece in compound microscope	$M_2 = 1 + \frac{d}{f_e}$	
Magnification of compound microscope	$M = M_1 \times M_2$	$M = \frac{q}{p}\left[1 + \frac{d}{f_e}\right]$
Length of telescope under normal adjustment	$L = f_o + f_e$	
Magnification of telescope	$M = \frac{f_o}{f_e}$	
Speed of light	$c = 16\ fd$	
Refractive index	$n = \frac{c}{v}$	
Snell's law	$n_1 \sin\theta_1 = n_2 \sin\theta_2$	$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1}$

Multiple Choice Questions

- *Four possible answers to each statement are given below. Tick (✓) the correct answer:*

1. The objective of telescope has large aperture to:
   (a) Reduce spherical aberration (b) Increase span of observation
   (c) Have high resolution (d) Have low dispersion
2. The sky appears blue because:
   (a) Real light is absorbed (b) Blue light is scattered most
   (c) Blue light is absorbed (d) It is sky's natural colour
3. The magnifying power of telescope can be increased by:
   (a) Increasing the length of telescope (b) Increasing the focal length of objective
   (c) Increasing the diameter of objective (d) Increasing the length of eyepiece
4. When light ray travels from one medium to another medium, the characteristic which does not change is:
   (a) Velocity (b) Wavelength
   (c) Frequency (d) Amplitude
5. The magnifying power of an astronomical telescope is 10. If the focal length of objective is 100 cm. What is focal length of eyepiece?
   (a) 10 cm (b) 100 cm
   (c) 1000 cm (d) 5 cm
6. For normal adjustment, what is length of astronomical telescope if focal lengths of objective and eyepiece are 100 cm and 20 cm respectively?
   (a) 100 cm (b) 20 cm
   (c) 5 cm (d) 120 cm
7. Two convex lenses of focal lengths 10 cm and 5 cm are placed in contact, then their combined focal lengths are:
   (a) 15 cm (b) 5 cm
   (c) $\frac{10}{3}$ cm (d) $\frac{3}{10}$ cm
8. The image formed by eyepiece of compound microscope is:
   (a) Real and magnified (b) Real and diminished
   (c) Virtual and enlarge (d) Virtual and diminished
9. Which of the following quantity is not related to the wavelength of incident light?
   (a) Focal length (b) Radius of curvature
   (c) Power (d) Chromatic aberration
10. Which of the following is used to obtain a virtual and diminished image?
   (a) Concave mirror (b) Convex lens

(c) Convex mirror (d) Plane mirror

11. Two convex lenses of equal focal length f are placed in contact, the resultant focal length of the combination is:

(a) Zero (b) f

(c) 2f (d) f / 2

12. Final image produced by the compound microscope is:

(a) Real and inverted (b) Real and erect

(c) Virtual and erect (d) Virtual and inverted

13. For normal adjustment, length of telescope is:

(a) $f_o + f_e$ (b) $f_o - f_e$

(c) $\frac{f_o}{f_e}$ (d) $\frac{f_e}{f_o}$

14. A spectrometer is used to find:

(a) Wavelength of light (b) Refractive index of prism

(c) Wavelength of different colours (d) All of above

15. Light rays coming from a distant object are considered to be:

(a) Parallel to each other (b) Parallel to principal axis

(c) Parallel to lens (d) Parallel to source

16. Critical angle is that angle of incidence in denser medium for which angle of refraction in rare medium is:

(a) 45° (b) 90°

(c) 0° (d) 180°

17. Michelson calculated the speed of light using:

(a) Spectrometer (b) Galvanometer

(c) Interferometer (d) None of these

18. In optical fibre transmission system _______ are used to regenerate the dim light signal.

(a) Diode (b) Repeaters

(c) Laser (d) Transformer

19. The least distance of distinct vision is 25 cm. The focal length of a convex lens is 5 cm. It can act as a simple microscope of magnifying power:

(a) 4 (b) 5

(c) 6 (d) 8

20. Photodiode converts the light signals into

(a) Electric signals (b) Sound signals

(c) Either of these (d) None of these

ANSWERS

1. c	2. b	3. b	4. c	5. a	6. d	7. d	8. a	9. d	10. c
11. d	12.	13. a	14. d	15. a	16.b	17. d	18. b	19. c	20. a

Short Questions of Exercise

Q.10.1 What do you understand by linear magnification and angular magnification? Explain how a convex lens is used as a magnifier?

(Mtn 2003, D.G.Khan 2005, Fsd 2005-2008, Bwp 2006, Mir Pur 2009, Grw 2009, Lhr 2009-2010-2011, Grw2011)

Ans. Linear Magnification

The ratio of the ***size of image to the size of object*** is called linear magnification. $M = \frac{I}{O} = \frac{q}{p}$

Angular Magnification

The ratio of the ***angle subtended by the image*** as seen through the optical device to the ***angle subtended by the object*** at naked eye is called angular magnification. $M = \frac{\theta_i}{\theta_o} = \beta/\alpha$

Convex lens as a Magnifier

When we place the object ***within the focal length*** of a double convex lens then a magnified, erect and virtual image is obtained.

Q.10.2 Explain the difference between angular magnification and resolving power of an optical instrument. What limits the magnification of an optical instrument?

(Rwp 2005, Bwp 2007, Lhr 2008-2011)

Ans. difference between angular magnification and resolving power

Angular magnification increases ***apparent size of image*** of the object whereas the resolving power of an instrument ***separates the images*** of two very close objects

Limitation

The ***chromatic and spherical aberrations*** are the two main defects in lenses which limits the magnification of optical instrument.

Q.10.3 Why would it be advantageous to use blue light with a compound microscope?

(Bwp 2004-2009, Mtn 2005, Lhr 2004-2005, Mir Pur 2006-2009, Fsd 2005-2008, Grw 2009-2011, Lhr 2010)

Ans Reason

We know that resolving power, $R = \frac{1}{\alpha_{min}} = \frac{D}{1.22\lambda}$

Since the blue light is of shorter wave length. It produces ***less diffraction***. Hence, it ***increases the resolving power*** of compound microscope.

Q.10.4 One can buy a cheap microscope for use by the children. The images seen in such a microscope have coloured edges, Why is this so?

(D.G.Khan 2005, Mir Pur 2009)

Ans Reason

It is due to ***chromatic aberration***.

The white light will disperse after passing through the lens. Such lenses cannot bring all the rays of white light from the object to a single point (focal point) which will give coloured edge.

Chromatic aberration is a type of distortion in which a lens ***fails to focus*** all colors to the same focus point. Chromatic aberration shows "fringes" of color along boundaries that separate dark and bright parts of the image.
Reason It occurs because lenses have a different refractive index for different wavelengths of light. The ***refractive index decreases with increasing wavelength***. Since the focal length f of a lens is dependent on the refractive index n, different wavelengths of light will be focused on different positions.

Q.10.5 Describe with help of diagrams, how (a) a single biconvex lens can be used as a magnifying glass. (b) biconvex lenses can be arranged to form a microscope.

(Mtn 2006, Fsd 2006)

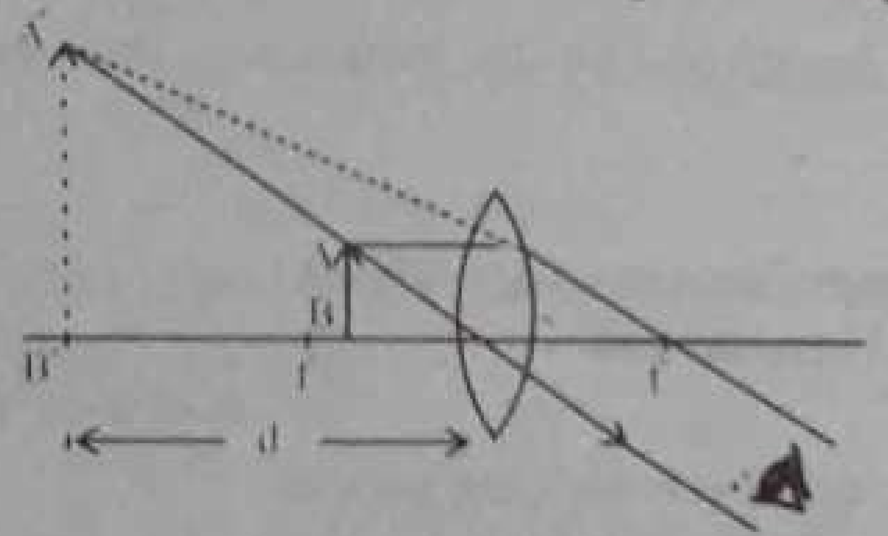
Ans. **For single biconvex lens:**

When object is placed within the focal length of the lens then an erect, virtual and magnified image is obtained.

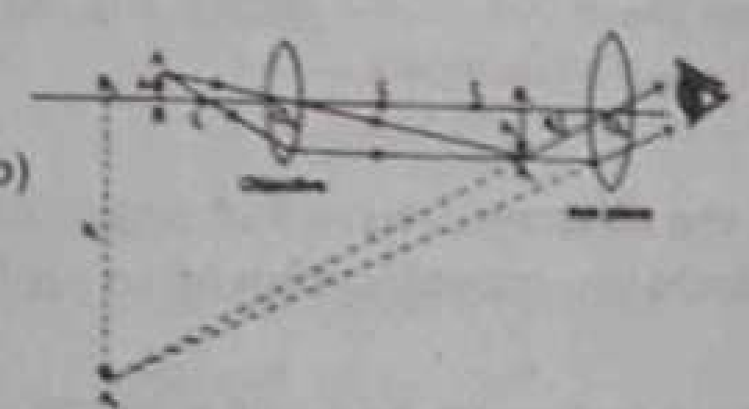
For biconvex lenses:

In compound microscope, when the image formed by the objective is within the focal length of the eyepiece then a virtual, inverted and magnified image is obtained.

(a)



(b)



(a) Ray diagram of single biconvex lens used as magnifying glass

(b) Biconvex lenses used as to form a microscope

Q.10.6 If a person were looking through a telescope at the full moon, how would the appearance of the moon be changed by covering half of the objective lens.

(Sgd 2003, Fsd 2004, Lhr 2010-2011)

Ans. The apparent size of image of moon ***does not change***. It looks ***dim*** only.

Reason

- Magnification of telescope depends upon the ***focal length*** of the eye piece. $M = f_o/f_e$
- When half of the objective lens is covered, it does not change its focal length. So only the intensity of the light ***reduces to half***.

Q.10.7 A magnifying glass gives a five times enlarged image at a distance of 25 cm from the lens. Find, by ray diagram, the focal length of the lens.

Ans. Scale: (along x – axis)

5 cm = 1 cm

25 cm = 5 cm

From fig. focal length = OF = 1.25 cm

Thus f = 1.25 × 5 = 6.2 cm

$M = 4$

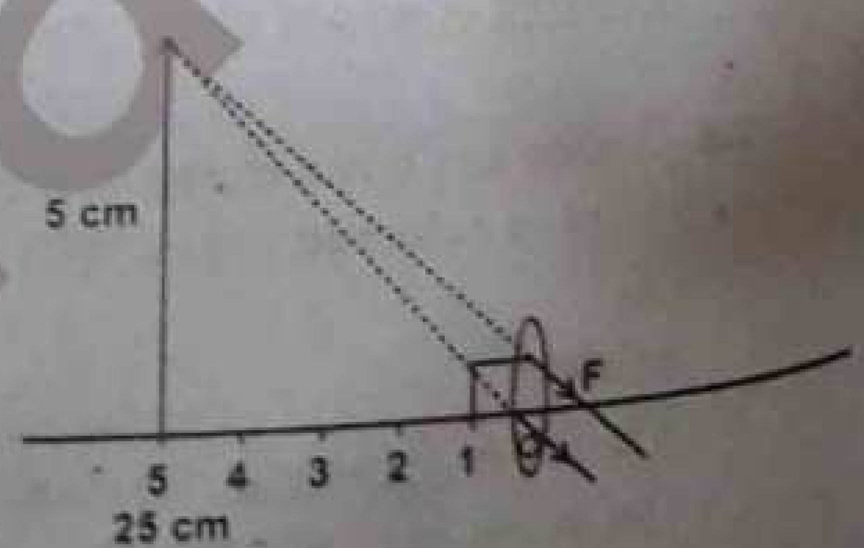
$D = 25$ cm

$f = ?$

$M = 1 + \frac{d}{f}$

$5 = 1 + \frac{25}{f}$

$4 = \frac{25}{f}$, $f = \frac{25}{4} = 6.2$cm



Q.10.8 Identify the correct answer:

(i) The resolving power of a compound microscope depends on:

(a) The refractive index of the medium in which the object is placed.

(b) The diameter of the objective lens.

(c) The angle subtended by the objective lens at the object.

(d) The position of an observer's eye with regard to the eye lens.

(ii) The resolving power of an astronomical telescope depends on:

(a) The focal length of the objective lens.

(b) The least distance of distinct vision of the observer.

(c) The focal length of the eye lens.

(d) The diameter of objective lens.

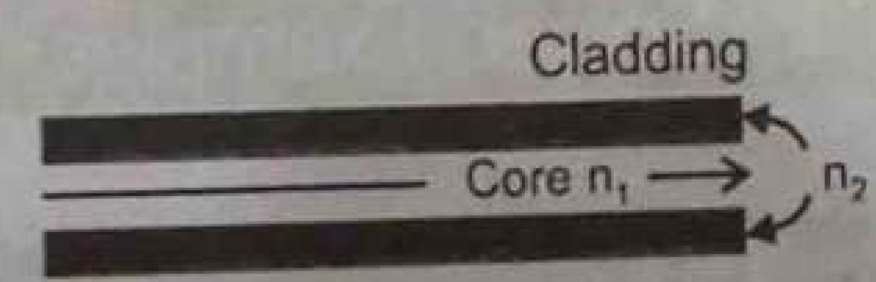
Ans. (i) (b) Diameter of objective lens.

(ii) (d) Diameter of objective lens

Q.10.9 Draw sketches showing the different light paths through a single-mode and a multi mode fibre. Why is the single-mode fibre preferred in telecommunications?

Ans. The different light paths through single mode and multi mode fibre are shown below.

Ans. The path through a single-mode fibre.



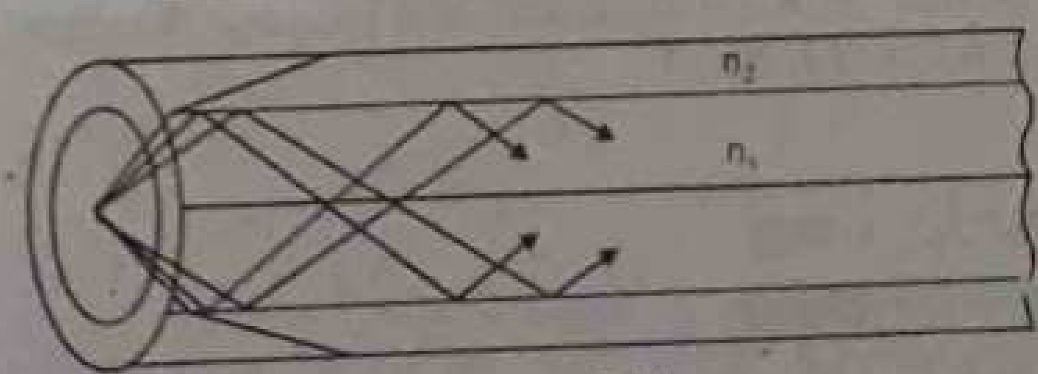


Fig. 10.14
Light propagation through Multi-mode step-index fibre.

Preference of single-mode fibre

Single mode is preferred in telecommunication because

- A strong ***mono-chromatic*** source is used in single mode fibre.
- There is ***no dispersion*** of light and hence no signal is lost.

Q.10.10 How the light signal is transmitted through the optical fibre?

(Bwp 2003-2008, D.G.Khan 2005, Fsd 2005, Grw 2010)

Ans. The signal is transmitted through the optical fibre by:

(i) ***Total internal reflection***

(ii) ***Continuous refraction***

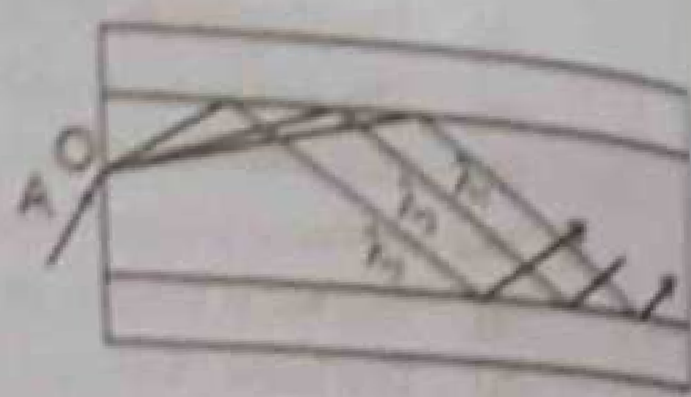
In ***multimode step index fibre***, the signal is transmitted by mean of total internal reflection while in case of ***multimode graded index fibre***, the signal is transmitted by total internal reflection and continuous refraction.

Q.10.11 How the power is lost in optical fibre through dispersion? Explain.

(Lhr 2004, Mir Pur 2004, Fsd 2005, Sgd 2005, Grw 2005, ...)

Ans. **Power loss by dispersion**

When light signal is not perfectly monochromatic, then light will ***disperse*** on passing through the core of the optical fibre into different wavelengths λ_1, λ_2 and λ_3 etc. as shown in figure.



- ~~Each wavelength meets core cladding boundary at different critical angles.~~
- Each wavelength will have different ***path length***.
- So the light of different wavelengths reaches the other end of fibre at ***different times***.
- Therefore, the signal received is distorted or ***faulty***.

> **Dispersion**
> When the light passes into a material at an angle, the light beam is bent or refracted according to Snell's Law and the index of refraction of the material. Thus, each wavelength is refracted at a slightly different angle when passing through a material. This spreading out of the beam of light is called **dispersion** or chromatic dispersion.

Solved Examples

Example 10.1

A microscope has an objective lens of 10 mm focal length, and an eye piece of 25.0 mm focal length. What is the distance between the lenses and its magnification, if the object is in sharp focus when it is 10.5 mm from the objective?

Given Data.

Focal length of objective = f_o = 10mm

Focal length of eye-piece = f_e = 25mm

Distance of object from objective lens = p = 10.5mm

To Find:

(i) Distance between lens = L = q + p' = ?

(ii) Total magnification = M = ?

Calculation:

(i) If we consider the objective alone

$$\frac{1}{f_0} = \frac{1}{p} + \frac{1}{q}$$

Putting values, we get

$$\frac{1}{10} = \frac{1}{10.5} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{10} - \frac{1}{10.5}$$

$$\frac{1}{q} = \frac{10.5-10}{(10)(10.5)}$$

$$\frac{1}{q} = \frac{0.5}{105}$$

$$q = \frac{105}{0.5}$$

$$q = 210 \text{ mm}$$

If we consider the eye-piece, than

$$\frac{1}{f_e} = \frac{1}{p'} + \frac{1}{q'}$$

Where

q' = Distance of the image from eye-piece = least distance of distinct vision = d = –25 cm = –250 mm

Negative sign is because of virtual image.

Thus $$\frac{1}{f_e} = \frac{1}{p'} + \frac{1}{d}$$

Putting values, we get

$$\frac{1}{25} = \frac{1}{p'} - \frac{1}{250}$$

$$\frac{1}{p'} = \frac{1}{25} + \frac{1}{250}$$

$$\frac{1}{p'} = \frac{250+25}{(25)(250)}$$

$$\frac{1}{p'} = \frac{275}{6250}$$

$$p' = \frac{6250}{275}$$

$$\boxed{P' = 22.72\text{mm}}$$

Distance between lens = L = q + P'

Putting values, we get

$$L = 210 + 22.72$$

$$L = 232.72\text{mm}$$

$$\boxed{L = 233\text{mm}}$$

(ii)Total Magnification = (Magnification of objective) (Magnification of eye-piece)

$$M = \left(\frac{q}{p}\right) \times \left(\frac{q'}{p'}\right)$$

Putting values, we get

$$M = \left(\frac{210}{10.5}\right) \times \left(\frac{-250}{22.72}\right)$$ As q' = d = –250mm

$$M = -\frac{52500}{238.56}$$

$$\boxed{M = -220}$$

Negative sign indicates that the image is virtual.

Example 10.2

Calculate the critical angle and angle of entry for an optical fibre having core of refractive index 1.50 and cladding of refractive index 1.48

Given Data:

Refractive index of core = n_1 = 1.50

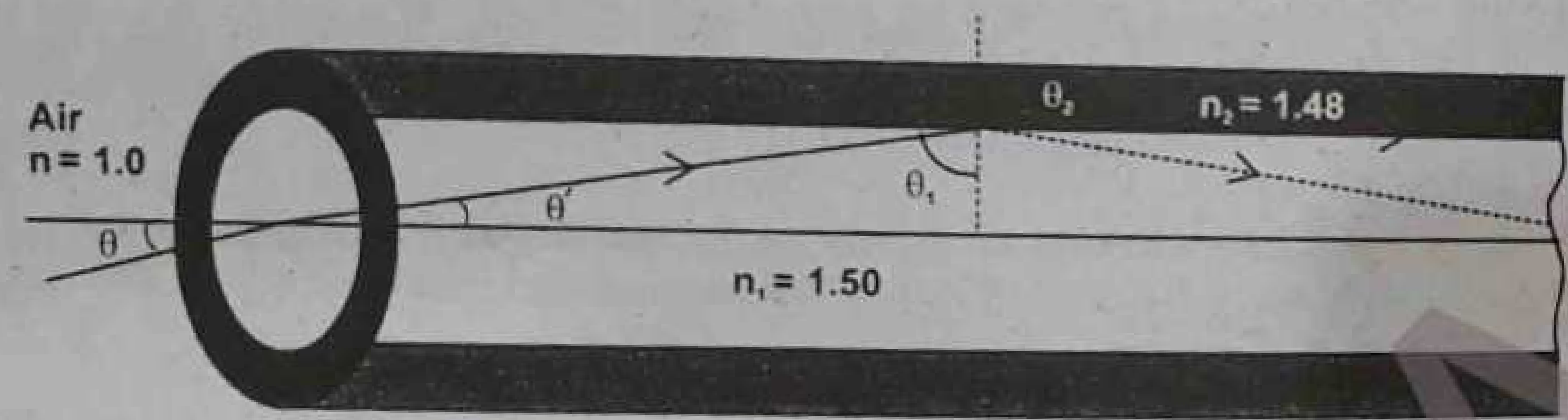
Refractive index of cladding = n_2 = 1.48

To Find.

(i) Critical angle = θ_c = ?

(ii) Angle of entry of light = θ = ?

Calculation.



(i) According to Snell's law (For core-cladding interface)

$$n_1 Sin\theta_1 = n_2 Sin\theta_2$$

When $\theta_1 = \theta_c$ then $\theta_2 = 90°$

Thus $1.5 \times \sin\theta_c = 1.48 \times Sin90° = 1$

$1.5 \times \sin\theta_c = 1.48$

Or $\sin\theta_c = \frac{1.48}{1.50}$

$\theta_c = Sin^{-1}(0.987)$

$\boxed{\theta_c = 80.6°}$

(ii) From Figure

$\theta' = 90° - \theta_c$

$\theta' = 90° - 80.6°$

$\theta' = 9.4°$

Again using Snell,s law (For air-core boundary)

$nSin\theta = n_1 Sin\theta'$ n = refractive index of air = 1.0

$(1.0)Sin\theta = (1.50)\ Sin\ 9.4°$

$Sin\ \theta = 1.50 \times 0.163$

$Sin\ \theta = 0.2445$

$\theta = Sin^{-1}(0.2445)$

$\boxed{\theta = 14.2°}$

Exercise Problems

10.1 A converging lens of focal length 5.0 cm is used as a magnifying glass. If the near point of the observer is 25cm and the lens is held close to the eye, calculate (i) the distance of the object from the lens (ii) the angular magnification. What is the angular magnification when the final image is formed at infinity ?

Given data:

Focal length = f = 5.0cm

Near point = least distinct vision = d = 25cm

To find:

(i) Distance of the object from lens = P = ?

(ii) Angular magnification = M = ?

(iii) Angular magnification, when image is at infinity = M_∞ = ?

Calculation:

(i) Using the lens equation

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

As image is virtual , so q = d = -25cm

Thus

$$\frac{1}{5} = \frac{1}{p} - \frac{1}{25}$$

$$\frac{1}{p} = \frac{1}{5} + \frac{1}{25}$$

$$\frac{1}{p} = \frac{6}{25}$$

$$p = \frac{25}{6}$$

$$p = 4.2cm$$

(ii) Angular magnification = $M = 1+\frac{d}{f}$

$$M=1+\frac{25}{5}$$

$$M=1+5$$

$$M=6$$

(iii) Now when image is at infinity, object must be at focus, hence p = f

$$M=\frac{q}{p}$$

$$M=\frac{q}{f}$$

Hence $M_\infty=\frac{d}{f}$

Putting values, we get

Or $M_\infty=\frac{25}{5}$

$$M = 5.0$$

10.2 A telescope objective has focal length 96cm and diameter 12cm. calculate the focal length and minimum diameter of a simple eye piece lens for use with the telescope , if the linear magnification required is 24 times and all the light transmitted by the objective from a distance point on the telescope axis is to fall on the eye piece.

Given data:

Focal length of objective = f_o = 96cm

Diameter of objective = d_o = 12cm

Linear magnification = M = 24

To find:

Focal length of the eye- piece = f_e = ?

Diameter of the eye – piece = d_e = ?

Calculation:

As magnification = $M=\frac{f_o}{f_e}$

Or $f_e=\frac{f_o}{M}$

Putting the values, we get

$$f_e=\frac{96}{24}$$

$$f_e = 4cm$$

As focal length is directly proportional to diameter, so the ratio of the diameter of the two lenses is equal to the ratio of their focal lengths. Thus

$$\frac{d_o}{d_e}=\frac{f_o}{f_e}$$

Or $d_e=d_o\times\frac{f_e}{f_o}$

Putting the values, we get

$$d_e=12\times\frac{4}{96}$$

$$\boxed{d_e=0.5cm}$$

10.3 A telescope is made of an objective of focal length 20cm and an eye piece of 5.0 cm, both convex lenses. Find the angular magnification.

Given data:

Focal length of objective = f_o = 20 cm

Focal length of eye-piece = f_e = 5.0cm

To find:

Angular magnification = M = ?

Calculation:

Angle magnification = $M=\frac{f_o}{f_e}$

Putting values, we get

$$M=\frac{20}{5}$$

$$\boxed{M=4}$$

10.4 A simple astronomical telescope in normal adjustment has an objective of focal length 100 cm and an eye piece of focal length 5.0 cm. (i) where is the final image formed? (ii) calculate the angular magnification.

Given data:

Focal length of objective = f_o = 100cm

Focal length of eye-piece = f_e = 5.0cm

To find:

(i) Distance of the final image = q' = ?

(ii) Angular magnification = M = ?

calculation:

(i) In normal adjustment, the image due to objective is formed at the focus of the eye – piece, this image acts as an object for eye – piece . therefore

$$p'=f_e=5.0cm$$

$$\frac{1}{f_e}=\frac{1}{p'}+\frac{1}{q'}$$

$$\frac{1}{5}=\frac{1}{5}-\frac{1}{q'}$$ (As image is virtual)

$$\frac{1}{q'}=\frac{1}{5}-\frac{1}{5}$$

$$\frac{1}{q'}=0$$

$$q'=\frac{1}{0}$$

$q'=\infty$ thus final image is formed at infinity

(ii) Angular Magnification

$$M=\frac{f_o}{f_e}$$

$$M=\frac{100}{5}$$

Or $\boxed{M=20}$

10.5 A point object is placed on the axis of and 3.6 cm from a thin convex lens of focal length 3.0cm. A second thin convex lens of focal length 16.0cm is placed coaxial with the first and 26.0 cm from it on the side away from the object. Find the position of the final image produced by the two lenses.

Given data:

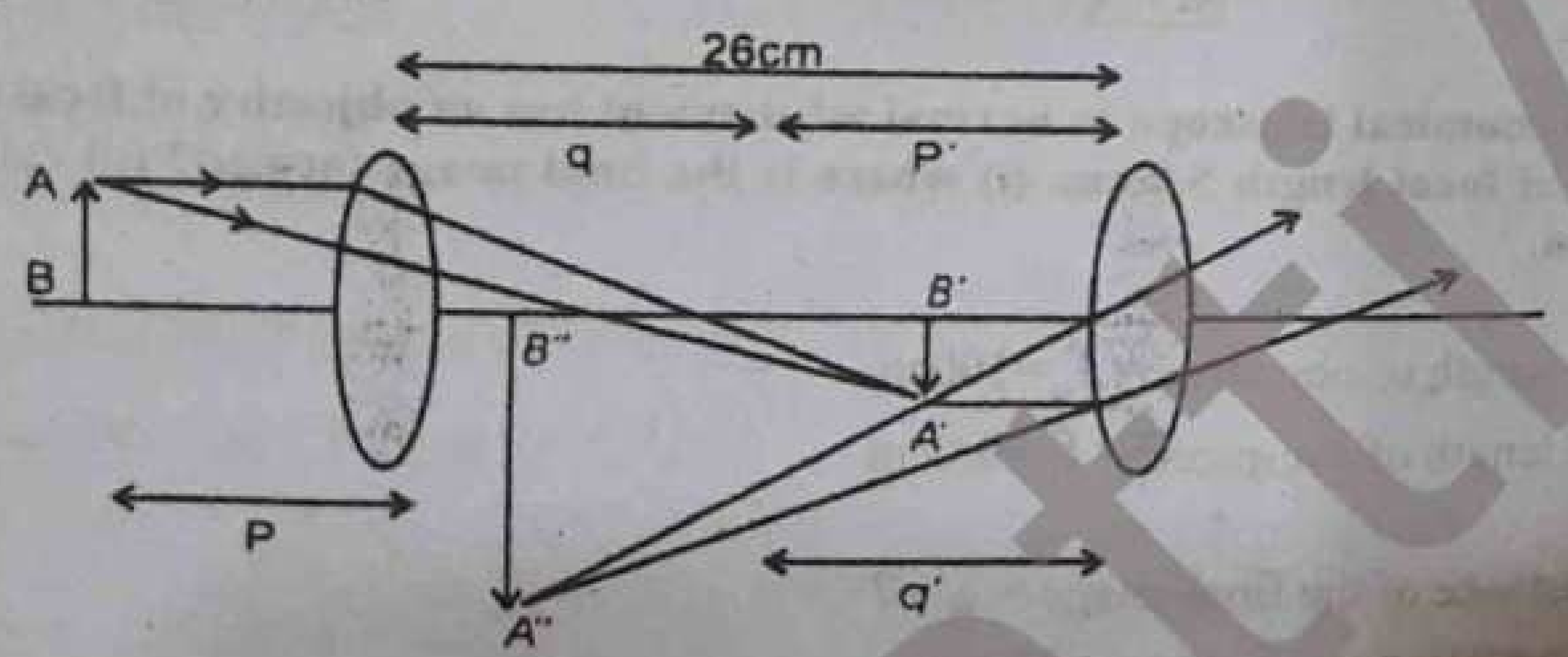
Distance of object = p = 3.6 cm
Focal length of first lens = f = 3.0 cm
Focal length of second lens = f' = 16.0 cm
Distance between two lenses = L = 26 cm

To find:

Position of the final image = q' = ?

Calculation:

Using the lens formula



$$\frac{1}{q}=\frac{1}{f}-\frac{1}{p}$$

Or $$\frac{1}{q}=\frac{1}{3}-\frac{1}{3.6}$$

Putting values, we get

$$\frac{1}{q}=\frac{3.6-3}{3\times3.6}$$

$$\frac{1}{q}=\frac{0.6}{10.8}$$

$$q=\frac{10.8}{0.6}$$

$$\boxed{q=18cm}$$

Thus the image $A'B'$ is formed 18cm away from the first lens. This image will act as an object for the second lens, which is 26cm from the first lens

Thus

$p'=L-q=26-18=8.0cm$

For second lens

$p'=8cm, f'=16cm, q'=?$

$$\frac{1}{f'}=\frac{1}{p'}+\frac{1}{q'}$$

$$\frac{1}{q'}=\frac{1}{f'}-\frac{1}{p'}$$

$$\frac{1}{q'}=\frac{1}{16}-\frac{1}{8}$$

$$\frac{1}{q'}=\frac{1-2}{16}$$

$$\frac{1}{q'}=\frac{-1}{16}$$

$$\boxed{q'=-16cm}$$

Negative sign shows that image is virtual.

10.6 A compound microscope has lenses of focal length 1.0cm and 3.0 cm. An object is placed 1.2 cm from the object lens. If a virtual image is formed, 25 cm from the eye, calculate the separation of the lenses and the magnification of the instrument.

Given data:

Focal length of objective = f_o = 1.0 cm
Focal length of eye-piece = f_e = 3.0 cm
Distance of object = p = 1.2 cm
Distance of image = q' = -25cm (virtual image)

To find:

Separation of lens = L = ?
Magnification = M = ?

Calculation:

As separation of lens = length of microscope = $L = q + p'$

So first we have to calculate q and p'

As lens formula is

$$\frac{1}{f_o}=\frac{1}{p}+\frac{1}{q}$$

Or $\frac{1}{q}=\frac{1}{f_o}-\frac{1}{p}$

Or $\frac{1}{q}=\frac{1}{1}-\frac{1}{1.2}$

$$\frac{1}{q}=\frac{1.2-1}{1.2}$$

$$\frac{1}{q}=\frac{0.2}{1.2}$$

$$\boxed{q=6cm}$$

This image will act as an object for the eye-piece

Now for eye-piece

$q'=-25$ cm , $f_e=3$cm , $p'=?$

Applying lens formula

$$\frac{1}{f_e}=\frac{1}{p'}+\frac{1}{q'}$$

$$\frac{1}{p'}=\frac{1}{f_e}-\frac{1}{q'}$$

$$\frac{1}{p'}=\frac{1}{3}-(-\frac{1}{25})$$

$$\frac{1}{p'}=\frac{1}{3}+\frac{1}{25}$$

$$\frac{1}{p'}=\frac{25+3}{75}$$

$$\frac{1}{p'}=\frac{28}{75}$$

$$p'=\frac{75}{28}$$

$$p'=2.7cm$$

Thus , separation between lens = L = q + p'

$$L=6+2.7$$

$$\boxed{L=8.7\ cm}$$

Magnification$=\frac{q}{p}(1+\frac{d}{f_e})$

Putting values, we get

$$M=\frac{6}{1.2}(1+\frac{25}{3})$$

$$M=\frac{6}{1.2}(\frac{3+25}{3})$$

$$M=5\times\frac{28}{3}$$

$$M=\frac{140}{3}$$

$$M=46.66$$

$$\boxed{M=47\text{ approx.}}$$

10.7 **Sodium light of wavelength 589nm is used to view an object under a microscope . If the aperture of the objective is 0.90 cm. (i) find the limiting angle of resolution , (ii)using visible light of any wavelength , what is the maximum limit of resolution for this microscope .**

Given data:

Wavelength of sodium light = λ =589 nm = $589\times10^{-9}m$

Aperture of the objective = D = 0.90 cm = $0.90\times10^{-2}m$

To find:

(i) Limiting angle of resolution = α_{min} = ?

(ii) Maximum limit of resolution = α'_{min} = ?

Calculation:

(i) **Limiting angle**

Using the relation for limiting angle of resolution.

$$\alpha_{min}=1.22\frac{\lambda}{D}$$

Putting values, we get

$$\alpha_{min}=1.22\times\frac{589\times10^{-9}}{0.90\times10^{-2}}$$

$$=\frac{718.58\times10^{-9+2}}{0.90}$$

$$=798.4\times10^{-7}$$

$$=7.98\times10^{-5}$$

$$\boxed{\alpha_{min}=8.0\times10^{-5}\text{radian}}$$

(ii) **Maximum limit of resolution**

For maximum limit of resolution α'_{min} , the shortest wavelength of the visible spectrum is used and it is 400nm for violet colour light.

Thus $\lambda'=400nm=400\times10^{-9}m$

So $\alpha'_{min}=1.22\frac{\lambda}{D}$

$$=1.22\times\frac{400\times10^{-9}}{0.90\times10^{-2}}$$

$$=\frac{4.88\times10^{-7}}{0.90\times10^{-2}}$$

$$\boxed{\alpha'_{min}=5.4\times10^{-5}\text{radian}}$$

10.8 An astronomical telescope having magnifying power of 5 consists of two thin lenses 24 cm apart. Find the focal length is of the lenses.

Given data:

Magnifying power = M = 5
Separation between the lens = L = 24cm

To find:

Focal length of objective $= f_o = ?$

Focal length of eye-piece $= f_e = ?$

Calculation:

Magnification M is

$$M = \frac{f_o}{f_e}$$

$$\text{Or} \quad 5 = \frac{f_o}{f_e}$$

$$\text{Or} \quad f_o = 5f_e \quad \ldots\ldots\ldots\ldots(1)$$

Distance between two lens is

$$L = f_o + f_e$$

$$24 = 5f_e + f_e$$

Using equation (1) in it

$$24 = 6f_e$$

$$f_e = \frac{24}{6}$$

$$f_e = 4\text{cm}$$

Putting this value of f_e in equation (1), we get

$$f_o = 5 \times 4$$

$$\boxed{f_o = 20\text{ cm}}$$

10.9 A glass light pipe in air will totally internally reflect a light ray if its angle of incidence is at least 39°. What is the minimum angle for total internal reflection if pipe is in water ? (refractive index of water = 1.33)

Given data:

Angle of incidence for glass in air $= \theta_c = 39°$

To find:

Angle of incidence for water $= \theta'_c = ?$

Calculation:

For refractive index of glass light pipe using the formula

$$n = \frac{1}{\sin\theta_c}$$

Putting the values, we get

$$n = \frac{1}{\sin 39°} = \frac{1}{0.629} = 1.59$$

Now using Snell's law for **glass – water interface**

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Where

n_1 = refrective index of glass = 1.59

n_2 = refrective index of water = 1.33

θ_1 = angle of incidence or critical angle $= \theta'_c = ?$

$\theta_2 = 90°$ = for total internal reflection

Thus above equation becomes

$$1.59 \times \sin\theta'_c = 1.33 \times \sin 90°$$

$$\sin\theta'_c = \frac{1.33 \times 1}{1.59} = 0.84$$

$$\theta'_c = \sin^{-1}(0.84)$$

$$\boxed{\theta'_c = 57°}$$

10.10 The reflective index of the core and cladding of an optical fibre are 1.6 and 1.4 respectively. Calculate (i) the critical angle for the interface (ii) the maximum angle of incidence in the air, of a ray, Which enters the fibre and is incident at the critical angle on the interface.

Given data:

Refractive index of core $= n_1 = 1.6$

Refractive index of cladding $= n_2 = 1.4$

To find:

(i) critical angle for core-cladding interface $= \theta_c = ?$

(ii) maximum angle of incidence $= \theta' = ?$

Calculation:

(i) **Snell's law for core-cladding interface**

$$n_1 \text{ Sin } \theta_1 = n_2 \text{ Sin } \theta_2$$

where – $n_1 = 1.6$

$n_2 = 1.4$

$\theta_1 = \theta_c = ?$

$\theta_2 = 90°$

Putting values, we get

$$1.6 \sin\theta_c = 1.4 \text{ Sin } 90°$$

$$\sin\theta_c = \frac{1.4}{1.6}$$

$$\sin\theta_c = 0.875$$

$$\theta_c = \sin^{-1}(0.875)$$

$$\boxed{\theta_c = 61°}$$

(ii) **Maximum angle of incidence**

From figure, for $\theta_c = 61°$,

the angle of refraction θ'_2 in

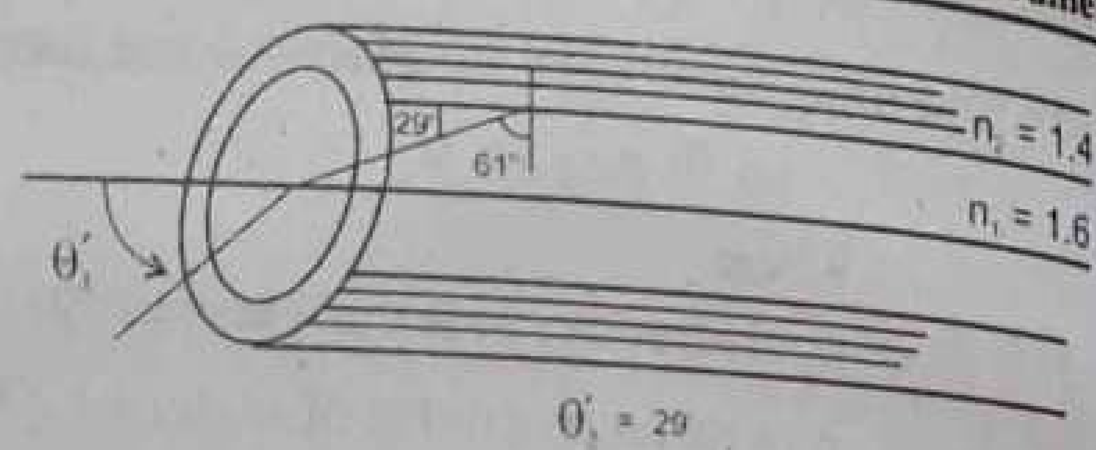
the core should be

$\theta_2' = 90° - \theta_c$

$\theta_2' = 90° - 61° = 29°$

Snell's law ,

for **air – core interface**



$n_1 \sin\theta_1' = n_2 \sin\theta_2'$

$1 \times \sin\theta_1' = 1.6 \times \sin 29°$

$\sin\theta_1' = 1.6 \times 0.485$

$\sin\theta_1' = 0.776$

$\theta_1' = \sin^{-1}(0.776)$

$\boxed{\theta_1' = 51°}$

For air, $n_1 = 1$

For core $n_2 = 1.6$

$\theta_2' = 29°$

****

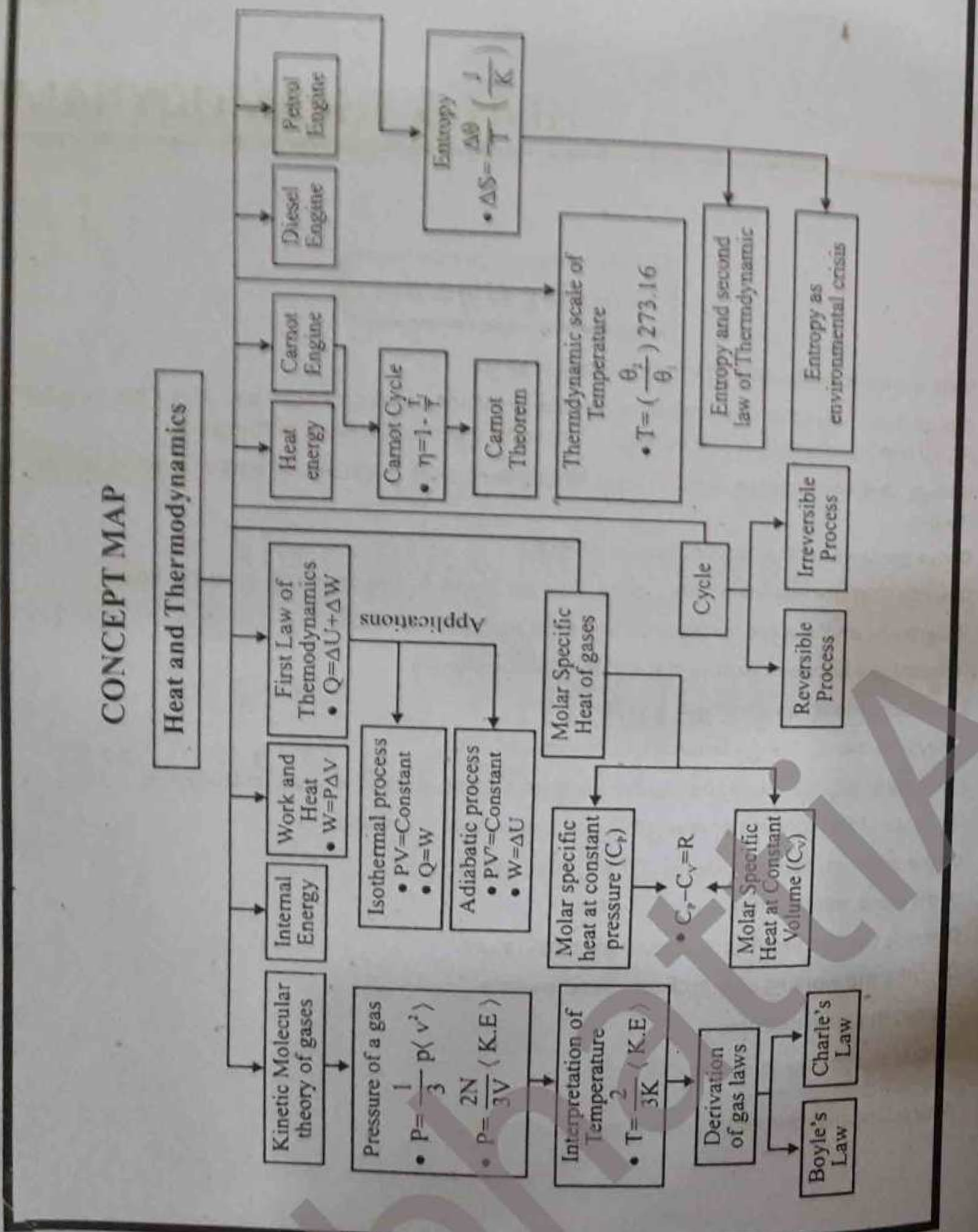
Chapter 11

HEAT & THERMODYNAMICS

Learning Objectives

1. State the basic postulates of kinetic theory of gases.
2. Explain how molecular movement causes the pressure exerted by a gas and derive the equation P = 2/3 N_o, <1/2mv^2>, where N_o is the number of molecules per unit volume of the gas.
3. Deduce that the average translational kinetic energy of molecules is proportional to temperature of the gas.
4. Derive gas laws on the basis of kinetic theory.
5. Describe that the internal energy of an ideal gas is due to kinetic energy of its molecules.
6. Understand and use the terms work and heat in thermodynamics.
7. Differentiate between isothermal and adiabatic processes.
8. Explain the molar specific heats of a gas.
9. Apply first law of thermodynamics to derive C_P-C_V=R.
10. Explain the second law of thermodynamics and its meaning in terms of entropy.
11. Understand the concept of reversible and irreversible processes.
12. Define the term heat engine.
13. Understand and describe Carnot theorem.
14. Describe the thermodynamics scale of temperature.
15. Describe the working of petrol and diesel engines.
16. Explain the term entropy.
17. Explain the change in entropy$\Delta S = \frac{\Delta Q}{T}$
18. Appreciate environmental crisis as an entropy crisis.

Chapter No. 11



Thermodynamics *is a branch of physics that deals with the relationships and conversions between heat and other forms of energy.*

An example of such transformation is the process of converting heat into mechanical work.

Role of Thermodynamics

It plays a central role in technology. All the raw energy available for our use is liberated in the form of heat.

Q.1 State the postulates of kinetic molecular theory of gases.

Ans.

Kinetic Theory of Gases

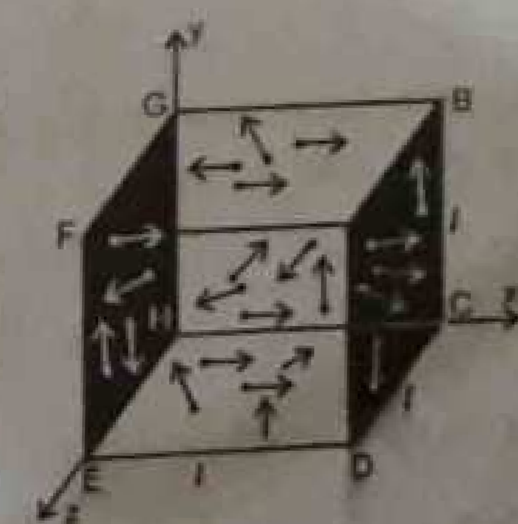
The behaviour of gases is well described by the kinetic theory. It relates **macroscopic** properties (T, P, and V etc) of gases to **microscopic** properties (K.E etc.) It provides a mathematical model to study the behavior of gases.

Postulates

- A **finite volume** of gas consists of very large number of molecules.
- The **size** of the molecules is much smaller than the separation between molecules.
- The gas molecules are in **random motion** and may change their direction of motion after every collision.
- Collisions between gas molecules themselves and with walls of container are assumed to be perfectly **elastic**.
- Molecules exert **no force** on each other except during a collision.

Q.2 Derive the relations for pressure and temperature in terms of average kinetic energy of the molecules?

Ans.



Pressure of Gas

According to kinetic theory, *the pressure exerted by a gas is* merely *the momentum transferred to the wall of the container per sec per unit area due to the continuous collision of molecules of the gas.*

Expression for pressure of gas

Consider a cubical box as shown in figure.

Let

Length of side of box = ℓ

Number of molecules of a gas = N

Mass of each molecule of a gas = m

Mass of N molecule = mN.

Velocity of a molecule moving along positive x-axis = v_{1x}

Initial momentum of molecules before striking the face ABCDA = mv_{1x}

If the collision is elastic, the molecule will rebound from ABCDA with same speed. So

Final momentum= $-mv_{1x}$

Change in momentum = $-mv_{1x} - mv_{1x} = -2mv_{1x}$

After recoiling the molecule travels to opposite face EFGHE and collides with it. The molecules again rebounds back to the face ABCDA after covering a distance 2ℓ.

Let, time between two successive collisions = Δt

Then $\Delta t = \dfrac{2\ell}{v_{1x}}$ $[\because S = vt]$

The number of collisions per second (or collision frequency) that the molecule makes with this face is,

$$f = \frac{1}{\Delta t} = \frac{v_{1x}}{2\ell}$$

Rate of change of momentum of the molecules

due to collision with face ABCDA= $\dfrac{-2mv_{1x}}{\Delta t}$

Putting value of Δt, we get

Rate of change of momentum $= -2mv_{1x} \times \dfrac{v_{1x}}{2\ell} = \dfrac{-mv_{1x}^2}{\ell}$

Now, according to the 2nd law of motion in terms of momentum:
~~By second law of motion,~~ rate of change of momentum of molecule is equal to the force applied by the wall.

From Newton's third law of motion Force F_{1x} exerted by the molecule on the face ABCDA is equal but opposite.

So,

$$F_{1x} = -\left(-\frac{mv_{1x}^2}{\ell}\right)$$

$$F_{1x} = \frac{mv_{1x}^2}{\ell}$$

Thus, the total force acting along x-axis due to N molecules of the gas moving with velocities $v_{1x}, v_{2x}, v_{3x}, \ldots\ldots\ldots, v_{Nx}$ is

$$F_x = F_{1x} + F_{2x} + F_{3x} \ldots\ldots\ldots + F_{Nx}$$

OR $F_x = \dfrac{mv_{1x}^2}{\ell} + \dfrac{mv_{2x}^2}{\ell} + \dfrac{mv_{3x}^2}{\ell} + \ldots\ldots\ldots \dfrac{mv_{Nx}^2}{\ell}$

As pressure is normal force per unit area

Hence $P_x = \dfrac{F_x}{A} = \dfrac{F_x}{\ell^2}$ $[\because \text{Area} = \ell \times \ell = \ell^2]$

OR $P_x = \dfrac{1}{\ell^2}\left[\dfrac{mv_{1x}^2}{\ell} + \dfrac{mv_{2x}^2}{\ell} + \dfrac{mv_{3x}^2}{\ell} + \ldots\ldots\ldots + \dfrac{mv_{Nx}^2}{\ell}\right]$

OR $P_x = \dfrac{m}{\ell^3}\left[v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ldots\ldots\ldots\ldots + v_{Nx}^2\right]$ ______ (1)

Since Density = $\dfrac{\text{mass}}{\text{Volume}}$

OR $\rho = \dfrac{mN}{\ell^3}$ $[\because V = \ell \times \ell \times \ell = \ell^3]$

OR $\dfrac{m}{\ell^3} = \dfrac{\rho}{N}$

Substituting the value of $\dfrac{m}{\ell^3}$ in eq. (1), we get

$$P_x = \frac{\rho}{N}\left[v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ldots\ldots\ldots\ldots + v_{Nx}^2\right]$$

$$P_x = \rho\left[\frac{v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ldots\ldots\ldots\ldots + v_{Nx}^2}{N}\right] \text{______ (2)}$$

Where $\left[\dfrac{v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ldots\ldots\ldots\ldots + v_{Nx}^2}{N}\right] = \langle v_x^2\rangle$ is called the mean of **square velocity** of molecules moving along x–direction So equation (2) becomes

$$P_x = \rho\langle v_x^2\rangle \text{______ (3)}$$

Similarly, pressure on the faces perpendicular to y and z axes will be

$$P_y = \rho\langle v_y^2\rangle$$

$$P_z = \rho\langle v_z^2\rangle$$

As the motion of molecules is random, therefore mean square velocity of all the component velocities will be equal. i.e.

$$\langle v_x^2\rangle = \langle v_y^2\rangle = \langle v_z^2\rangle$$

Since $\langle v^2\rangle = \langle v_x^2\rangle + \langle v_y^2\rangle + \langle v_z^2\rangle$

$$\langle v^2\rangle = \langle v_x^2\rangle + \langle v_x^2\rangle + \langle v_x^2\rangle$$

$$\langle v^2\rangle = 3\langle v_x^2\rangle$$

$$\langle v_x^2\rangle = \frac{1}{3}\langle v^2\rangle$$

Putting the value of $\langle v_x^2\rangle$ in eq. (3), we get

$$P_x = \frac{1}{3}\rho\langle v^2\rangle$$

According to Pascal Law, pressure of the gas every where inside the vessel will be the same provided the gas is of uniform density.

So,

$$P_x = P_y = P_z = \frac{\rho}{3}\langle v^2\rangle$$

Thus in general

$$P = \frac{1}{3}\rho\langle v^2\rangle \text{______ (4)}$$

This is the relation for pressure in terms of mean square velocity.

Relation between pressure and average K.E.

As $\rho = \dfrac{mN}{V}$

Putting value of ρ in equation (4)

$$P = \frac{1}{3}\frac{mN}{V}\langle v^2\rangle$$

$$P = \frac{2}{3}\left(\frac{1}{2}\frac{mN}{V}\langle v^2\rangle\right)$$

$$P = \frac{2}{3}\frac{N}{V}\left\langle\frac{1}{2}mv^2\right\rangle$$

$$P = \frac{2}{3}N_o < \frac{1}{2}mv^2 >$$

Where $\frac{N}{V} = N_o$ = number of molecules per unit volume

$$P = \text{constant} < \frac{1}{2}mv^2 > \qquad [\because \frac{2}{3}N_o = \text{Constant}]$$

OR $$P \propto < \frac{1}{2}mv^2 >$$

OR $$P \propto <K.E>$$

Conclusion

Pressure exerted by the gas is directly proportional to the average translational K.E of gas molecules.

Q.3 Prove that, $T \propto < \frac{1}{2}mv^2 >$

Ans

Interpretation of Temperature

According to ideal gas law

$$PV = nRT \quad \text{\_\_\_\_\_\_ (1)}$$

Where n is the number of moles

V is the volume, T is the absolute temperature and R is the universal gas constant ($R = 8.314\ Jmol^{-1}K^{-1}$)

If N_A is Avogadro number then number of mole can be expressed as

$$n = \frac{N}{N_A}$$

Thus equation (1) can be written as

$$PV = \frac{NRT}{N_A}$$

or $$PV = NkT \quad \text{\_\_\_\_\_\_ (2)}$$

Where $k = \frac{R}{N_A}$ is the Boltzmann's constant. Its value is $1.38 \times 10^{-23}\ J\ K^{-1}$ it is gas constant per molecule

Also we know that

$$P = \frac{2}{3}\frac{N}{V} < \frac{1}{2}mv^2 >$$

OR $$PV = \frac{2N}{3} < \frac{1}{2}mv^2 > \quad \text{\_\_\_\_\_\_ (3)}$$

Comparing the equations(2) and (3), we have

$$NkT = \frac{2}{3}N < \frac{1}{2}mv^2 >$$

OR $$T = \frac{2}{3K} < \frac{1}{2}mv^2 > \qquad [\because \frac{2}{3K} = \text{Constant}]$$

$$T = \text{constant} < \frac{1}{2}mv^2 >$$

$$T \propto < \frac{1}{2}mv^2 >$$

OR $$T \propto <K.E.>$$

Absolute temperature of an ideal gas is directly proportional to the average translational kinetic energy of the gas molecules.

Q.4 Derive the gas laws on the basis of kinetic molecular theory of gases.

Ans

Boyle's Law

Statement

The volume of a given mass of a gas at constant temperature is inversely proportional to the pressure applied to the gas.

From kinetic molecule theory of gases

$$P = \frac{2}{3}\frac{N}{V} < \frac{1}{2}mv^2 >$$

OR $$PV = \frac{2N}{3} < \frac{1}{2}mv^2 >$$

Since at constant temperature, the average kinetic energy of the molecules $< \frac{1}{2}mv^2 >$ always remains constant.

In above equation, the remaining factors are also constant. So

$$PV = \text{constant}$$

OR $$P = \text{constant}\ (\frac{1}{V})$$

OR $$P \propto \frac{1}{V}$$

Hence proved, it is the Boyle's law.

Charles Law

The volume of the given mass of a gas is directly proportional to the absolute temperature when the pressure is kept constant.

From kinetic molecular theory of gases.

$$P = \frac{2}{3}\frac{N}{V} < \frac{1}{2}mv^2 >$$

OR $$V = \frac{2}{3}\frac{N}{P} < \frac{1}{2}mv^2 >$$

As the pressure is constant, $\frac{2}{3}N$ is also constant

Thus,

$$V = \text{constant} < \frac{1}{2}mv^2 >$$

OR $$V \propto < \frac{1}{2}mv^2 >$$

Also $$< \frac{1}{2}mv^2 > \propto T$$

So, $V \propto T$.

Hence verified, it is Charles law.

Q.5 Write a note on Internal Energy and show that it is independent of the path.

Ans

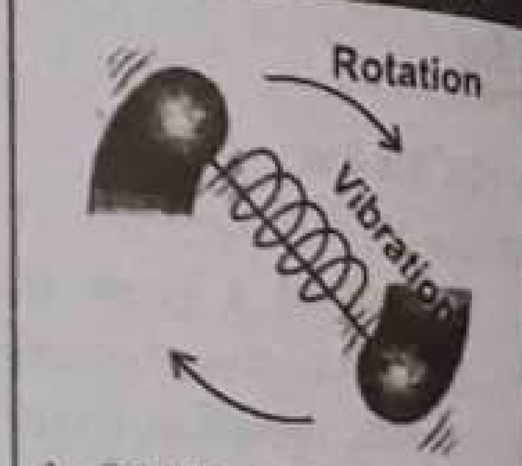
Internal Energy

The sum of all the forms of molecular energies (such as kinetic and potential energy) of a substance is called internal energy.

Explanation

In the study of thermodynamics, an ideal gas is usually considered as a working substance. The molecules of an ideal gas are mere **point mass** which exerts no force on one another. So *the internal energy of an ideal gas system is generally the translational K.E of its molecules.* Since $T \propto \langle K.E. \rangle$, thus the internal energy of an ideal gas is directly proportional to its temperature.

Do You Know?



A diatomic gas molecule has both translational and rotational energy. It also has vibrational energy associated with the spring like bond between its atoms.

How can we Increase the internal Energy

1) By heating

When we heat a substance, energy associated with its atoms or molecules is increased. i.e., *heat is converted to internal energy.*

2) By doing mechanical work

When two objects are rubbed together, their *internal energy increases because of mechanical work.* The increase in temperature of the object indicates an increase in the internal energy.

Note

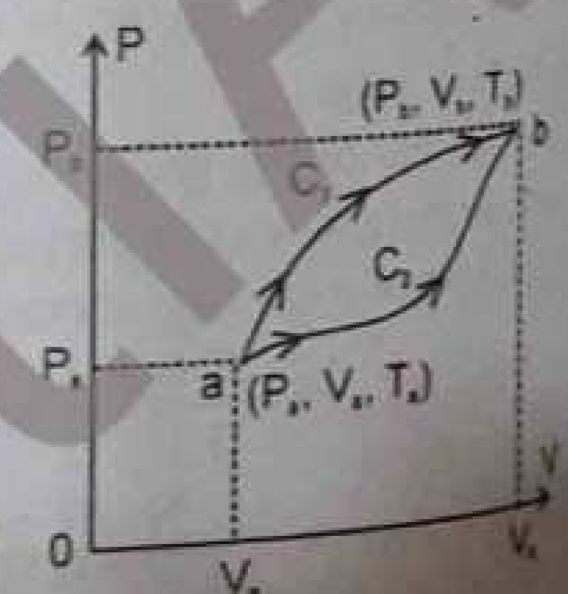
Similarly, when an object slides over any surface and comes to rest because of frictional forces, the mechanical work done on or by the system is partially converted into internal energy.

Internal energy is a state function

In thermodynamics, *internal energy is function of state.* Consequently, it does not depend on the path but depends on initial and final states of the system.

Explanation

Consider a system which undergoes a pressure and volume change from P_a and V_a to P_b and V_b respectively, regardless of the process by which the system changes from initial to final state. By experiment it has been seen that the change in internal energy is always the same and is independent of the paths C_1 and C_2.



Internal energy is similar to the gravitational P.E. So like the gravitational P.E we take the change in internal energy and not its absolute value, which is important.

Q.6 Discuss transfer of energy into work and heat. Also calculate the work done by a thermodynamic system during the volume change.

Ans

Work and Heat

Both heat and work correspond to transfer of energy by some means.

This idea was first applied to the steam engine where it was natural to transfer heat in and get work out.

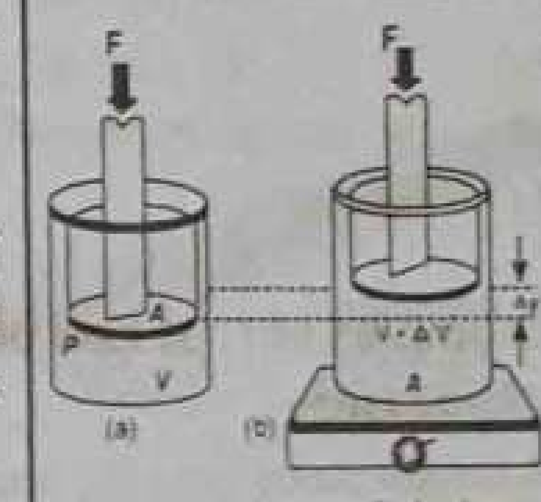
Positive Work done

Work done by the system (gas) on its environment is considered as positive.

Negative work done

Work done by the environment on the system is considered as negative.

If an amount of heat Q enters the system it either appears as an increase in internal energy of the system or is used up in doing work by the system on its environment.



A gas sealed in a cylinder by a weightless, frictionless piston. The constant downward applied force F equals PA, and when the piston is displaced, downward work is done on the gas.

Expression for Work in terms of directly measurable variables

Consider a gas enclosed in the cylinder with a moveable, -frictionless piston of cross-sectional area 'A'. In equilibrium, the system occupies volume 'V', and exerts a pressure 'P' on the walls of the cylinder and its piston. As pressure is defined, force per unit area i.e.,

$$P = \frac{F}{A}$$

or $$F = PA$$

This is the force exerted by gas on piston.

The gas expands through ΔV very slowly so that it remains in the equilibrium. As the piston moves up through a small distance $= d = \Delta y$

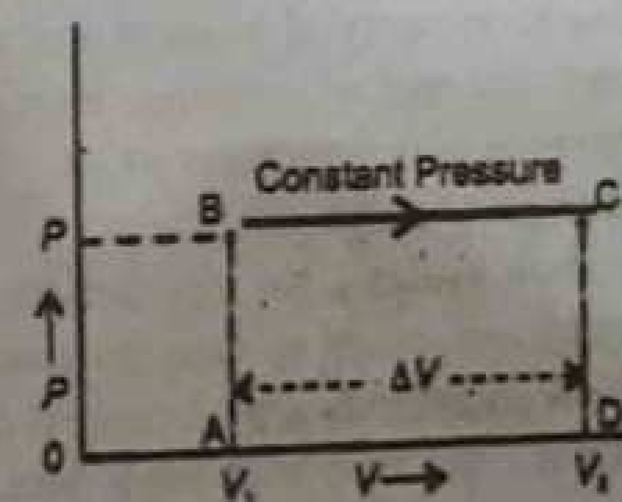
Work done by the gas is

$$W = F\Delta y$$

$$W = PA\Delta y$$

Since $A\Delta y = \Delta V$ (change in volume)

$$\boxed{W = P\Delta V}$$



This is the work done by gas on piston.

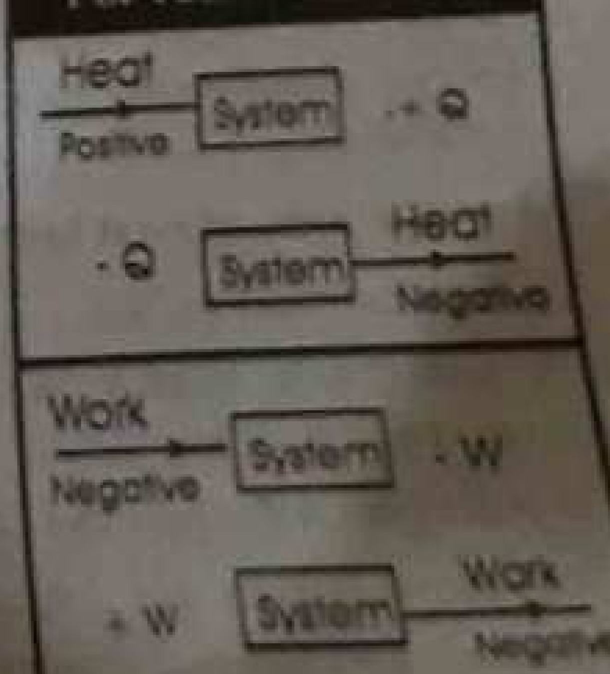
We can express work in terms of directly measurable quantities.

Graphical Representation

Work done can be calculated from the area under P-V graph.

By the details of change in internal energy and the mechanical work done, we can describe the general principles which deal with heat energy transformation into mechanical energy. These principles are known as laws of thermodynamics.

For Your Information



Q.7 Explain the first law of Thermodynamics and its consequences?

Ans

First Law of Thermodynamics

When the heat Q is added to a system, this energy appears as an increase in the internal energy ΔU stored in the system plus the work done W by the system on its surroundings.

Mathematically

$$Q = \Delta U + W$$

Explanation

When heat is added to a system; there is an increase in the internal energy from U_1 to U_2 due to the rise in temperature and an increase in pressure or change in the state. If at the same time, a substance is allowed to expand, then W is the work done on its environment

$$Q = (U_2 - U_1) + W$$

OR $$Q = \Delta U + W \quad \text{______} \quad (1)$$

Thus, the change in internal energy $\Delta U = U_2 - U_1$

From equation (1)

$$\Delta U = Q - W \quad \text{______} \quad (2)$$

EXPLANATION: Heat entering a system is positive and leaving a system is negative. The work done by the system is positive and the work done on the system is negative, as shown in the figure.

Examples of First Law of Thermodynamics

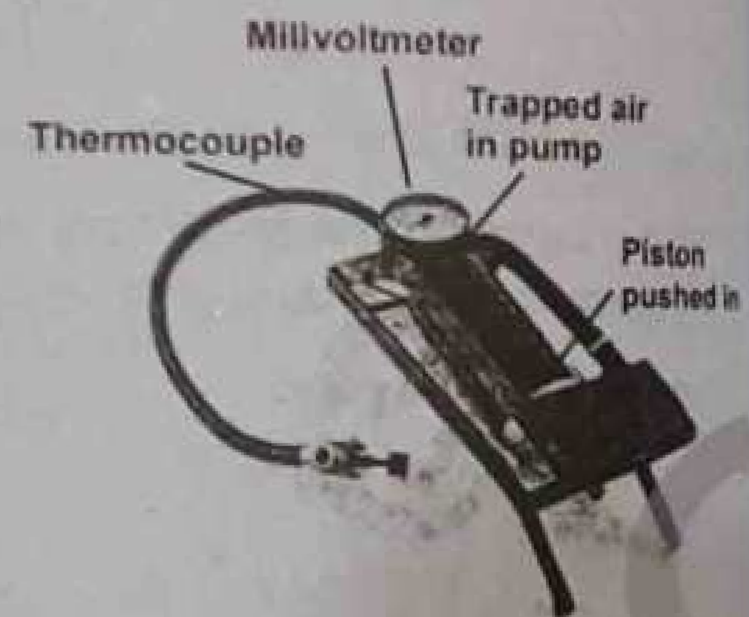
Bicycle Pump

A bicycle pump provides a good example. When we pump on the handle rapidly, it becomes hot due to mechanical work done on the gas, in this way; it increases its internal energy.

Note

The arrangement consists of;

Bicycle pump with a blocked outlet. A thermocouple connected through the blocked outlet to note the temperature of air When piston is rapidly pushed, thermometer shows a temperature rise due to increase of internal energy of the air. The push force does work on the air, thereby increasing its internal energy, by the increase in temperature of air.



Human Metabolism

Human metabolism also provides an example of energy conservation. Human being and other animals do work when they walk, run, or move heavy objects. Work requires energy. Energy is also needed for growth to make new cells and to replace old cells that have died. Work done will result in decrease in internal energy of the body.

According to 1st law of thermodynamics, to an organism of human body

$$\Delta U = Q - W$$

Hence, the body temperature or internal energy is maintained by the food we eat

Note (Metabolism)

Energy transforming processes that occur within an organism are named as metabolism.

Applications of First law of thermodynamics

(1) Isothermal Process

A process in which the temperature of the system is constant is called isothermal process.

In isothermal process the condition for the application of Boyle's Law is fulfilled.

Therefore, when gas expands or compresses isothermally, the product of its pressure and volume during the process remains constant.

Explanation

If P_1, V_1 are the initial pressure and volume where as P_2 and V_2 are pressure and volume after the isothermal change takes place. Then

$$P_1V_1 = P_2V_2$$

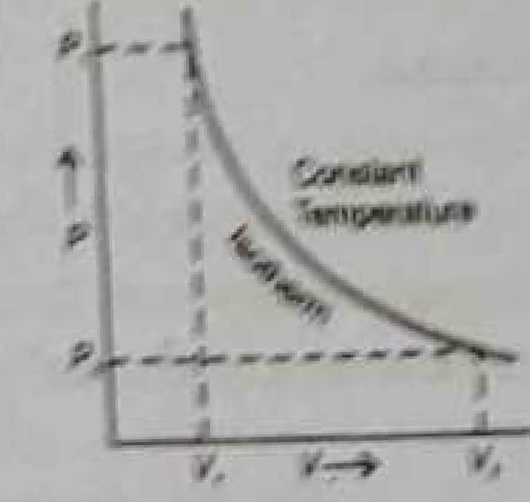


Fig.11.8(a)

As the internal energy of an ideal gas depends only on its temperature, which in this case is constant.

Therefore

$$\Delta U = 0$$

Hence first law of thermodynamics reduces to

$$Q = \Delta U + W$$
$$Q = 0 + W$$
$$Q = W$$

Isothermal Expansion

If a gas expands and does external work W, an amount of heat Q has to be supplied to the gas in order to produce an isothermal change.

Since, transfer of heat from one place to another requires time; hence to keep temperature of the gas constant, the expansion must take place slowly.

Isotherm

The curve representing an isothermal process is called an Isotherm.

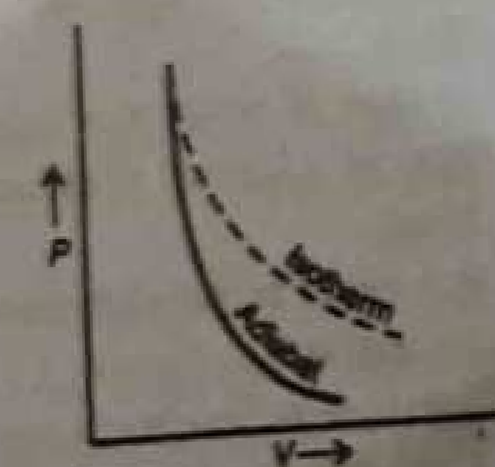
(2) Adiabatic Process

A process in which no heat enters or leaves the system is called adiabatic process.

Explanation

Since in adiabatic process no heat enters or leaves the system i.e., Q = 0

Hence the first law of thermodynamics becomes



$$Q = \Delta U + W$$
$$0 = \Delta U + W$$

OR $$W = -\Delta U$$

Adiabatic Expansion

If gas expands and does external work, it is does at the expense of internal energy of its molecules and hence, the temperature of the gas falls.

Adiabatic compression

If gas is compressed, work is done on the gas; it increases the temperature of the gas

$$-W = \Delta U$$

Condition for adiabatic change

Adiabatic change occurs when the gas expands or is compressed rapidly. Particularly when the gas is contained in an isolated cylinder. In case of adiabatic changes, as the temperature of the gas does not remain constant,

So $$PV^{\gamma} = \text{constant}$$

Where, γ is the ratio of the molar specific heat of the gas at constant pressure to the molar specific heat at constant volume. i.e.,

$$\gamma = \frac{C_p}{C_v}$$

Adiabat

The curve representing an adiabatic process is called an adiabat

An adiabat is steeper than an isotherm.

Examples of Adiabatic Process

The examples of adiabatic processes are

1. The rapid escape of air from a burst tyre.
2. The rapid expansion and compression of air through which a sound wave is passing.
3. Cloud formation in the atmosphere.

Q.8 (a) Define the following terms:

(i) molar specific heat

(ii) molar soecific heat at constant volume (C_v)

(iii) molar specific heat at constant preasure (C_p)

(b) Prove that $C_p - C_v = R$

Ans

Specific heat

The amount of heat required to raise the temperature of one kilogram of a substance up to one Kelvin is called specific heat.

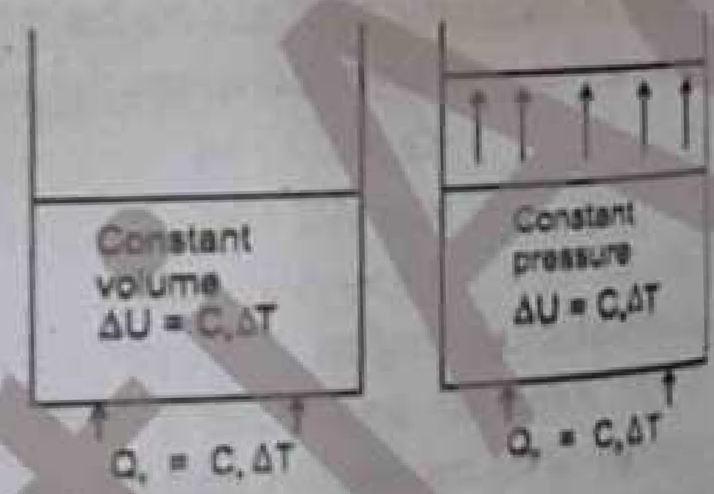
One kilogram of different substances contains different number of molecules. Sometimes it is preferred to consider a quantity called mole. One mole of any substance contains same number of molecules.

Molar Specific Heat of a Gas

Molar specific heat of the substance is defined as the heat required to raise the temperature of one mole of a substance through 1K.

Note

In case of solids and liquids the change of volume and hence work done against external pressure during a change of temperature is negligibly small.



But gases suffer variation in pressure as well as in volume with the rise in temperature. Hence, to study the effect of heating the gases, either pressure or volume is kept constant. We can define molar specific heat of a gas in two ways.

(1) Molar specific heat at constant volume

The molar specific heat at constant volume is the amount of heat required to raise the temperature of one mole of the gas though 1K at constant volume.

It is symbolized by C_v. Its SI unit is J Mole^{-1} K^{-1}.

(2) Molar specific heat at constant pressure

The molar specific heat at constant pressure is the amount of heat required to raise the temperature of one mole of the gas through 1k at constant pressure.

It is represented by symbol C_p.

(3)Derivation of $C_p - C_v = R$

At constant volume

If one mole of an ideal gas is heated at constant volume so that its temperature rises by ΔT then the heat transferred Q_v is given by

$$Q_v = C_v\Delta T \quad \text{——— (1)}$$

Applying first law of thermodynamics.

$$Q_v = \Delta U + W$$

$$C_v\Delta T = \Delta U + W$$

$$C_v\Delta T = \Delta U + P\Delta V \quad [\because W = P\Delta V]$$

Since volume remains constant (i.e $\Delta V = 0$), so work done by the system is zero Thus the first law of thermodynamics

$$C_v\Delta T = \Delta U$$

Hence

OR

$$\Delta U = C_v\Delta T \quad \text{——— (2)}$$

At constant pressure

If one mole of an ideal gas is heated at constant pressure so that its temperature rises by ΔT then the heat transferred Q_p is given by

$$Q_p = C_p\Delta T \quad \text{——— (3)}$$

the internal energy increases by the same amount as at constant volume for the same rise in temperature ΔT.

Thus

$$\Delta U = Cv\Delta T \quad \text{——— (4)}$$

Since, the gas expands to keep the pressure constant, so the work done by the gas is

$$W = P\Delta V \quad \text{——— (5)}$$

according to first law of thermodynamics

$$Q_p = \Delta U + W$$

OR $$C_p\Delta T = C_v\Delta T + P\Delta V \quad \text{——— (6)} \quad \text{[using equations(3), (4) \& (5)]}$$

According to general gas equation.

$$PV = nRT$$

For one mole of an ideal gas, n = l

$$PV = RT$$

At constant pressure P, amount of work done by one mole of a gas due to expansion ΔV caused by the rise in temperature ΔT is given by

$$P\Delta V = R\Delta T$$

Putting value of $P\Delta V$ in equation (6), we get

$$C_p\Delta T = C_v\Delta T + R\Delta T$$

OR

$$C_p = C_v + R$$

OR

$$C_p - C_v = R$$

It is clear that $C_p > C_v$ by an amount equal to universal gas constant R.

Q.9 Write a note on Reversible and Irreversible Processes.

Ans

Reversible Process

A reversible process is one which can be retraced in exactly reverse order, without producing any change in the surroundings.

Explanation

In the reverse process, the working substance passes through the same stages as in the direct process, but thermal and mechanical effects at each stage are

exactly reversed. If heat is absorbed in the direct process, it will be given out in the reverse process. If work is done by the substance in the direct process, work will be done on the substance in the reverse process. Hence, the working substance is restored to its original conditions.

Cycle

A succession of events which bring the system back to its initial condition is called a cycle.

Examples of Reversible Process

(1) The process of liquefaction and the evaporation of a substance performed **slowly** are reversible processes.

(2) Slow compression of a gas in a cylinder is reversible process as the compression can be changed to expansion by decreasing the pressure on the piston.

Irreversible Process

A reversible process is one which can not be retraced in exactly reverse order, without producing any change in the surroundings.

Explanation

All changes which occur **suddenly** or which involve **friction** or **dissipation** of energy through conduction, convection and radiation are irreversible.

Examples

(i) Explosion is an example of highly irreversible process.

(ii) Work done against friction

Q.10 Write a note on Heat Engine?

Ans.

Heat Engine

Heat engine is a device which converts heat energy into mechanical work

Introduction

The earliest heat engine was the steam engine. It was developed on the fact that when water is boiled in a vessel covered with a lid, the steam inside tries to push the lid off, showing the ability to do work. This observation helped to develop a steam engine.

Do You Know?

The steam engine is a Thermodynamics system.

Construction

A heat engine consists of;

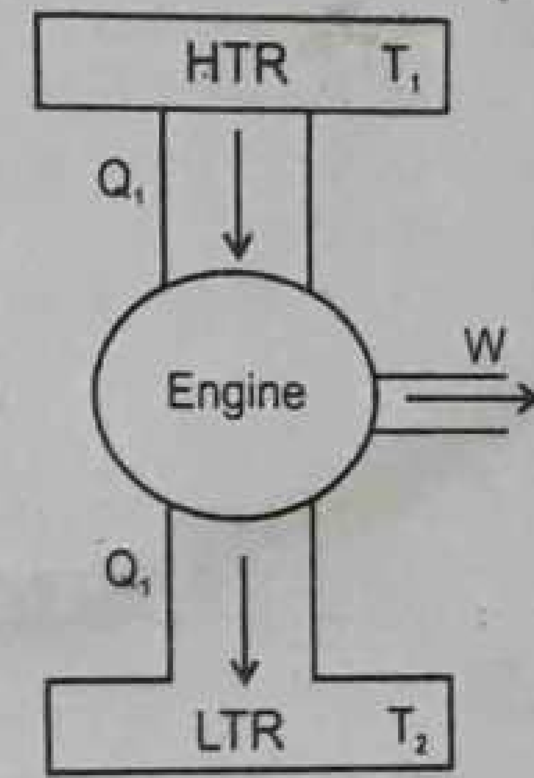
- hot reservoir or source which can supply heat at high temperature
- a cold reservoir or sink into which heat is rejected at a lower temperature.
- a working substance is needed which can absorb heat Q_1 from source, convert some of it into work W by expansion and rejects the rest heat Q_2 to cold reservoir or sink.

Working

A heat engine is made cyclic to provide a continuous supply of work. Working substance absorbs heat Q_1 from source, converts some of it into work W by expansion and rejects the rest heat Q_2 to cold reservoir or sink.

Efficiency of heat engine

The efficiency of heat engine is defined as the ratio of work done to the heat supplied. It is denoted by η.



Where $W = Q_1 - Q_2$

$$\text{efficiency} = \frac{W}{Q_1}$$

Thus,

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

OR

$$\eta = \frac{Q_1}{Q_1} - \frac{Q_2}{Q_1}$$

$$\boxed{\eta = 1 - \frac{Q_2}{Q_1}}$$

This is the expression for efficiency of heat engine.

Q.11 State and explain Second Law of Thermodynamics

Ans.

Second Law of Thermodynamics

According to Lord Kelvin's statement for working of heat engine,

It is impossible to make a heat engine which converts all the heat absorbed from a hot reservoir into work without rejecting any heat to sink.

OR

There is no perfect heat engine

Explanation

Let, the engine absorbs a quantity of heat Q_1 from the heat source at temperature T_1. It does work W and rejects heat Q_2 to low temperature reservoir at temperature T_2. As the working substance goes through a cyclic process, in which the substance eventually returns to its initial state, thus the change in internal energy is zero $\Delta U = 0$

Hence, according to First law of thermodynamics, the net work done is

$$Q = \Delta U + W$$

Therefore $Q_1 - Q_2 = 0 + W$ $[\because W = Q_1 - Q_2$ and $Q = Q_1 - Q_2]$

OR $Q_1 - Q_2 = W$

$$W = Q_1 - Q_2$$

Consequence

As a consequence of second law of thermodynamics *two bodies at different temperature are essential for the conversion of heat into work.*

A single heat reservoir, no matter how much energy it contains cannot be made to perform any work. Hence for the working of heat engine there must be a source of heat at high temperature and a sink at low temperature to which heat may be expelled. Thus it is true for oceans and our atmosphere which contains a large amount of heat energy but cannot be converted into useful mechanical work.

The reason for our inability to utilize the heat contents of oceans & atmosphere is that there is *no reservoir at a temperature lower* than anyone of the two.

Note

In practice, the petrol engine of a motor car extract heat from the burning fuel and converts a fraction of this energy to mechanical energy or work & expels the rest to atmosphere.

Petrol engine converts roughly **25%** & diesel engine **35% to 40%** available heat energy into work.

Tit - Bits

According to the Kelvin statement of the 2nd law of thermodynamics the process pictured here is impossible. Heat from a source at a single temperature cannot be converted entirely into work.

Q.12 What is Carnot's Engine? Explain its working and calculate its efficiency. Also state Carnot's theorem.

Ans.

Carnot's Engine

A Carnot heat engine is a **hypothetical** engine that operates on the reversible **Carnot cycle.** Sadi Carnot in 1824 proposed this ideal engine using only isothermal and adiabatic process.

He showed that a heat engine operating in an ideal reversible cycle between two heat reservoirs at different temperatures would be the most efficient engine.

Principle

Carnot's engine works on the same principle as that of cyclic heat engine. It takes heat from hot body, convert a part of it into work and reject the remaining part to cold body.

Working

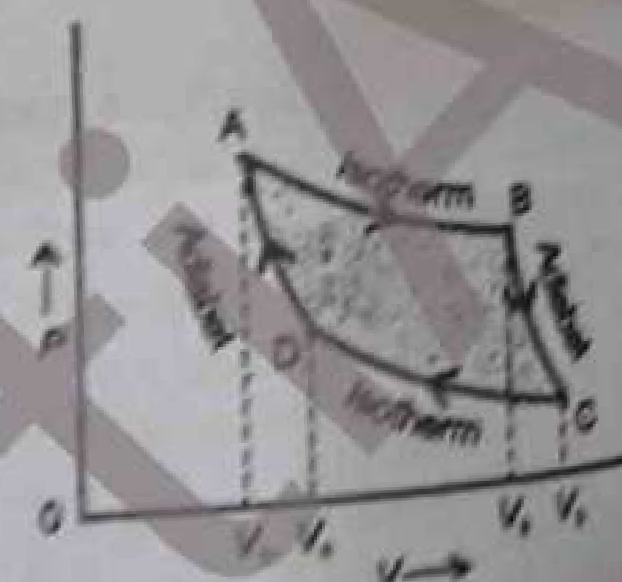
A Carnot cycle using an ideal gas as a working substance is shown in PV-diagram. It consists of following four steps.

1. **Isothermal expansion**

The gas is allowed to expand isothermally at temperature T_1, absorbing heat Q_1 from the hot reservoir. The process is represented by curve AB.

2. **Adiabatic Expansion**

The gas is then allowed to expand adiabatically until its temperature drop to T_2. The process is represented by BC curve.



3. **Isothermal compression**

The gas at this stage is compressed isothermally at temperature T_2 rejecting heat Q_2 to the cold reservoir. The process is represented by curve CD.

4. **Adiabatic compression**

Finally the gas is compressed adiabatically to restore its initial state at temperature T_1. The process is represented by curve DA.

Thermal and mechanical equilibrium is maintained all the time so that each process is perfectly reversible.

Expression for Efficiency

As the working substance returns to the initial state, there is no change in its internal energy i.e., $\Delta U = 0$.

The net work done during one cycle equals to the area enclosed by the path ABCDA of the PV diagram.

It can also be estimated from net heat ΔQ absorbed in one cycle.

$$\Delta Q = Q_1 - Q_2$$

From 1st law o thermodynamics,

$$\Delta Q = \Delta U + W \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

Putting value of ΔQ and ΔU in equation (1), we get

$$Q_1 - Q_2 = 0 + W$$

OR

$$W = Q_1 - Q_2$$

Efficiency η (eta) of heat engine is defined as

$$\eta = \frac{\text{output (work)}}{\text{Input (energy)}}$$

$$\eta = \frac{W}{Q_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1} \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

The energy transfer in an isothermal expansion or compression turns out to be proportional to Kelvin temperature.

i.e. Q_1 and Q_2 are proportional to Kelvin temperature T_1 and T_2 respectively.

Hence $\dfrac{T_2}{T_1} = \dfrac{Q_2}{Q_1}$

Thus eq. (2) becomes

$$\boxed{\eta = 1 - \frac{T_2}{T_1}}$$

The efficiency is usually taken in percentage.

$$\text{Percentage efficiency} = \eta = \left[1 - \frac{T_2}{T_1}\right] \times 100\,\%$$

Interesting Information

A waterfall analogy for the heat engine.

EXPLANATION:

Some water is lost in running a turbine. Similarly, some heat is lost to the sink in running a heat engine.

Dependence of Efficiency

Efficiency of a Carnot Engine depends on the **temperature of hot & cold reservoir**. It is independent of the nature of the working substance. The larger the temperature difference of the two reservoirs, the greater is the efficiency. In most practical cases the cold reservoir is near room temperature. So the efficiency can be increased by **raising the temperature of hot reservoir**

Can efficiency of heat engine be 100%?

It can never be one or 100% unless cold reservoir is at absolute zero temperature. Such reservoirs are not available & hence maximum efficiency is always less than one.

Carnot's Theorem

Statement

No heat engine can be more efficient than a Carnot engine operating between the same two temperatures.

Extended statement

All Carnot's engines operating between the same two temperatures have the same efficiency, irrespective of the nature of working substance.

Note

All real heat engines are less efficient than Carnot engine due to friction & heat losses.

Q.13 Describe the thermo-dynamical temperature scale.

Ans.

Thermodynamical Scale of Temperature

A temperature scale is defined by using thermometric property of certain working substance. If the working substance is not pure then its thermometry property is changed and reading of temperature measuring instrument becomes unreliable.

Thermodynamic scale of temperature is **independent** of nature of working substance.

According to this scale, the ratio $\frac{Q_1}{Q_2}$ is equal to the ratio of temperature of source and sink.

So, $$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad \text{——— (A)}$$

Thermodynamic scale of temperature is defined by choosing 273.16K as the absolute temperature of triple point of water as one fixed point (upper fixed point) and absolute zero as the other (lower fixed point).

Kelvin

One Kelvin is defined as $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water.

Triple Point

Triple point of water is the temperature at which water, ice and water vapours are in equilibrium state which is obtained at particular temperature and pressure.

Let

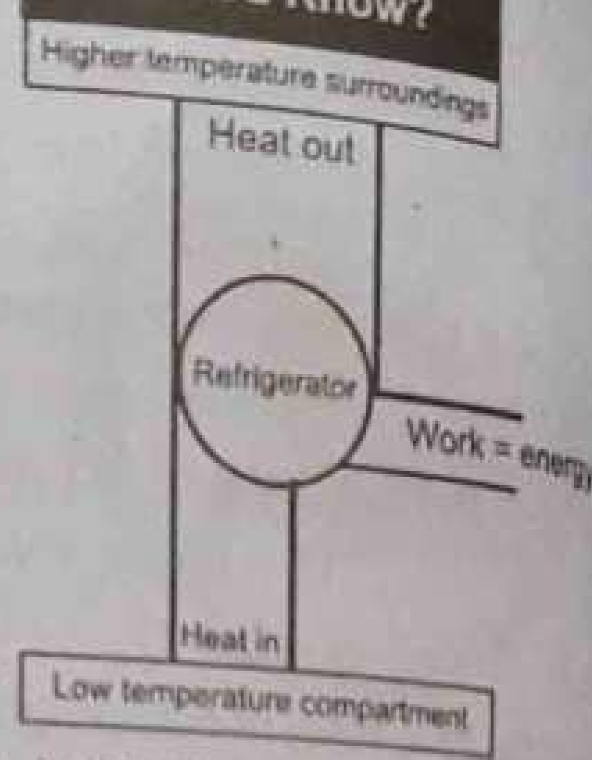
Q_1 = the heat absorbed or rejected by the system at unknown temperature T.

Q_2 = the heat absorbed or rejected by the system at temperature of triple point of water.

Then, according to Carnot cycle

$$\frac{Q_1}{Q_2} = \frac{T}{273.16}$$

Do You Know?

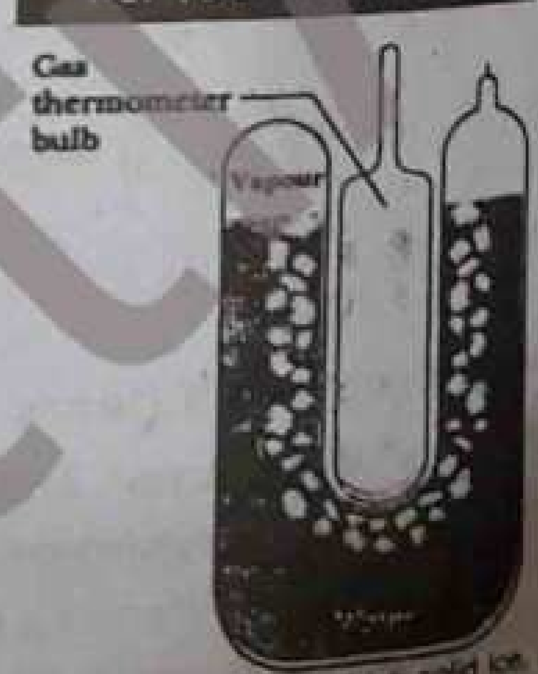


A refrigerator transfers heat from a low-temperature compartment to higher-temperature surroundings with the help of external work. It is a heat engine operating in reverse order

EXPLANATION:

A heat engine takes heat from a source, converts a part of it into work and rejects the remaining part to the sink. But a refrigerator takes heat from a cold body with the help of external work and rejects the heat, to hot body. Thus a refrigerator works in reverse order of heat engine.

For Your Information



A triple-point cell, in which solid ice, liquid water, and thermal equilibrium. By international agreement, the temperature of this mixture has been defined to be 273.16 K. The bulb of a constant-volume gas thermometer is shown inserted into the well of the cell.

OR $$\boxed{T = (273.16)\frac{Q_1}{Q_2}}$$

This formula is used to calculate the absolute temperature. Since this scale is independent of working substance. So it can be applied at very low temperature. Platinum resistance thermometer and gas thermometer are based upon thermodynamical scale of temperature.

$$T = 273.16\frac{R}{R_t} \quad \text{(For platinum resistance thermometer)}$$

$$T = 273.16\frac{P}{P_t} \quad \text{(For gas thermometer)}$$

Q.14 Explain the working of petrol engine and diesel engine.

Ans.

Petrol Engine

Although different engines may differ in their construction technology but they are based on the principle of a Carnot cycle.

Construction

Petrol engine consists of pistons, crankshaft, sparking plug and valves. It is shown in figure.

Working

A typical four stroke petrol engine also undergoes four successive processes in each cycle.

1. Intake stroke

The cycle starts on the intake stroke in which piston moves outward and petrol air mixture is drawn through an inlet valve into the cylinder from the carburetor at atmosphere pressure.

2. Compression Stroke

On the compression stroke, the inlet valve is closed and the mixture is compressed adiabatically by inward movement of the piston.

3. Power stroke

On the power stroke, a spark fires the mixture causing a rapid increase in pressure & temperature. The burning mixture expands adiabatically and forces the piston to move outward. This is the stroke which delivers power to crankshaft to derive fly wheels,

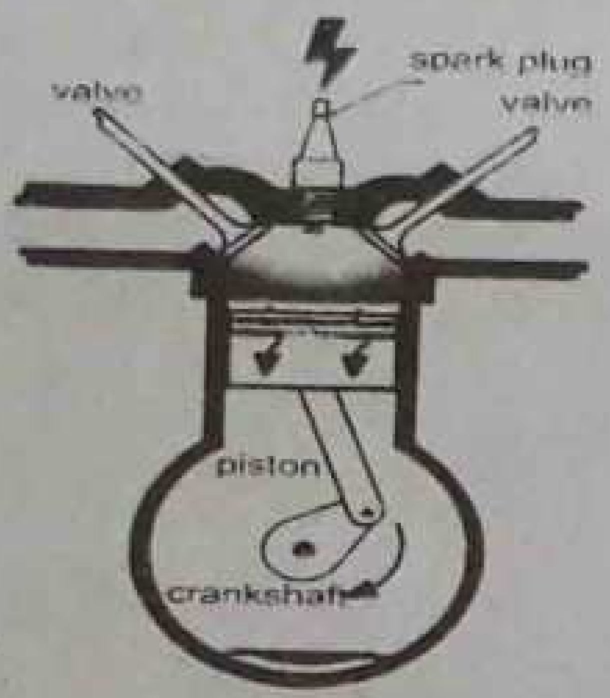
4. Exhaust Stroke

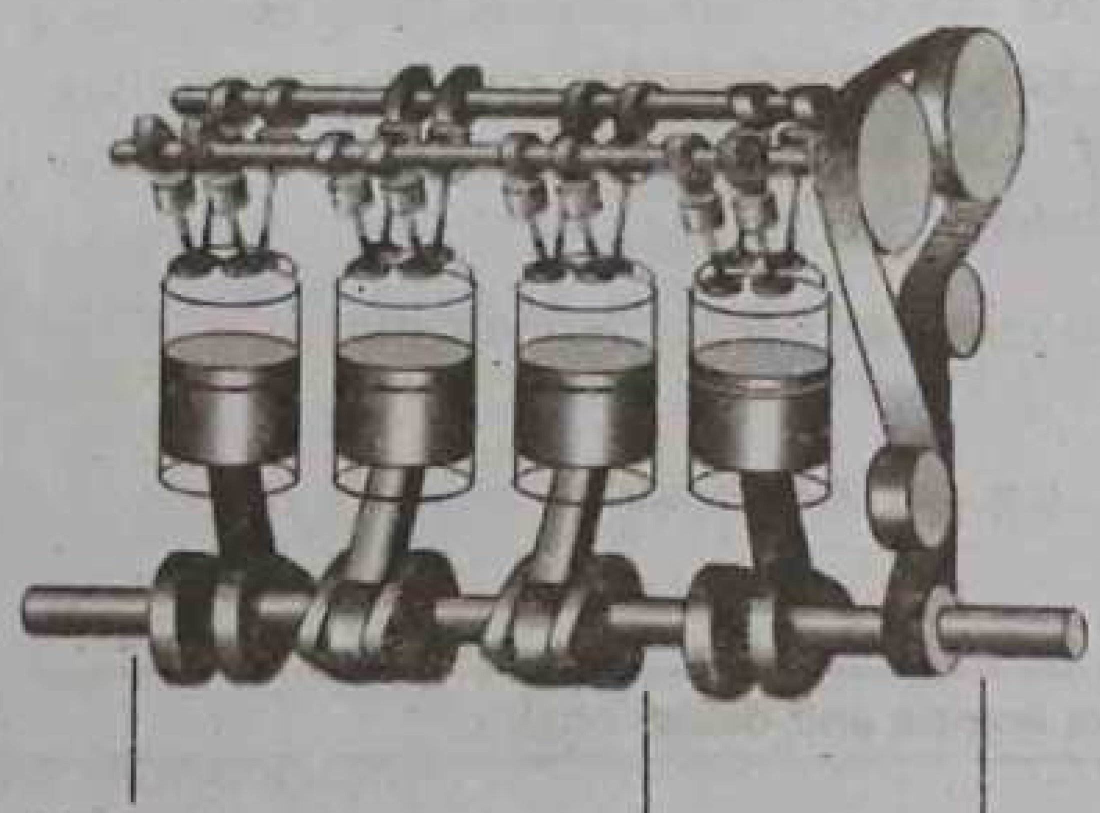
On the exhaust stroke the outlet valves opens. The gases are expelled & piston moves inward. Most motorbikes have one cylinder engine but cars usually have four cylinders on same crankshaft.

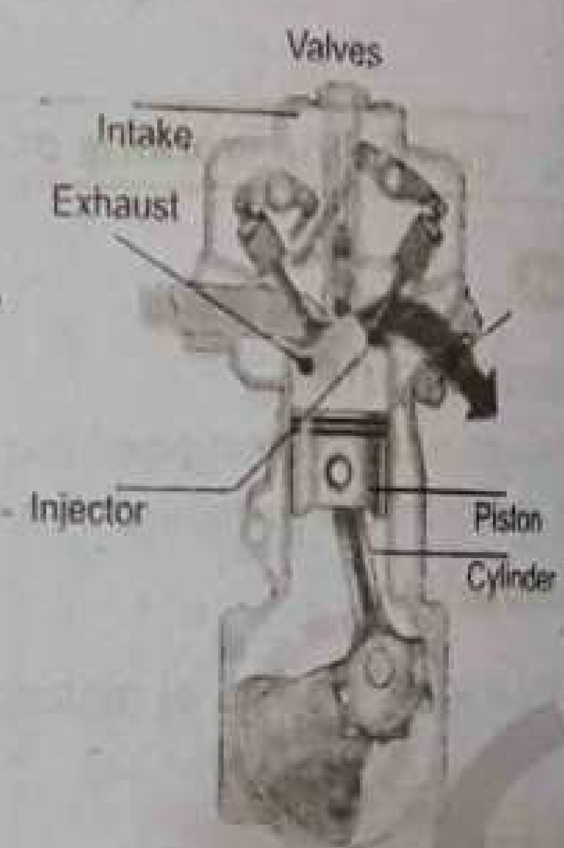
The cylinders are timed to fire turn by turn in succession for a smooth running of the car.

Efficiency

The actual efficiency of tuned engine is not more than 25 to 30% because of heat & friction losses.







Diesel Engine

No spark plug is needed in the diesel engine. Diesel is sprayed into cylinder at maximum compression. Because air is at high temperature after compression the, fuel mixture ignites on contact with air in cylinder & pushes the piston outward.

Efficiency:

The efficiency of diesel engine is about 35% to 40%.

Q.15 Define and explain the term entropy.

Ans.

Entropy

Entropy is state variable of thermodynamically system. It was introduced by Rudolph Claudius in 1856.This gives quantitative basis or mathematical formula for second law of thermodynamics.

The physical significance of entropy is that ***it is a measure of disorder of molecules of a system.*** Change in entropy is denoted by ΔS

If ΔQ is the quantity of heat absorbed by the system at temperature T. Then change in entropy (state variable) of the system is,

$$\Delta S = \frac{\Delta Q}{T} \quad \text{(for reversible process)}$$

Just like internal energy and potential energy, it is change in entropy which is more important than its absolute value.

Sigh Convention

The change in entropy is positive (means that entropy increases) when heat is added to a system.

Change of entropy is negative (means that entropy decreases) when heat is taken out of this system.

Unit

The SI unit of change of entropy or entropy is joule/Kelvin (JK^{-1}).

Principle of Increase of Entropy

This principle states that *entropy of a system plus it surroundings increases due to natural process done on or by the thermo-dynamical system.*

Let T_1 and T_2 are the temperature of hot body and cold body respectively (i.e. $T_1 > T_2$) and Q be the amount of heat conducted. Then

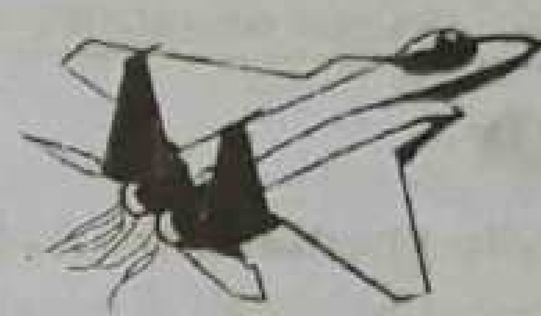
$$\text{Decrease of entropy of HTR} = \frac{Q}{T_1}$$

$$\text{Increase of entropy of LTR} = \frac{Q}{T_2}$$

$$\text{Net change in entropy} = \frac{Q}{T_2} - \frac{Q}{T_1} = \text{positive}$$

As $T_1 > T_2$ so the sign of net change of entropy is +ve or we can say that net entropy of the system is increased.

This proves that there is **net increase of entropy due to a natural process** (i.e. flow of heat from higher to lower temperature). This is also called another statement of second law of thermodynamics.



DO YOU KNOW?

The jet engine on this aircraft converts thermal energy to work, but the visible exhaust clearly shows that a considerable amount of thermal energy is lost as waste heat.

EXPLANATION:

The remaining part of thermal energy is rejected to the sink, according to second law of thermodynamics.

Second Law of Thermodynamics in terms of Entropy

If a system undergoes a natural process, it will go in the direction that the entropy of system plus the environment increases.

For example, an irreversible heat flows from a hot body to a cold body to increases the disorder. So we can say that the entropy is increased.

Addition of heat increases the disorder; hence the entropy is also increased.

Entropy as Unavailability of Mechanical Work (i.e., Degradation of energy):

Let us consider two water tank of different temperature, so the average K.E of molecules in higher temperature water is greater than lower temperature. The two water tanks can be used as source and sink of a heat engine, which could be operated, between them and mechanical work can be obtained.

But if these two tanks are connected with a conducting rod then heat starts to flow from hot body towards the cold body until thermal equilibrium is reached. So no mechanical work is done due to the absence of heat engine, which results unavailability of mechanical work. Hence, *increase in entropy means the degradation of energy.*

According to principle of increase of entropy, the entropy of the universe increases after to get mechanical work from heat. It would be called "heat death" or ending up of thermal energy.

Heat death of Universe

When the entropy of the universe will reach at maximum value, everything will be at same temperature and there will be no way to convert heat into useful work and it is called heat death.

Do You Know?

Approximate efficiencies of various devices

Device	Efficiency (%)
Electric generator	70-99
Electric motor	50-93
Dry cell battery	90
Domestic gas furnace	70-85
Storage battery	72
Hydrogen-oxygen fuel cell	60
Liquid fuel rocket	47
Steam turbine	35-46
Fossil-fuel power plant	30-40
Nuclear power plant	30-35
Nuclear reactor	39
Aircraft gas turbine engine	36
Solid-state laser	30
Internal combustion gasoline engine	20-30
Gallium arsenide solar cells	>20
Fluorescent lamp	20
Silicon solar cell	12-16
Steam locomotive	8
Incandescent lamp	5
Watt's steam engine	1

Q.16 Describe environmental crisis as entropy crisis.

Ans.

Environmental Crisis As Entropy Crisis

According to 2nd law of thermodynamics, every real process causes to increase the disorder or entropy of the universe. Any increase in the disorder of a system produces and even greater increase in the disorder of the environment, which is called "environmental crisis.

The disorder producing activities due to all industries may result a great increases of disorder which affect the overall life support system.

Our mechanical energy producing processes are not efficient. For example petrol engine has its efficiency about 30% and diesel engine's about 40%. Hence most of energy is transferred into the environment in form of heat, which causes to increase the entropy of it.

The second law of thermodynamics impose limit on the efficiency of mechanical energy produces by engines, which says that ***thermal pollution is an inevitable result of second law of thermodynamics***. Due to the thermal pollution in environment, temperature change may occur. But a small change in environment may serious effects on metabolic rate in plants and animals. This may disturb ecological balance.

The imperative from thermodynamics is that whenever you do anything, you should be sure about its present and future impact on your environment. This is an ecological imperative that we must keep in view now if we are to prevent a strong degradation of life on our beautiful but fragile earth.

FOR YOU INFORMATION

Diesel fuel evaporates more slowly because it is heavier.

It contains more carbon atoms in longer chains than gasoline does (gasoline is typically C9H20, while diesel fuel is typically C14H30).

It takes less refining to create diesel fuel, which is why it used to be cheaper than gasoline.

Diesel fuel has a higher energy density than gasoline. On average, 1 gallon (3.8 L) of diesel fuel contains approximately 155x106 joules (147,000 BTU), while 1 gallon of gasoline contains 132x106 joules (125,000 BTU).

This, combined with the improved efficiency of diesel engines, explains why diesel engines get better mileage than equivalent gasoline engines.

FORMULAE

Pressure of gas	$P=\frac{1}{3}\rho<v^2>$	$P=\frac{2}{3}\frac{N}{V}<\frac{1}{2}mv^2>$	$P=\frac{2}{3}N_o<\frac{1}{2}mv^2>$
Number of moles	$n=\frac{N}{N_A}$		
Boltzman's constant/gas constant per molecule	$k=\frac{R}{N_A}$		
Temperature of gas	$T=\frac{2}{3K}<\frac{1}{2}mv^2>$		
Work done	$W=P\Delta V$		
First law of thermodynamics	$Q=\Delta U+W$	$Q=(U_2-U_1)+W$	$\Delta U=Q-W$
Isothermal process	$Q=W$		
Adiabatic process	$W=-\Delta U$ (expansion)	$-W=\Delta U$ (compression)	
Relation between and C_p and C_v	$C_p-C_v=R$	$\gamma=\frac{C_p}{C_v}$	
Efficiency of heat engine	$\eta=\frac{W}{Q_1}$	$\eta=\frac{Q_1-Q_2}{Q_1}$	$\eta=1-\frac{Q_2}{Q_1}$
Efficiency of carnot's engine	$\eta=\frac{T_1-T_2}{T_1}$	$\eta=1-\frac{T_2}{T_1}$	
Thermodynamic temperature scale	$\frac{T_2}{T_1}=\frac{Q_2}{Q_1}$	$T=\frac{Q_1}{Q_2}(273.16)$	
Entropy	$\Delta S=\frac{\Delta Q}{T}$	(JK^{-1})	

unit

Multiple Choice Questions

- *Four possible answers to each statement are given below. Tick (✓) the correct answer:*

1. The average speed of the gas molecules in a container is:
   (a) Zero (b) 273
   (c) 100 (d) Infinite
2. The mean K.E. of a gas is zero at:
   (a) $0^\circ C$ (b) $-273^\circ C$
   (c) 100 K (d) $100^\circ C$
3. Gas molecules exerts pressure on the walls of container because they:
   (a) Have momentum (b) Collide with one another
   (c) Collide with wall (d) None of these
4. If pressure is increased, melting point of ice:
   (a) Decreases (b) Increases
   (c) Remains same (d) None of these
5. At constant temperature, if pressure is halved then its volume is:
   (a) Constant (b) Halved
   (c) Four times (d) Doubled
6. The internal energy of a compressed real gas, as that of the ideal gas at same temperature, is:
   (a) Less (b) More
   (c) Sometimes less, sometimes more (d) None of these
7. At boiling point, _______ remains constant.
   (a) Pressure (b) Volume
   (c) Temperature (d) Both (a) and (c)
8. Which of following properties is same for all gases at the particular temperature?
   (a) Momentum (b) Mass
   (c) Velocity (d) <K.E.>
9. Unit of entropy is:
   (a) J K (b) $J K^{-1}$
   (c) Nm/s (d) $kg\ m^2/s^2\ K$
10. In which process internal energy of system remains same, even work is done:
    (a) Isothermal (b) Isobaric
    (c) Adiabatic (d) Isochoric
11. When temperature of source and sink of a heat engine become equal, the entropy change will be:
    (a) Zero (b) Minimum
    (c) Maximum (d) Negative
12. The internal energy of a mono-atomic ideal gas is:

    (a) Only kinetic (b) Only potential
    (c) Partly kinetic and partly potential (d) None of these
13. The direction of flow of heat between two bodies is determined by:
    (a) Internal energy (b) Kinetic energy
    (c) Total energy (d) Atmospheric pressure
14. Which of the following is a best container for gas during adiabatic process?
    (a) Wood vessel (b) Thermos flask
    (c) Copper vessel (d) Glass vessel
15. In which of the following process, the system always return to original thermodynamic state?
    (a) Isobaric (b) Adiabatic
    (c) Cyclic (d) Isothermal
16. A carnot engine has an efficiency of 50% when its sink temperature is at $27^\circ C$. The temperature of source is:
    (a) $300^\circ C$ (b) $327^\circ C$
    (c) $373^\circ C$ (d) $273^\circ C$
17. A carnot engine works between ice point and steam point. Its efficiency will be:
    (a) 85.42% (b) 71.23%
    (c) 53.36% (d) 26.81%
18. If the temperature of sink is absolute zero, the efficiency of heat engine should be:
    (a) 100% (b) 50%
    (c) 0 (d) Infinity
19. When the temperature difference between source and sink increases, the efficiency of heat engine will:
    (a) Increase (b) Decrease
    (c) Is not effected
    (d) May increase/decrease depend upon working substance
20. The internal energy of an isolated system:
    (a) Keeps on changing (b) Remains constant
    (c) Zero (d) None of these

ANSWERS

1. a	2. b	3. c	4. a	5. d	6. b	7. c	8. d	9. b	10. a
11. c	12. a	13. a	14. c	15. c	16. b	17. d	18. a	19. a	20. b

Short Questions of Exercise

Q.11.1 Why the average velocity of the molecules in a gas container is zero but the average of the square of velocities is not zero?

(Fsd 2005, Rwp 2006, D.G.Khan 2006, Lhr 2010-2011)

Ans. Reason

Average velocity of the molecules

The motion of gas molecules is ***random.*** So if a molecule is moving in one direction, another will be moving in opposite direction. Thus the vector sum of these velocities is zero.

i.e $$\langle v\rangle = \frac{v+(-v)}{2} = 0$$

Average of the square of velocities

As the square of negative velocity is ***positive***, so the average of square of velocities is not equal to zero.

i.e $$\langle v^2\rangle = \frac{v^2+(-v)^2}{2} \neq 0$$

Q.11.2 Why does the pressure of gas in a car tyre increase when it is driven trough some distance?

(Mir Pur 2004-2009, Mtn 2006, Bwp 2007-2008, Fsd 2008, Grw 2009-2011)

Ans. Reason

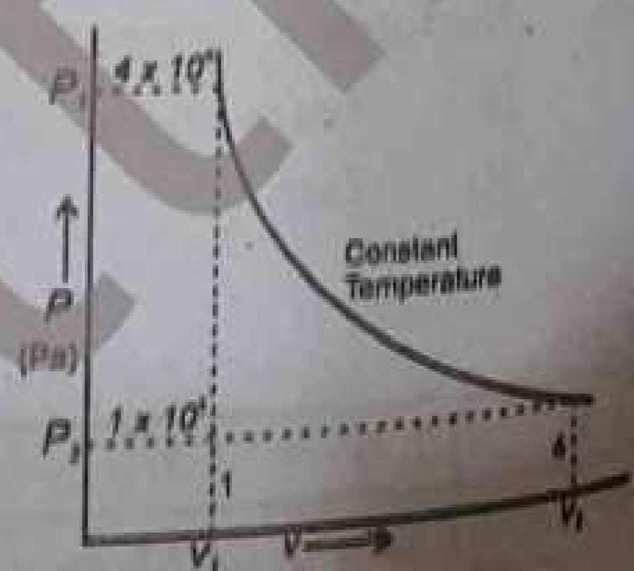
- This is due to the ***work done against friction*** between road and tyre.
- This work done appears as ***heat.***
- This heat is absorbed by the gas molecule.
- It increases the average kinetic energy of molecules.
- As pressure ***$P \propto \langle K.E.\rangle$*** ,so pressure of gas is increased

Q.11.3 A system undergoes from state P_1V_1 to state P_2V_2 as shown in fig. What will be the change in internal energy?

Ans. The change in internal energy of the system is ***zero.***

Reason

- It is clear from figure that temperature of system is ***constant.***
- It means that internal energy is also constant as it depends upon temperature
- So, there will be no change in internal energy



Q.11.4 Variation of volume by pressure is given in Fig. 11.13. a gas is taken along the paths ABCDA, ABCA and A to A. what will be the change in internal energy?

(D.G.khan 2005, Fsd 2005)

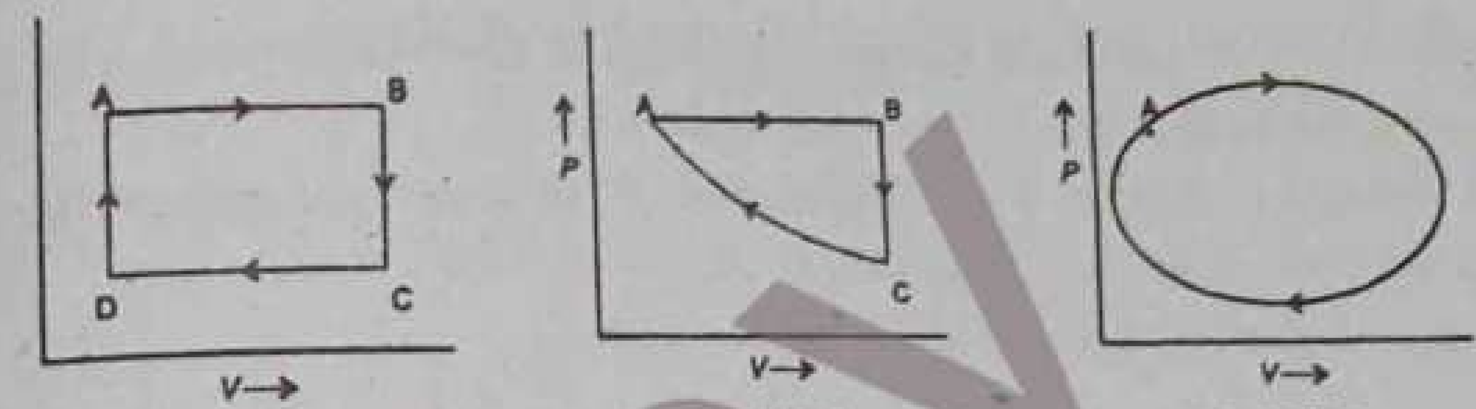


Fig.11.13(a) Fig.11.13(b) Fig.11.13(c)

Ans. The change in internal energy of the system is ***zero in all cases.***

Reason

As all the processes are ***cyclic.*** So the system returns to its ***initial state*** after each cycle. Hence, the internal energy of the system does not change.

Q.11.5 Specific heat of a gas at constant pressure is greater than specific heat at constant volume. Why?

(Mtn 2003, Lhr 2003, Rwp 2004, D.G.khan 2005, Fsd 2008, Bwp 2007-2008, Mir Pur 2009, Lhr 2010-2011)

Ans. Reason

When gas is heated at constant pressure then;

(i) a part of heat is used to do ***work on piston***

(ii) rest of heat is used to ***increase the temperature*** through 1K

When gas is heated at constant volume then all the heat absorbed is used to increase temperature through 1K.

That is why the molar specific heat at constant pressure is greater than molar specific heat at constant volume. ($C_p > C_v$).

Q.11.6 Give an example of a process in which no heat is transferred to or from the system but the temperature of the system changes.

(Grw 2009, Lhr 2009)

Ans. In ***adiabatic process*** no heat enters or leaves the system take place but temperature of the system changes.

$$Q = \Delta U + W$$

For adiabatic process $Q = 0$

So $$0 = \Delta U + W$$

$$-\Delta U = W \quad (\text{adiabatic expension})$$

Or $$\Delta U = -W \quad (\text{adiabatic compression})$$

So by doing so whole mechanical energy is converted into heat energy.

Examples

- ***Rapid escape of air*** from a burst tyre
- ***Rapid expansion and compression*** of air through which sound wave is passing.
- ***Cloud formation*** in the atmosphere

Q.11.7 Is it possible to convert internal energy into mechanical energy? Explain with an example.

(Bwp 2003, Sgd 2003, Mtn 2004, Grw 2005, Fsd 2005-2006, Lhr 2010-

Ans. ***Yes***, it is possible to convert internal energy into mechanical energy.

Example

When a gas is allowed to expand adiabatically, the gas does work on the surrounding by using internal energy, due to which its internal energy decreases. i.e $W = \Delta U$

Gases cam be liquefied by this process.

Q.11.8 Is it possible to construct a heat engine that will not expel heat into the atmosphere?

(Sgd 2004, Mtn 2004, Grw 2005)

Ans. ***No***, it is not possible.

Reason

If it is possible, then it will be ***violation of second law*** of thermodynamics, which states that it is not possible to construct a heat engine that will not expel heat into atmosphere with out leaving any change on the working substance.

Q.11.9 A thermos flask containing milk, as a system is shaken rapidly. Does the temperature of milk rise?

(Lhr 2005, Fsd 2005, Mir Pur 2009)

Ans. ***Yes***, the temperature of the milk rises.

Reason

We know that

$$T \propto \langle K.E \rangle$$

When we rapidly shake the thermos flask we do some work on it, this ***work done increases the K.E*** of molecules of milk. Hence, the temperature of milk rises.

Q.11.10 What happens to the temperature of the room, when an air conditioner is left running on a table in the middle of the room?

(Bwp 2003, Fsd 2005, Mir Pur 2006-2009, Lhr 2005-2006-2009, Grw 2010)

Ans. The temperature of the room will not ***decrease*** even it increases ***slightly***.

Reason

As air conditioner is running at th middle of room it absorbs as well as rejects heat in the same room at the ***same rate***. So temperature of the room remains unchanged. But due to working of compressor somee heat is produced due to friction. So temperature of the room will increase slightly.

Q.11.11 Can the mechanical energy be converted completely into heat energy? If so give an example.

(Mtn 2005, Grw 2005-2006)

Ans. ***Yes***, it can be converted completely into heat energy.

Reason

During isothermal compression *work done* 'W' on the system is converted into heat 'Q'.

$$Q = \Delta U + W$$

For isothermal process $\Delta U = 0$

So

$$-Q = 0 + (-W)$$

$$Q = W$$

Or

$$W = Q$$

Example: During slow compression all the heat rejected is used as

So by doing so whole mechanical energy is converted into heat energy.

Q.11.12 Does entropy of a system increases or decreases due to friction?

(Mtn 2004-2005, Bwp 2006, D.G.Khan 2006, Sgd 2005, Lhr 2006-2008, Rwp 2008)

Ans. ***Yes***, the entropy of the system increases due to friction.

Reason

Since $\Delta S = \dfrac{\Delta Q}{T}$

Due to friction, some mechanical energy is ***converted into heat*** (i.e heat is added up into the system) which increases the entropy of system.

Q.11.13 Give an example of a natural process that involves an increase in entropy?

(Sgd 2005, Grw 2005, Lhr 2006, Bwp 2006, Lhr 2009,Grw2011)

Ans. We know that entropy is measured by following equation.

$$\Delta S = \frac{\Delta Q}{T}$$

Examples

The ***melting of ice*** involves the increase in entropy. Ice absorbs the heat from its surrounding and changes its state.(i.e. from solid into liquid). Thus, entropy increases.

All natural processes in which friction is involved, the entropy of the system increases.

Q.11.14 An adiabatic change is the one in which

(a) No heat is added to or taken out of a system. (b) No change of temperature takes place.
(c) Boyle's law is applicable. (d) Pressure and volume remains constant.

Ans. (a) No heat is added to or taken out of a system

Q.11.15 Which one of the following process is irreversible?

(a) Slow compressions of an elastic spring.
(b) Slow evaporation of a substance in an isolated vessel.
(c) Slow compression of a gas. (d) A chemical explosion.

Ans. (d) A chemical explosion is irreversible.

Q.11.16 An ideal reversible heat engine has:

(a) 100% efficiency. (b) highest efficiency.
(c) an efficiency which depends on the nature of working substance.
(d) none of them.

(Rwp 2004, Sgd 2005, Fsd 2005, Mtn 2006)

Ans. (b) highest efficiency.

Solved Examples

Example 11.1

What is the average translational Kinetic energy of molecules in a gas at temperature 27C°?

Given Data.

Temperature = T = 27C°

= 27 + 273 = 300K

To Find.

Average translational kinetic energy = $<K.E> = ?$

Calculation.

As $T = \frac{2}{3K} <K.E.>$

Or $<K.E.> = \frac{3}{2} KT$

Where K = Boltzman constant = $1.38 \times 10^{-23} JK^{-1}$

Thus $<K.E.> = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$

$<K.E.> = \frac{3}{2} \times 1.38 \times 10^{-23} \times 3 \times 10^2$

$\boxed{<K.E.> = 6.21 \times 10^{-21} J}$

Example 11.2

Find the average speed of oxygen molecule in the air at S.T.P.

Given Data.

At ST.P. , we have

Temperature = T = 0°C = 0 + 273 = 273K

Pressure = P = $1.01 \times 10^5 Nm^{-2}$

To Find.

Average speed of oxygen molecules = $<v> = ?$

Calculation.

As $T = \frac{2}{3k} <\frac{1}{2} mv^2>$

Or $<v^2> = \frac{3kT}{m}$ .... (1)

Where $k = 1.38 \times 10^{-23} JK^{-1}$

To find mass m, of one molecule of oxygen, we use the formula

Or $m = \frac{\text{Molecular mass of oxygen}}{\text{Avogadro 's number}}$

$m = \frac{M}{N_A}$

Where Molecular mass of oxygen = 32g = 32×10^{-3}kg

and N_A = Avogadro's number = 6.022×10^{23}

Thus $m = \frac{32 \times 10^{-3}}{6.022 \times 10^{23}}$

$m = \frac{32}{6.022 \times 10^{26}}$

$m = 5.31 \times 10^{-26}$kg

Putting values in equ. (1), we get

$<v^2> = \frac{3 \times 1.38 \times 10^{-23} \times 273}{5.31 \times 10^{-26}}$

Where $<v^2> = \frac{1130.22}{5.31} \times 10^3$

$<v^2> = 212.84 \times 10^3$

$<v^2> = 212840$

Or $\boxed{v_{rms} = \sqrt{<v^2>} = 461 ms^{-1}}$

Example 11.3

A gas is enclosed in a container fitted with a piston of cross-sectional area 0.10 m². The pressure of the gas is maintained at 8000 Nm⁻². When heat is slowly transferred, the piston is pushed up through a distance of 4.0 cm. If 42 J heat is transferred to the system during the expansion, what is the change in internal energy of the system?

Given Data.

Cross – sectional area of the piston = A = $0.10m^2$

Pressure of the gas = P = $8000 Nm^{-2}$

Distance moved by the piston = Δy = 4.0 cm = 4×10^{-2}m

Heat supplied to the system = Q = 42J

To Find.

Change in internal energy = $\Delta U = ?$

Calculation.

As we know that

work done by the gas = $W = P\Delta V$

So $W = PA\Delta y$ As $\Delta V = A\Delta y$

Putting values, we get

$W = 8000 \times 0.10 \times 4 \times 10^{-2}$

$W = 32J$

Now Applying first law of thermodynamics

$Q = \Delta U + W$

Or $\Delta U = Q - W$

Where ΔU is the change in internal energy

Putting values, we get

$\Delta U = 42 - 32$

$\boxed{\Delta U = 10J}$

Example 11.4

The turbine in a steam power plant takes steam from a boiler at 427°C and exhausts into a low temperature reservoir at 77°C. What is the maximum possible efficiency?

Given Data.

Temperature of hot body (boiler) = $T_1 = 427C° = 427 + 273 = 700K$

Temperature of cold body (reservoir) = $T_2 = 77C° = 77 + 273 = 350K$

To Find.

Efficiency = η = ?

Calculation.

As $$\eta = (1 - \frac{T_2}{T_1})$$

Putting values, we get

$$\eta = (1 - \frac{350}{700})$$

$$\eta = (1 - \frac{1}{2})$$

$$\eta = \frac{1}{2} = 0.5$$

$$\% \eta = 0.5 \times 100$$

$$\boxed{\% \eta = 50\%}$$

Example 11.5

Calculate the entropy change when 1.0kg ice at 0°C melts into water at 0°C. Latent heat of fusion of ice $L_f = 3.36 \times 10^5 \text{ J kg}^{-1}$

Given Data.

Mass of ice = m = 1.0 kg

Temperature = T = 0°C = 273K

Latent heat of fusion of ice = $L_f = 3.36 \times 10^5 \text{Jkg}^{-1}$

To Find.

Change in entropy = ΔS = ?

Calculation.

Change in entropy is given by

$$\Delta S = \frac{\Delta Q}{T}$$

Here

ΔQ = Heat added to ice = mL_f

So $$\Delta S = \frac{mL_f}{T}$$

Putting values, we get

$$\Delta S = \frac{1.0 \times 3.36 \times 10^5}{273}$$

$$\Delta S = 0.01230 \times 10^5 \text{ JK}^{-1}$$

Or $$\boxed{\Delta S = 1.23 \times 10^3 \text{ JK}^{-1}}$$

Exercise Problems

11.1 **Estimate the average speed of nitrogen molecules in air under standard conditions of pressure and temperature.**

Given data:

Under standard conditions

Temperature = T = 0°C = 0 + 273 = 273k

Pressure = P = 1atm. = 1.01×10^5 Pa

To find:

Average speed of nitrogen molecules = <v> =?

Calculation:

Using the formula

$$T = \frac{2}{3k} < \frac{1}{2} mv^2 >$$

$$3kT = < mv^2 >$$

$$< v^2 > = \frac{3kT}{m} \quad \text{.................(1)}$$

Where k = Boltzman constant = $1.38 \times 10^{-23} \text{JK}^{-1}$

Now the mass of one molecules of nitrogen (N_2) is

$$m = \frac{\text{Molar mass}}{\text{Avogadro number}}$$

$$m = \frac{28g}{6.02 \times 10^{23}} = \frac{28 \times 10^{-3} kg}{6.02 \times 10^{23}} = 4.65 \times 10^{-26} kg$$

Putting values in eqution (1), we get

$$< v^2 > = \frac{3 \times 1.38 \times 10^{-23} \times 273}{4.65 \times 10^{-26}}$$

$$< v^2 > = \frac{1.13022 \times 10^{-20}}{4.65 \times 10^{-26}}$$

$$< v^2 > = 2.43058 \times 10^5$$

$$\boxed{v_{rms} = \sqrt{< v^2 >} = 493 ms^{-1}}$$

11.2 **Show that ratio of the root mean square speeds of molecules of two different gases at a certain temperature is equal to the square root of the inverse ratio of their masses.**

To find:

$$\frac{v_{1rms}}{v_{2rms}} = ?$$

Calculation:

Using the formula

$$T = \frac{2}{3k} < \frac{1}{2} mv^2 >$$

For first gas

$$T=\frac{2}{3k}<\frac{1}{2}m_1v_1^2>\text{......................}(1)$$

For second gas

$$T=\frac{2}{3k}<\frac{1}{2}m_2v_2^2>\text{......................}(2)$$

Dividing equation (i) by (ii)

$$\frac{T}{T}=\frac{\frac{2}{3k}<\frac{1}{2}m_1v_1^2>}{\frac{2}{3k}<\frac{1}{2}m_2v_2^2>}$$

$$1=\frac{<m_1v_1^2>}{<m_2v_2^2>}$$

$$\frac{<v_1^2>}{<v_2^2>}=\frac{m_2}{m_1}$$

$$\sqrt{\frac{<v_1^2>}{<v_2^2>}}=\sqrt{\frac{m_2}{m_1}}$$

$$\boxed{\frac{v_{1rms}}{v_{2rms}}=\sqrt{\frac{m_2}{m_1}}}$$

This is the required proof.

11.3. A sample of gas is compressed to one half of its initial volume at constant pressure of 1.25×10^5 Nm^{-2}. During the compression, 100 J of work is done on the gas. Determine the final volume of the gas.

Given data:

Suppose initial volume of gas $=V_i=V$

Final volume of gas $=V_f=\frac{V}{2}$

Pressure $=P=1.25\times10^5 Nm^{-2}$

Work done $=W=-100$ J

To find:

Final volume of gas $=V_f=?$

Calculation:

As $W=P\Delta V$

or $W=P(V_f-V_i)$

$W=P\left(\frac{V}{2}-V\right)$ As $V_i=V$ and $V_f=\frac{V}{2}$

$W=P\left(\frac{V-2V}{2}\right)$

$-100=P\left(-\frac{V}{2}\right)$

Or $\frac{V}{2}=\frac{100}{P}$

So, $V_f=\frac{100}{1.25\times10^{5}}$ As $\frac{V}{2}=V_f$

$$\boxed{V_f=8\times10^{-4}\,m^3}$$

11.4. A thermodynamic system undergoes a process in which its internal energy decreases by 300J. If at the same time 120 J of work is done on the system, find the heat lost by the system.

Given data:

Decrease in internal energy $=\Delta U=-300J$

Work done on the system $=W=-120J$

To find:

Heat lost by the system $=Q=?$

Calculation:

According to first law of thermodynamics

$Q=\Delta U+W$

Putting the values, w get

$Q=-300-120$

$$\boxed{Q=-420J}$$

Negative sign shows the lost of heat.

11.5. A carnot engine utilizes an ideal gas. The source temperature is 227°C and the sink temperature is 127°C. Find the efficiency of the engine. Also find the heat input from the source and heat rejected to the sink when 10000 J of work is done.

Given data:

Temperature of source $=T_1=227\,^\circ C=227+273=500K$

Temperature of sink $=T_2=127\,^\circ C=127+273=400K$

Work done $=W=10000$ J

To find:

(i) Efficiency of Carnot engine $=\eta=?$

(ii) Heat input $=Q_1=?$

(iii) Heat rejected to sink $=Q_2=?$

Calculation:

(i) **As we know**

$$\eta=(1-\frac{T_2}{T_1})\times100$$

Putting the values, we get

$$\eta=\frac{100}{500}\times100$$

$$\boxed{\eta=20\%}$$

(ii) **To find Q_1**

As efficiency = $\dfrac{\text{output work}}{\text{input heat}}$

$$\eta = \frac{W}{Q_1}$$

Putting values, we get

$$\frac{20}{100} = \frac{10000}{Q_1}$$

$$Q_1 = \frac{10000 \times 100}{20}$$

$$Q_1 == 50000J$$

$$\boxed{Q_1 = 5.0 \times 10^4 J}$$

(iii) To find Q_2

As $W = Q_1 - Q_2$

Or $Q_2 = Q_1 - W$

Putting values, we get

$$Q_2 = 50000 - 10000$$

$$Q_2 = 40000$$

$$\boxed{Q_2 = 4.0 \times 10^4 J}$$

11.6 A reversible engine works between two temperatures whose difference is 100°C If it absorbs 746 J of heat from the source and rejects 546 J to the sink, calculate the temperature of the source and the sink.

Given data:

Temperature difference = $T_1 - T_2 = 100C° = 100K$

Amount of heat absorbed = $Q_1 = 746J$

Amount of heat rejected = $Q_2 = 546J$

Find data:

Temperature of source = $T_1 = ?$

Temperature of sink = $T_2 = ?$

Calculation:

As $\eta = 1 - \dfrac{Q_2}{Q_1}$

Putting values, we get

$$\eta = 1 - \frac{546}{746}$$

$$\eta = \frac{746 - 546}{746}$$

Or $\eta = 0.268 ....(1)$

$$\eta = 1 - \frac{T_2}{T_1}$$

Now

$$\eta = \frac{T_1 - T_2}{T_1}$$

$$0.268 = \frac{100}{T_1}$$

$$T_1 = \frac{100}{0.268}$$

$$T_1 = 373.13K$$

$$\boxed{T_1 = 100°C}$$

Also $T_1 - T_2 = 100K$

Or $373.13 - T_2 = 100$

Or $T_2 = 373.13 - 100$

$$T_2 = 273.13K$$

$$\boxed{T_2 = 0°C}$$

11.7 A mechanical engineer develops an engine, working between 327°C and 27°C claims to have efficiency of 52%. Does he claim correctly? Explain.

Given data:

Temperature of source = $T_1 = 327C° = 327 + 273 = 600K$

Temperature of sink = $T_2 = 27C° = 27 + 273 = 300K$

Claimed efficiency = 52%

To find:

Actual efficiency = $\eta = ?$

Calculation:

As $\eta = 1 - \dfrac{T_2}{T_1}$

Putting values, we get

$$\eta = 1 - \frac{300}{600}$$

$$\eta = 1 - \frac{1}{2}$$

$$\eta = \frac{1}{2}$$

$$\eta = 0.5$$

Percentage efficiency = 0.5×100

$\boxed{\%\eta = 50\%}$

So the claim of 52% efficiency is not correct.

11.8. A heat engine performs100 J of work and at the same time rejects 400 J of heat energy to the cold reservoirs. What is the efficiency of the engine?

Given data:

work done = W= 100 J

Heat rejected = Q_2=400J

To find:

Efficiency = η=?

Calculation:

As $W = Q_1 - Q_2$

$Q_1 = W + Q_2$

Or $Q_1 = 100 + 400$

$Q_1 = 500J$

Now $\%\eta = \frac{W}{Q_1} \times 100$

Putting the values

$\%\eta = \frac{100}{500} \times 100$

$\%\eta = 0.2 \times 100$

$\%\eta = 20\%$

11.9 A Carnot engine whose low temperature reservoir is at 7 °C has an efficiency of 50%. It is desired to increase the efficiency to 70%. By how many degrees the temperature of the source be increased?

Given data:

Temperature of low reservoir = $T_2 = 7C° = 273+7 = 280$ K

Initial efficiency =$\%\eta = 50\%$

Final efficiency = $\%\eta' = 70\%$

To Find:

Temperature of source for 50% efficiency = T_1=?

Temperature of source for 70% efficiency = T_1'=?

Then increase in temperature = $T_1' - T_1$=?

Calculation:

To find T_1	To Find T_1'
$\%\eta = 50\%$	$\%\eta' = 70\%$
$\eta = 0.5$	$\eta' = 0.7$
Now	Now
$\eta = 1 - \frac{T_2}{T_1}$	$\eta' = 1 - \frac{T_2}{T_1'}$
$\eta = 1 - \frac{T_2}{T_1}$	$\eta' = 1 - \frac{T_2}{T_1'}$
Putting values, we get	Putting values, we get
$0.5 = 1 - \frac{280}{T_1}$	$0.7 = 1 - \frac{280}{T_1'}$
$\frac{280}{T_1} = 1 - 0.5$	$\frac{280}{T_1'} = 1 - 0.7$
$\frac{280}{T_1} = 0.5$	$\frac{280}{T_1'} = 0.3$
$T_1 = \frac{280}{0.5}$	$T_1' = \frac{280}{0.3}$
$T_1 = 560K$	$T_1' = 933.33K$

Thus increase in temperature

$T_1' - T_1 = 933.3 - 560 = 373K$

$T_1' - T_1 = 373K$

$\boxed{T_1' - T_1 = 373C°}$

11.10 A steam engine has a boiler that operator at 450K. The heat changes water to steam, which drives the piston. The exhaust temperature of the outside air is about 300K. What is maximum efficiency of this steam engine?

Given data:

Temperature of boiler (source) = T_1 = 450K

Temperature of outside air (sink) = T_2 =300K

To find:

Maximum efficiency = η=?

Calculation:

As $\eta = 1 - \frac{T_2}{T_1}$

$\eta = 1 - \frac{300}{450}$

$$\eta = \frac{150}{450}$$

$$\eta = 0.33$$

$$\text{percentage efficiency} = 0.33 \times 100$$

$$\boxed{\eta = 33\%}$$

11.11 336 J of energy is required to melt 1 g of ice at 0° C. what is the change in entropy of 30 g of water at 0° C as it is changed to ice at 0° C by a refrigerator?

Given data:

Heat of fusion of ice = L_f = 336 J

Mass of water = m = 30g

Temperature = T = 0°C =273K

To find:

Change in entropy = ΔS = ?

Calculation:

Since heat is transferred from the water to freeze it into ice, therefore entropy of ice decreases and is denoted by negative sign

$$\text{Change in entropy} = \Delta S = -\frac{\Delta Q}{T}$$

Here ΔQ = heat energy taken out of water = mL_f

Thus
$$\Delta S = -\frac{mL_f}{T}$$

Putting values, we get

$$\Delta S = -\frac{30 \times 336}{273}$$

$$\boxed{\Delta S = -36.92 JK^{-1}}$$

***** THE END *****

Board Model paper

Model Paper Physics Objective

Intermediate Part – I (11th Class) Examination Session 2013-2014 and onward

Total marks: 17 Paper Code Time Allowed: 20 minutes

Q.1.	QUESTIONS	(A)	(B)	(C)	(D)
1.	The unit of Pressure in base units is	$Kgm^{-1} Sec^{-1}$	$Kgm^{-1} Sec^{-2}$	$Kgm^{-2} Sec^{-1}$	$Kgm^{-2} Sec^{-2}$
2.	The complete Equilibrium of a body implies that	$\sum F = 0$	$\sum Fx = 0$ $\sum Fy = 0$	$\sum F = 0$ $\sum \tau = 0$	$\sum \tau = 0$
3.	At highest point, the vertical component of velocity of Projectile becomes	Maximum	Zero	Minimum	$V_i \cos^{o}$
4.	Impulse has the same unit as that of	Force	Energy	Mass	Linear Momentum
5.	The Tidal Energy is due to gravitational pull of the	Sun	Moon	Earth	Mars
6.	The rotational K.E. of a disc is	½ mv^2	¼ mv^2	1/6 mv^2	1/8 mv^2
7.	Torque per unit Moment of Inertia is Equivalent to	Angular velocity	Angular Acceleration	Inertia	Radius of Gyration
8.	Escape velocity on surface of earth is 11.2 km/Sec-1. The escape velocity on the Surface of another planet of same mass as that of earth but of 1/4times the radius of earth is	5.6 Km sec^{-1}	11.2 Km sec^{-1}	22.4 Km sec^{-1}	44.8 Km sec^{-1}
9.	The SI unit of flow rate of fluid is	$m^3 sec^{-1}$	$m^2 sec^{-1}$	$m^2 sec^{-2}$	$M^3 sec^{-3}$
10.	For a spring mass system arranged horizontally, the instantaneous displacement is	$x = x_0 \sin \omega t$	$x = x_0 \cos \omega t$	$x = x_0 \sin^2 \omega t$	$x = x_0 \cos^2 \omega t$
11.	In the time required for the tuning fork to make one complete vibration, the wave in air will travel a distance equal to	$\lambda/4$	$\lambda/2$	λ	2λ
12.	Velocity of sound is independent of	Temperature	Density	Pressure	Medium
13.	Two tuning forks of frequencies 40Hz and 243Hz respectively are sounded together, the no. of beats produced per second is	Zero	'2'	'3'	'4'
14.	In young's Double slit experiment, the position of Bright fringes are given by Formula,	$Y_m = m\frac{\lambda L}{d}$	$Y_m = m\frac{\lambda d}{L}$	$Y_m = m\frac{Ld}{\lambda}$	$Y_m = \frac{md}{L\lambda}$
15.	Final image produced by the compound Microscope is	Real and inverted	Real and erect	Virtual and erect	Virtual and inverted
16.	Carnot cycle consists of	Two steps	Three steps	Four steps	Five steps
17.	The Internal energy of a piece of lead when beaten by a hammer will	Increase	Decrease	Remain constant	First increase then decrease

Subjective

Intermediate Part – I (11th Class) Examination Session 2013-2014 and onward

Total marks: 83 Time: 3:10 hours

SECTION ———— I

2. Write answers of any EIGHT questions. (8 x 2 = 16)

(i) Define dimension. Check the correctness of the equation $v=f\lambda$ by the principle of Homogeneity of dimensions.
(ii) Briefly explain the two drawbacks to use the period of simple pendulum as a time standard.
(iii) Assess the total uncertainty in the final result of a timing experiment with the help of an example.
(iv) Determine the dimensions of pressure and density.
(v) Under what condition would a vector have components that are equal in magnitude.
(vi) Justify the statement "A body cannot rotate about its centre of gravity under the action of its own weight".
(vii) If $\vec{A}.\vec{B}=\vec{O}$, Can it be concluded that $\vec{A}$ *and* $\vec{B}$ are perpendicular to each other? Support your answer with a proof.
(viii) Why fog droplets appear to be suspended in air?
(ix) Discuss the sign of acceleration due to gravity for a cricket ball thrown upward, for its upward and downward motion.
(x) Can the velocity of an object reverse the direction when acceleration is constant? Justify with an example.
(xi) It is advisable to fasten the seat belts during a fast drive. Why is it?
(xii) Explain how would a bouncing ball behave in each of an elastic and inelastic collision with floor of room.

3. Write answers of any EIGHT questions. (8 x 2 = 16)

(i) When a rocket enters the atmosphere, why does its nose cone become very hot? Where does this heat energy come from?
(ii) State the work energy principle. Express it in equation.
(iii) While calculating the Absolute Gravitational potential energy, why is the distance between infinity and surface of earth is divided into very small steps.
(iv) What is meant by moment of Inertia? Give its significance.
(v) How is artificial gravity created in an Artificial satellites.
(vi) Centripetal force and centrifugal reaction are equal in magnitude but opposite in direction. Why these forces do not balance each other.
(vii) What happens to the period of simple pendulum if
(a) its length is doubled
(b) its suspended mass is doubled.
(viii) Show that in SHM, the acceleration is zero when velocity is greatest and the velocity is zero when the acceleration is greatest?
(ix) Why can we not realize an ideal simple pendulum.
(x) What features do longitudinal waves have in common with transverse waves.
(xi) Why does sound travel faster in solids than in gases?
(xii) Justify the statement "Velocity of sound in a gas is independent of pressure of the gas"

4. Write answers of any SIX questions. (6 x 2 = 12)

(i) Define coherent sources of light. How two light beams can be made coherent.
(ii) How is the distance between interference fringes is affected by the separation between the slits of Young's double shit experiment?
(iii) How would you distinguish between unpolarized light and plane polarized light.
(iv) Name and explain any two of major components of a fiber optic communication system.
(v) How the resolving power of a compound microscope can be increased.
(vi) What happens to the temperature of the room, when an air conditioner is left running on a table in the middle of the room.
(vii) What is meant by tripple point of water. What is the value of Absolute temperature of tripple point of water.
(viii) Can the efficiency of a carnot engine be 100%? Justify your answer with proof.
(ix) Normal Human body temperature is 98.6 °F. Convert it into °C and K.

SECTION II (Essay Type)

Note:- Attempt any three questions. **(8 x 3 = 24)**

5. (a) Define Rectangular components of a vector. How two vectors can be added by Rectangular component method. 1+4

(b) A ball is thrown with a speed of 30 m sec-1 in a direction 30º above the horizontal. Determine the height to which it rises. 3

6.(a) What are geostationary orbits. Derive an expression for orbital radius of a Geostationary orbit 1+4

(b) How large a force is required to accelerate an electron ($m=9.1 \times 10^{-31}$ kg) from rest to a speed of 2×10^{7} msec^{-1} through a distance of 5.0 cm. 3

7. (a) What is the limitation of Newton's formula for speed of sound in air. How did Laplace correct it. 1+4

(b) A simple pendulum is 50cm long. What will be its frequency of vibration at a place where g=9.8m sec^{-2} 3

8.(a) Explain the principle, construction and Magnifying power of a compound microscope with the help of a ray diagram. 1+2+2

(b) A light is incident normally on a grating which has 2500 lines/cm. compute the wavelength of a spectral line for which the deviation in 2nd order is 15°. 3

9.(a) Explain the carnot cycle and calculate the efficiency of a carnot heat engine. 2+3 = 5

(b) Water flows through a hose whose internal diameter is 1cm at a speed of 1m sec^{-1}. What should be the diameter of the nozzle if the water is to emerge at 21m sec^{-1}. 3

SECTION III (PRACTICAL) **4 x 2 = 8**

Note:- Give answers to any Four Questions.

10.(a) (i) How does the electronic timer measure time of free fall accurately.
(ii) A student measured the diameter of cylinder as 2.45 cm by a vernier calliper having least count +0.01 cm. But later on he observes a zero error in the instrument and finds zero of the vernier scale lies to the right of the zero of principal scale and 4th division of vernier scale faces any division or the principal scale. Find the correct value of diameter of cylinder.
(iii) The wire of sonometer is stretched with a load of 4kg wt including the hanger and resonant length of wire is found to be 11cm by using a tuning fork having frequency 512 Hz. If diameter of the wire is doubled, find the resonant frequency of this wire for the same resonating length and same load.
(iv) Find clockwise torque from diagram. 0 10cm 50 cm 70cm 90cm 100cm 12 N 4N 5N

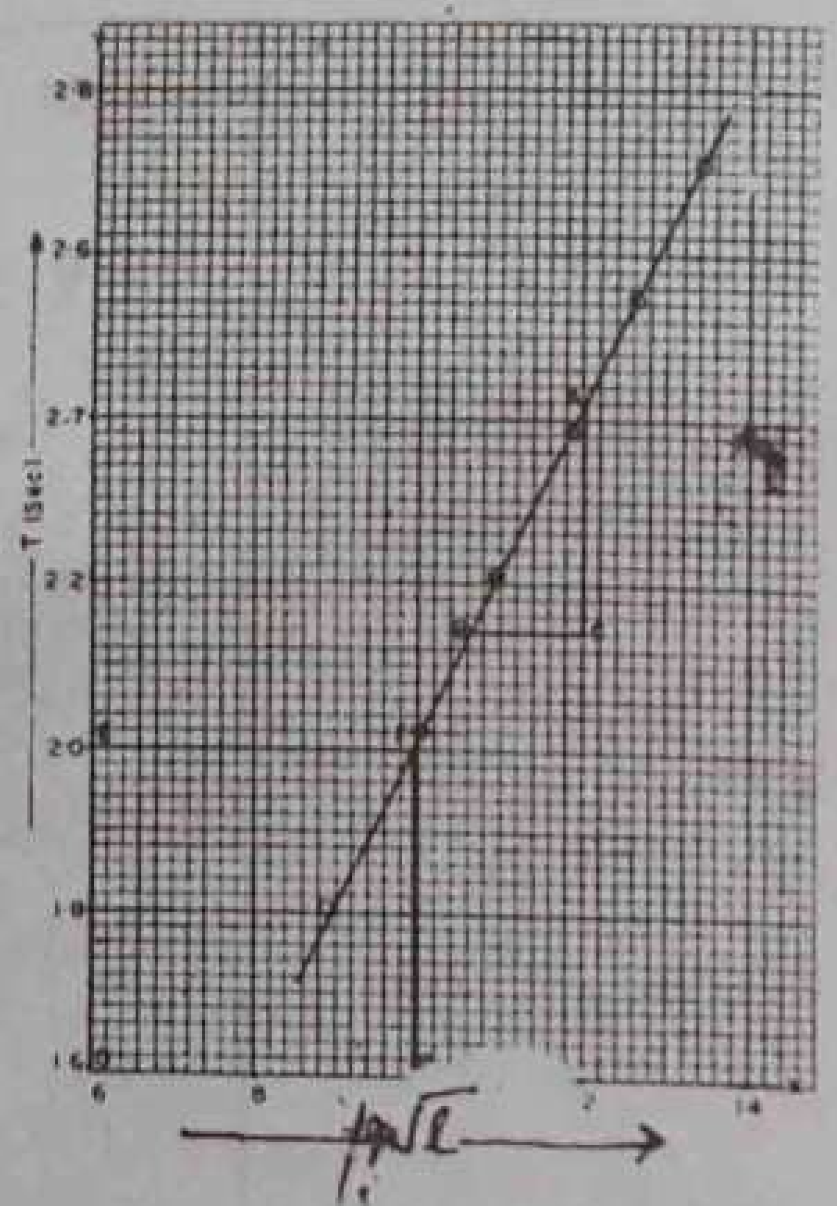
(v) How does the angle of deviation vary with the angle of incidence in case of prism.
(vi) Does the critical angle of a transparent material varies with the colour of light.
(vii) What are the sources of error during the experimental determination of mechanical equivalent of Heat by electrical method.
(viii) Design a table of observations/calculations to prove the law of length by using the vibrations in the string of sonometer.

10.(b) Write down the brief procedure to show experimentally that time period of simple pendulum is independent of amplitude . 3

OR

Write down the Brief procedure to determine experimentally the focal length of a convex lens by displacement method.

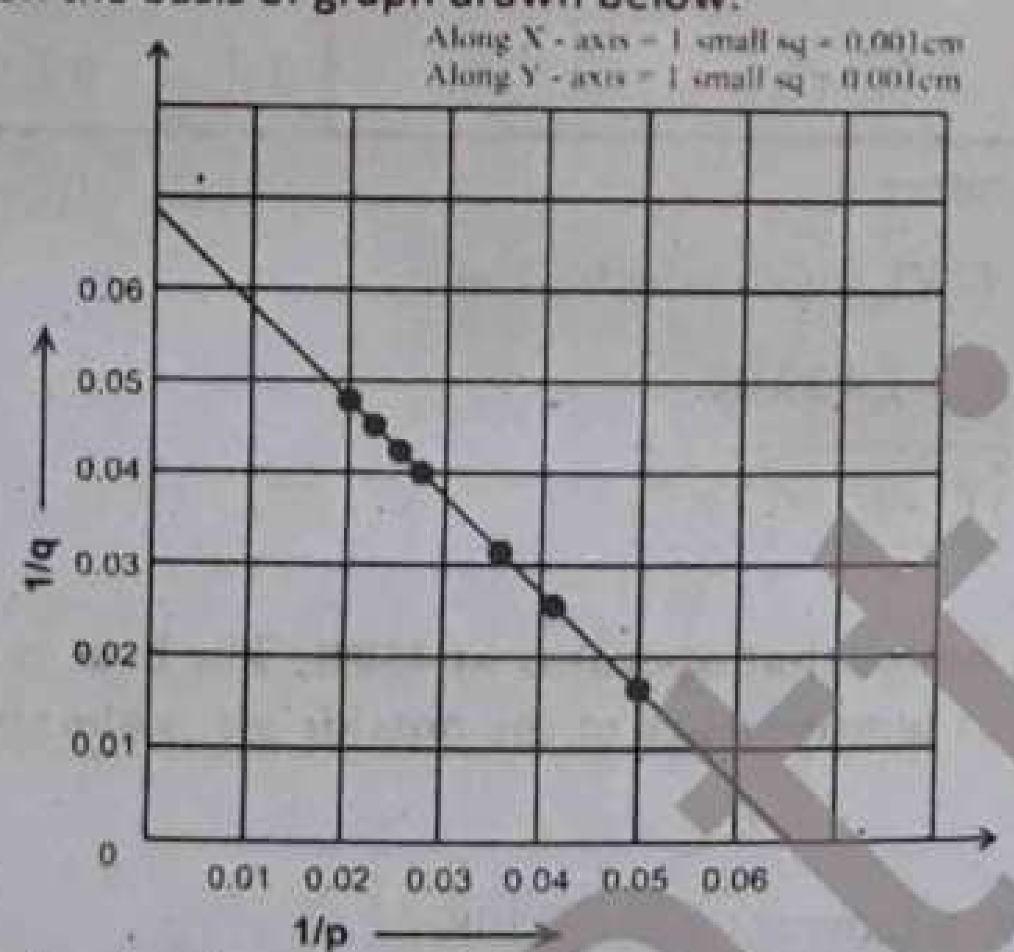
10.(c) Answer the following Question on the basis of graph drawn below.



(i) What can you conclude from the graph 1
(ii) Find the value of "g" from the graph 2
(iii) Measure the length of second's pendulum from the graph 1

OR

Answer the following Question on the basis of graph drawn below.



(i) What is value of "p" corresponding to $1/q = 0.5\ cm^{-1}$
(ii) Using a set of values of 1/p and 1/q from evaluate focal length.

MULTAN 2013

PHYSICS PAPER-I **2013 (A)** **Roll No:**
(NEW COURSE) **TIME ALLOWED: 3.10 Hours**
SUBJECTIVE MAXIMUM MARKS: 83

NOTE:- **Write same question number and its part number on answer book, as given in the question paper.**

SECTION-I

Q.No.2 **Attempt any eight parts.** **8 x 2 = 16**

(i) Give the drawbacks to use the period of a pendulum as a time standard.
(ii) The period of simple pendulum is measured by a stopwatch. What type of errors are possible in the time period?
(iii) How many seconds are there in 1 year?
(iv) Write the dimensions of (i) Pressure (ii) Density
(v) Can a vector have a component greater than the vector's magnitude?
(vi) Can you add zero to a null vector?
(vii) Can a body rotate about its centre of gravity under the action of its weight?
(viii) Can the velocity of an object reverse the direction when acceleration is constant? If so, give an example.
(ix) At what point or points in its path does a projectile have its minimum speed, its maximum speed?
(x) Find the velocity of a heavy body when it elastically collides with a stationary light body.
(xi) Derive a relation between impulse and linear momentum.
(xii) Explain, how the swing is produced in a fast moving cricket ball?

Q.No.3 **Attempt any eight parts.** **8 x 2 = 16**

(i) Calculate the work done in Kilo Joules in lifting a mass of 10kg through a vertical height of 10m.
(ii) A girl drops a cup from a certain height, which breaks into pieces. What energy changes are involved?
(iii) What is Escape Velocity?
(iv) What is meant by Moment of Inertia? Explain its significance.
(v) Show that orbital angular momentum $L_o = mvr$.
(vi) Explain Rotational K.E of a Disc and a Hoop.
(vii) Does frequency depend on amplitude for Harmonic Oscillators?
(viii) Can we realize an ideal simple pendulum?
(ix) What is Phase?
(x) Is it possible for two identical waves travelling in the same direction along a string to give rise to stationary waves?
(xi) Explain why sound travels faster in warm air than in cold air.
(xii) Define Doppler Effect.

Q.No.4 **Attempt any six parts.** **6 x 2 = 12**

(i) State Huygen's Principle.
(ii) Under what conditions two or more sources of light behave as coherent sources?
(iii) Why the Polaroid Sunglasses are better than ordinary sunglasses?
(iv) How the light signal is transmitted through optical fibre?
(v) When object lie within principal focus of convex lens what is the nature of image and where is it formed?
(vi) Is it possible to convert energy into mechanical energy? Explain with an example.
(vii) Why is the average velocity of the molecules in a gas zero but the average of the square of velocities is not zero?
(viii) State Carnot's Theorem.
(ix) A thermos flask is shaken which contains milk. Does the temperature of milk rise?

SECTION-II

NOTE:- Attempt any three questions of the following:-

5.(a) Define rectangular components of a vector. How two vectors can be added by rectangular components method? 5
(b) A football is thrown upward at an angle of 30^o with respect to the horizontal. To throw it 40m pass, what must be the initial speed of the ball? 3
6.(a) State and explain Work-Energy Principle. 5
(b) A 1000 kg car is turning round a corner at $10ms^{-1}$ as it travels along an arc of a circle. If the radius of the circular path is 10m, how large a force must be exerted by the pavement on the tyres to hold the car in the circular path? 3
7.(a) Define Simple Pendulum and derive relation for its time period. 1+4

(b) A pipe has a length of 1 m. Determine the frequencies of the fundamental and the first harmonics if the pipe is open at both ends. (speed of sound in air = $340 ms^{-1}$)

8.(a) Define Simple Microscope. Find its Magnification. 3

(b) In a double slit experiment the second order maximum occurs at $\theta = 0.25^o$. The wavelength is 650 nm. Determine the slit separation. 5

9.(a) Explain Isothermal and Adiabatic Processes. 3

(b) A water hose with an internal diameter of 20 mm at the outlet discharges 30 Kg of water in 60 seconds. Calculate the water speed at the outlet. Assume the density of water is $1000 kg/m^3$ and its flow is steady. 5

SECTION-III (PRACTICAL PART)

10.(A) Write answers of any four parts. **4 x 2 = 8**

(i) How does the electronic timer measures time of free fall accurately?
(ii) Why a screw gauge is more accurate than a vernier callipers?
(iii) How the weight of a metre rod is found when it is suspended to two spring balances?
(iv) The velocity of sound at 0^oC is $332 ms^{-1}$. What is its velocity at 25^oC?
(v) The index of refraction of glass with respect to air is 3/2.
What is the index of refraction of air with respect to glass?
(vi) What are the conditions to Total Internal Reflection?
(vii) What are the sources of error during the experimental verification of law of mass of vibrating stretched string using a sonometer?
(viii) Design a table of observations to find acceleration due to gravity by oscillating mass spring system.

(B) Write down the brief procedure to find experimentally the unknown weight of body by the method of vector addition of forces. 3

OR

Write down the brief procedure to find experimentally the refractive index of the material of a prism by critical angle method.

(C) Answer the following questions on the basis of graph drawn below:- 2+2

Graph A

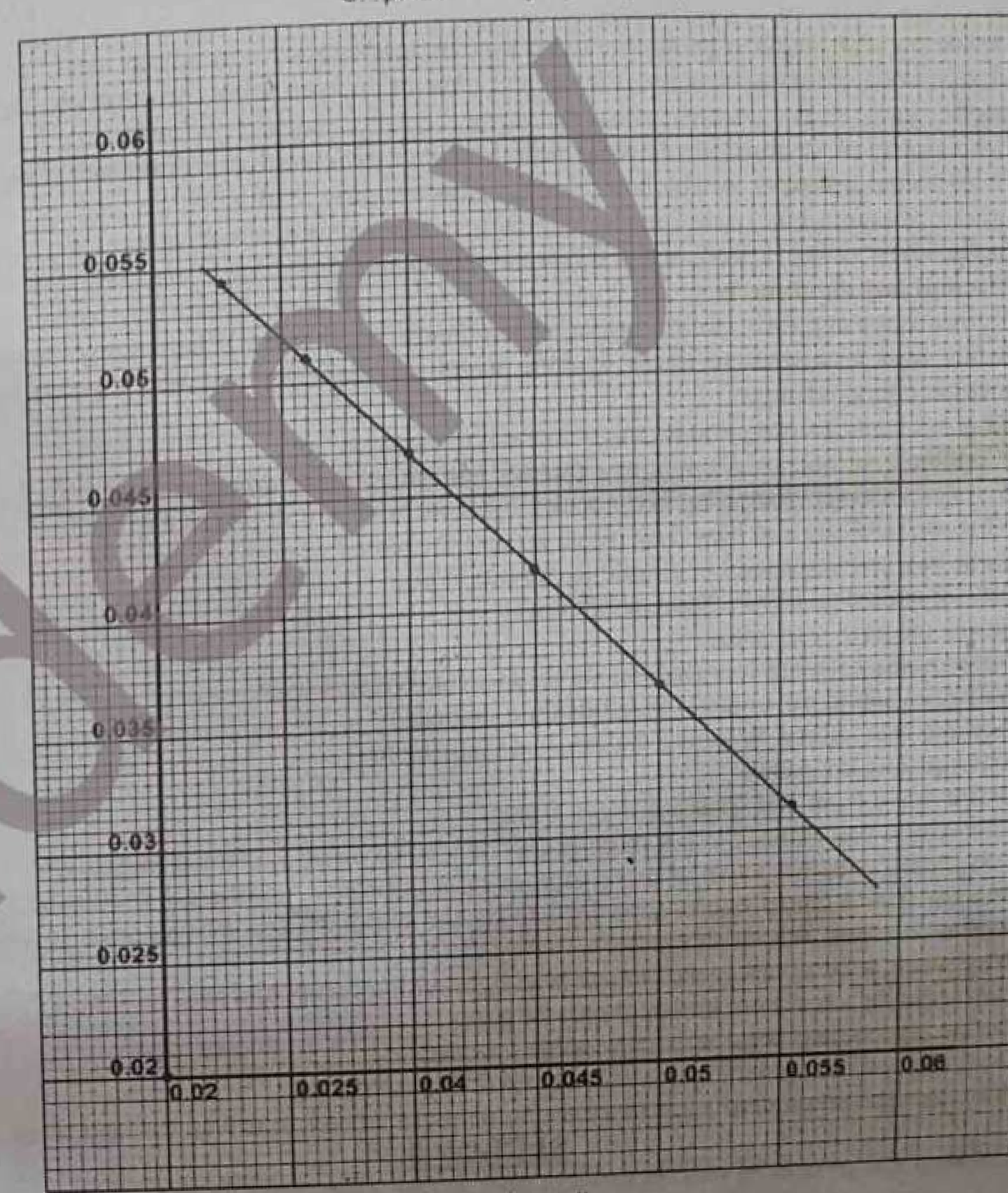
(i) What is the relation between 'F' and 'x'?
(ii) Find the value of spring constant

OR

Answer the following question on the basis of graph drawn below:-

Graph B

Graph between p+q and p x q



From the graph ploted , at point "p"

(i) Find the value of $\frac{1}{p}$ and $\frac{1}{q}$
(ii) Find the value of focal length of lens

GUJRANWALA 2013

PHYSICS PAPER-I
INTERMEDIATE PART-I (11th CLASS)
(NEW COURSE)
TIME ALLOWED: 3.10 Hours
SUBJECTIVE MAXIMUM MARKS: 83

NOTE:- Write same question number and its part number on answer book, as given in the question paper.

SECTION-I

Q.No.2 Attempt any eight parts. 8 x 2 = 16

(i) Name several repetitive phenomenon occurring in nature which could serve as reasonable time standards.
(ii) Why do we find it useful to have two units for amount of substance, kilogram and mole?
(iii) Distinguish between base and derived units.
(iv) How many nanoseconds in one year are there?

(v) Define the terms unit vector and position vector.
(vi) Can you add zero to a null vector?
(vii) Give two conditions of equilibrium.
(viii) Define impulse and give its relation with momentum.
(ix) Motion with constant velocity is a special case of motion with constant acceleration. Is this statement true?
(x) Can velocity of an object reverse the direction when acceleration is constant? If so, give an example.
(xi) Explain circumstances in which acceleration and velocity V of a car
(a) a is zero but v is not zero (b) v is zero but a is not zero
(xii) Explain how swing is produced in a fast moving cricket ball.

Q.No.3 Attempt any eight parts. $8 \times 2 = 16$
(i) A girl drops a cup form certain height which breaks into pieces. What energy changes are involved?
(ii) A boy uses a catapult to throw a stone which accidentally smashes a green house window. List the possible energy changes.
(iii) What is escape velocity and its mathematical expression?
(iv) When the mud flies off the tyre of a moving bicycle, in what direction does it fly?
(v) Why does a diver change his body position before and after diving in the pool?
(vi) What are satellites and how they move around the earth?
(vii) Name two characteristics of simple harmonic motion.
(viii) Can we realize an ideal simple pendulum?
(ix) Define the term resonance.
(x) Differentiate between longitudinal waves and transverse waves.
(xi) Why does sound travel faster in solids than in gases?
(xii) How are beats useful in tuning the musical instruments?

Q.No.4 Attempt any six parts. $6 \times 2 = 12$
(i) Define wave front and spherical wave front.
(ii) How would you manage to get more orders of spectra using a diffraction grating?
(iii) Under what conditions two or more sources of light behave as coherent sources?
(iv) One can buy a cheap microscope for use by the children. The images seen in such a microscope have couloured edges. Why is this so?
(v) Why would it be advantageous to use blue light with a compound microscope?
(vi) Derive Boyle's law from kinetic theory of gases.
(vii) Define reversible process and irreversible process.
(viii) Does entropy of a system increase or decrease due to friction?
(ix) Why does the pressure of a gas in a car tyre increase when it is driven through some distance?

SECTION-II

5.(a) Define scalar product of two vectors and give its four characteristics. 1,4
(b) A football is thrown along an angle of 30^o with respect to horizontal to throw 40 m pass. What must be the initial speed of the ball? 3
6.(a) Describe work energy principle. 5
(b) A 1000 kg car trave ling with a speed of 144 km h^{-1} round a curve of radius 100 m. Find necessary centripetal force. 3
7.(a) What is drawback of Newton's formula for speed of sound in air? How Laplace corrected it? 1,4
(b) A simple pendulum is 50.0 cm long. What will be its frequency of vibration at a place where g = 9.8 m/s^2? 3
8.(a) Describe the Michelson's experiment for calculation of speed of light. 5
(b) In a double slit experiment, the second order maximum occurs at $\theta = 0.25^o$. The wavelength is 650 nm. Determine the slit separation. 3
9.(a) Explain isothermal process and adiabatic process. 5
(b) What gauge pressure is required in the city mains for a stream from a fire hose connected to the mains to reach a vertical height of 15.0m?

SECTION III (PRACTICAL)

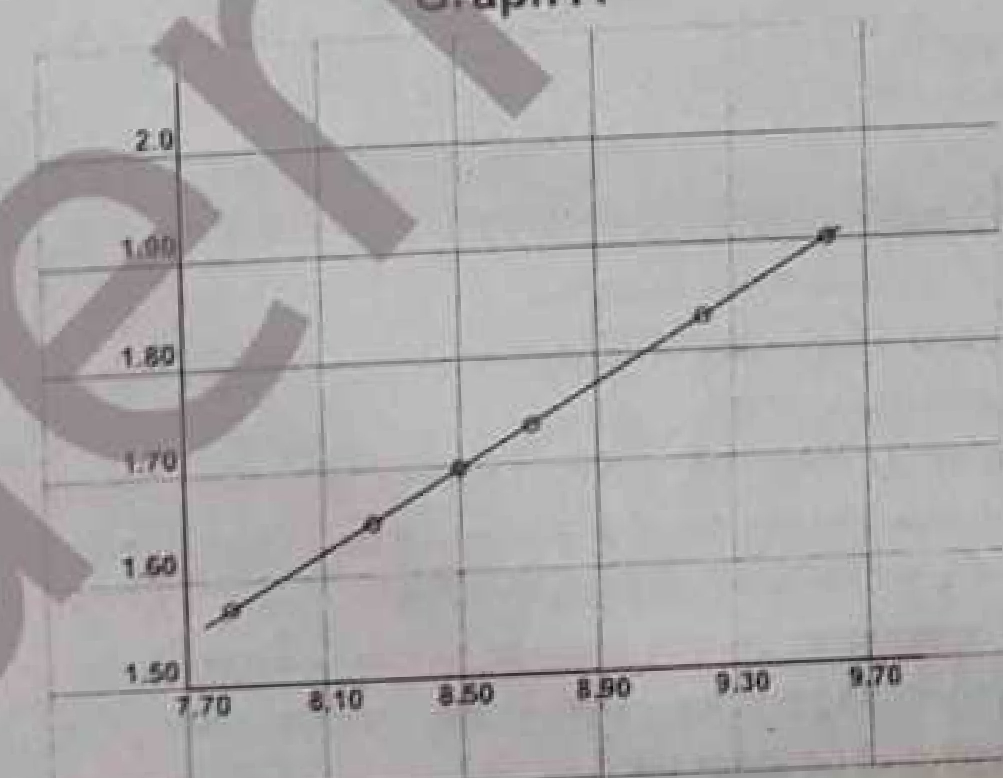
.(A) Write short answers to any four questions: $4 \times 2 = 8$
(i) What is vernier constant?

(ii) What do you know about zero error?
(iii) Define weight and write its S.I. unit.
(iv) What is second pendulum?
(v) What is the difference between a real image and a virtual image?
(vi) Define critical angle.
(B) Write down the brief procedure to show experimentally that time period of simple pendulum is independent of amplitude. 3

OR

Write down the brief procedure to find out the focal length of a convex lens experimentally. 4
(C)(i) What can you conclude from the graph?
(ii) Find the value of 'g' from the graph.

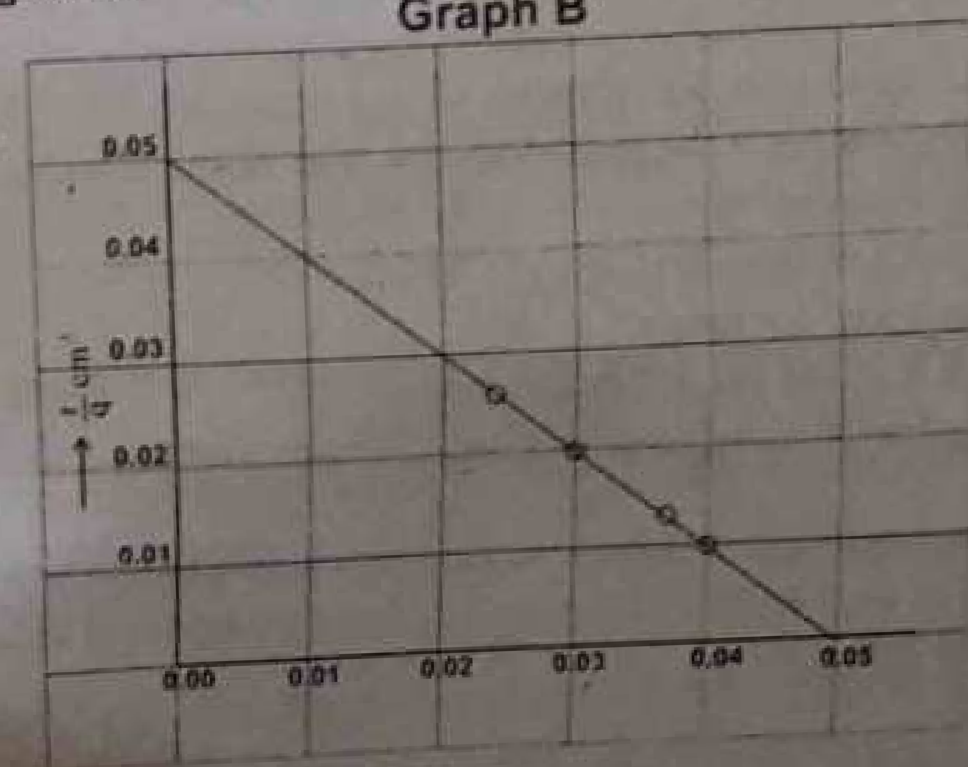
Graph A



OR

(i) What do you infer from the graph?
(ii) Find out the focal length of a lens from the graph.

Graph B



BAHAWALPUR 2013

INTERMEDIATE PART-I (11th CLASS)
(NEW COURSE)

PHYSICS PAPER-I TIME ALLOWED: 3.10 Hours

SUBJECTIVE MAXIMUM MARKS: 83

NOTE:- Write same question number and its part number on answer book, as given in the question paper.

Roll No.	808-	Inter (Part - I)	New Marks Scheme
Physics (Subjective)	Inter-A-2013	Session (2012-14)	Marks = 83 Time : 3.10 Hours

Note: It is compulsory to attempt (8) parts each from Q.No.2 and 3 while attempt any 6 parts from Q.No.4 and attempt any (03) questions from Part II. While attempt Part III (Practical Part) according to the given instructions.

Write same questions no. and its part no. as given in the question paper.

SECTION I

Q.No.2 Attempt any eight parts. 8 x 2 = 16

(i) Define Physics, significant figures.
(ii) Define Radian, Steradian.
(iii) Can you add zero to a null vector?
(iv) Define Displacement, instantaneous velocity.
(v) State Newton's First and Third Law of Motion.
(vi) Define Impulse and Isolate System.
(vii) Write the Dimension of (i) Pressure (ii) Density
(viii) Give the drawbacks to use the period of a pendulum as a time standard.
(ix) Can the magnitude of a vector have a negative value?
(x) Can a vector have a component greater than the vector's magnitude?
(xi) At what point or points in its path does a projectile have its minimum speed, its maximum speed?
(xii) Why Fog droplets appear to be suspended in air?

Q.No.3 Attempt any eight parts. 8 x 2 = 16

(i) Calculate the work done in kilo joules in lifting a mass of 10 kg through height of 10 m.
(ii) When a rocket re-enters the atmosphere it's nose cone becomes very hot? Where does this heat energy comes from?
(iii) What is meant by Moment of Inertia? Explain its significance?
(iv) Explain the difference between Tangential Velocity and Angular Velocity.
(v) What are Geo-Stationary Satellites?
(vi) Name two characteristics of Simple Harmonic Motion.
(vii) Explain the terms Node and Antinode.
(viii) Differentiate between free and Forced Oscillations.
(ix) What is total distance travelled by an object moving with SHM in a time equal to its period if amplitude is A?
(x) What features do longitudinal waves have in common with transverse waves?
(xi) How astronomours use the Doppler effect to calculate the speeds of different stars?
(xii) How Pollution can be reduced?

Q.No.4 Attempt any six parts. 6 x 2 = 12

(i) State Huyge's Principle?
(ii) How the power is lost in Optical Fibre?
(iii) Under what conditions two or more sources of light behave as Coherent Sources?
(iv) An oil film spreading over a wet foot-path shows colours. How does it happen?
(v) What is meant by Linear Magnification and Angular Magnification?
(vi) Derive Charles' Law from Kinetic Theory of Gases.
(vii) Is it possible to construct a heat engine that will not expell heat into the atmosphere?
(viii) What happen to the temperature of the room when an air conditioner is left running on at table in the middle of the room?
(ix) Is it possible to convert internal energy into mechanical energy. Explain with example.

SECTION II

5. (a) Define Projectile Motion. Derive expression for Height of Projectile and Time of Flight. 5
(b) The positions of two aeroplanes at any instant are represented by two points A (2, 3, 4) and B (5, 6, 7) in Km. Calculate the distance between the two aeroplanes. 3

6. (a) Explain interconversion of P.E. and K.E. 5
(b) A disc without slipping rolls down a hill of height 10.0 m. If the disc starts from rest at the top of the hill, what is its speed at the bottom? 3

7. (a) What is Doppler's Effect? What change in Frequency Receiving by observer when source is moving away and towards the observer at rest? Explain. 5
(b) A block of mass 4 kg is dropped from a height of 0.80 m on to a spring of spring constant K = 1960 $N.m^{-1}$. Find the maximum distance through which the spring will be compressed. 3

8. (a) What is Compound Microscope? Write down its working and magnification. 5
(b) X-rays of Wavelength 0.150 nm are observed to undergo a first order reflection at a Bragg angle of 13.3^o from a quartz (SiO_2) crystal. What is the interplaner spacing of the reflecting planes in the crystal? 3

9. (a) Explain CARNOT Engine and state CARNOT's Theorem. 5
(b) Water Flows through a hose, whose Internal Diameter is 1 cm at a speed of $1ms^{-1}$? What should be the diameter of the nozzle if the water is to emerge at $21ms^{-1}$? 3

SECTION-III (PRACTICAL PART)

10.(A) Write short answers to any four questions: 4 x 2 = 8

(i) Define Vernier Constant.
(ii) How Accurate is the Screw Gauge?
(iii) What is the use of inside (OR Upper) jaws and sliding strip in Vernier Caliper?
(iv) What function of sounding box of sonometer is?
(v) What kind of vibration is executed by Sonometer?
(vi) What is Principal axis of a Lens?
(vii) Define Refraction of Light.
(viii) When Zero Error is positive and when it is negative?

(B) Write down the procedure to show experimentally that time period of simple pendulum is independent of Amplitude. 3

OR

Write down the procedure to determine experimentally the focal length of a convex lens by displacement method.

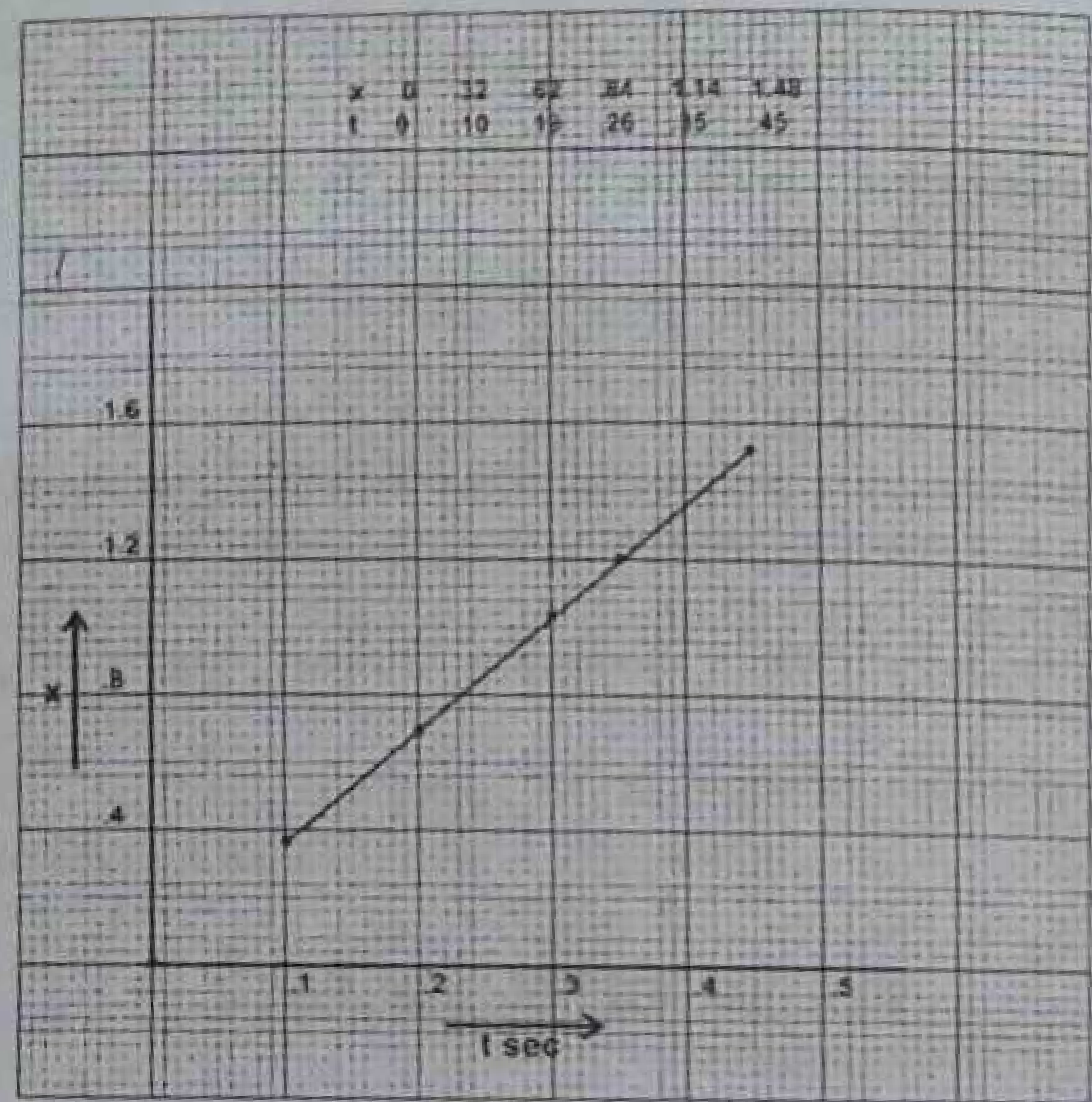
(C) Answer the following questions the basis of Displacement Time Graph.
(i) Find the Average Velocity at time t = .3 Sec
(ii) What this Graph infer?

OR

Graph between (p + q) and (p x q) and answer the following questions.
(i) Find the Slope of Graph.
(ii) What does Slope represent?

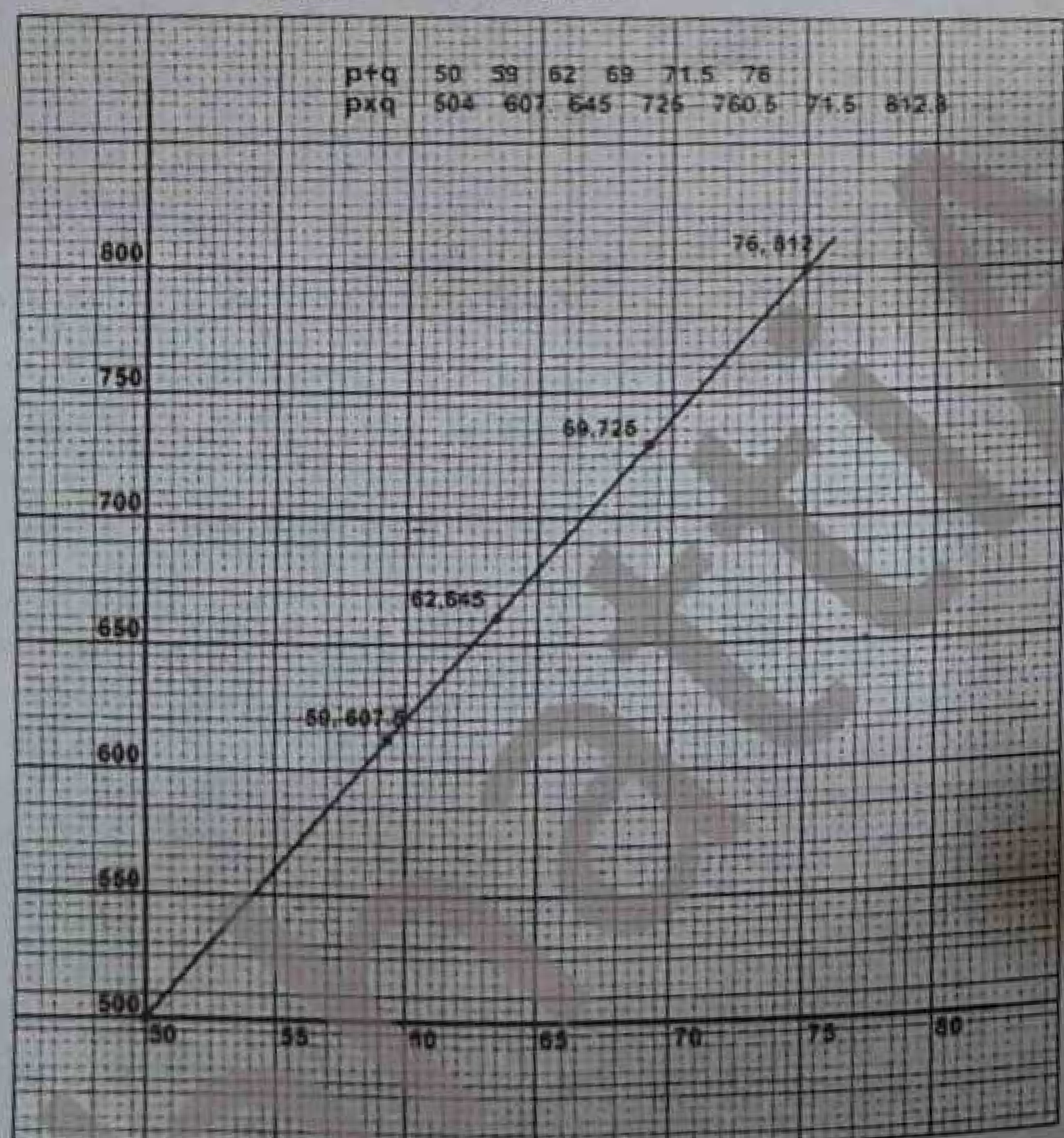
Graph A

Graph between displacement and Time



Graph B

Graph between p+q and p x q



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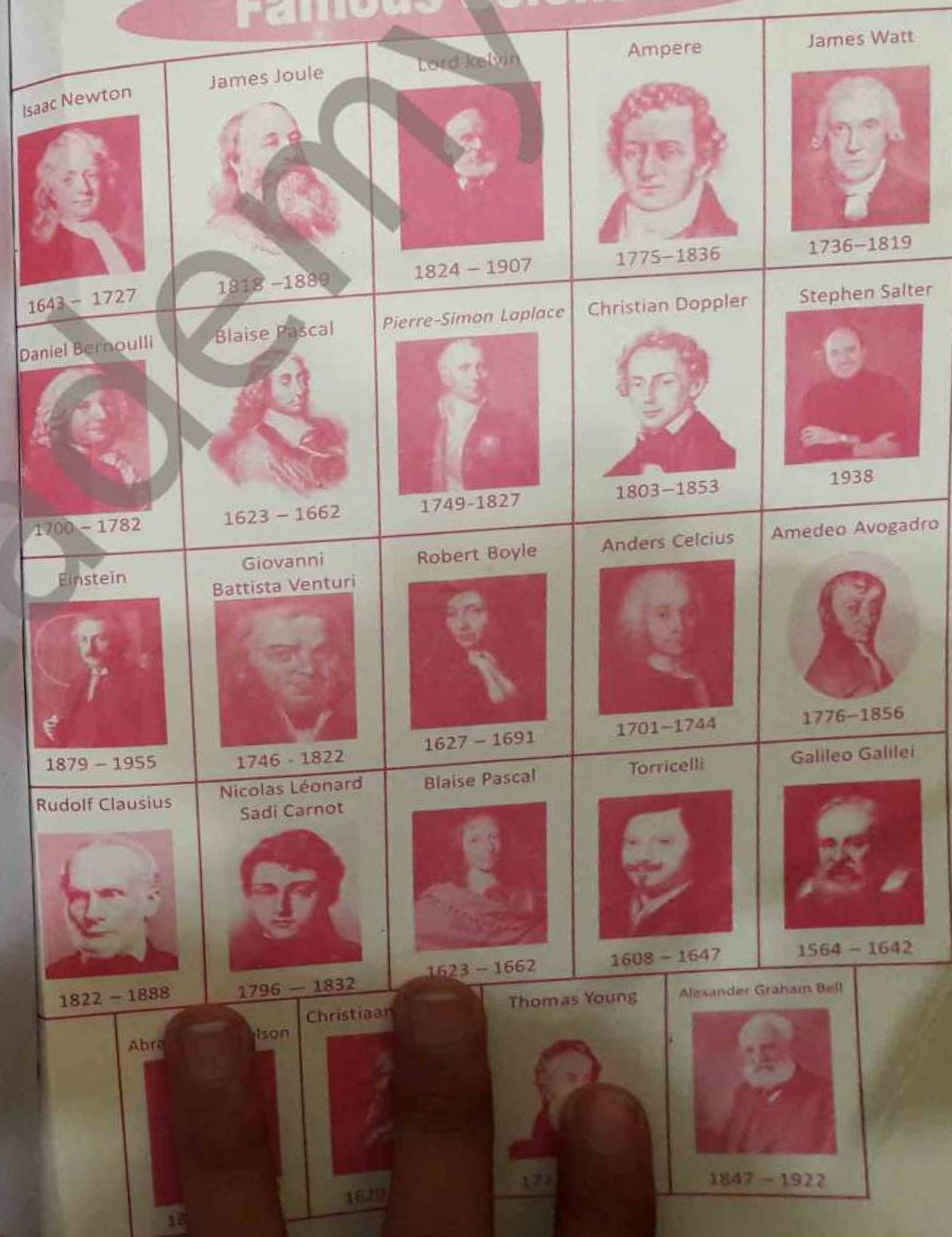
Famous Scientist



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